Math 1431

Section 16679

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Questions

Suppose we want to find the distance between two numbers a and b on the number line. How do we represent this distance mathematically?

When we discuss limits, we say we want x to be arbitrarily close to a but it doesn't have to equal a.

How can we represent this mathematically?

Once we are "close enough" to x, we find our limit by looking at what our y values are close to. (Remember, $y = f(x)$).

Let L represent our answer for the limit and let ϵ be our distance that is "close enough". In mathematical terms, we will write this as:

The formal definition of a limit

Let f be a function defined on the intervals $(c - \delta, c)$ and $(c, c + \delta)$, where $\delta > 0$.

$$\lim_{x \to c} f(x) = L$$

if and only if for each $\epsilon > 0$, there exists a $\delta > 0$ such that

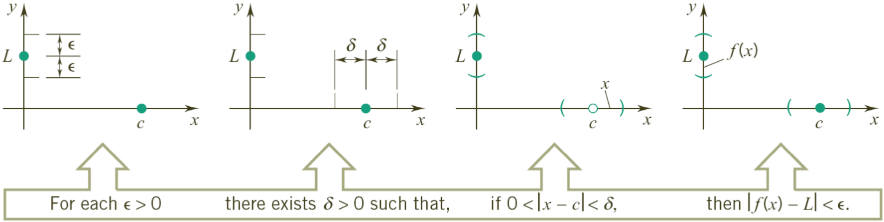
if

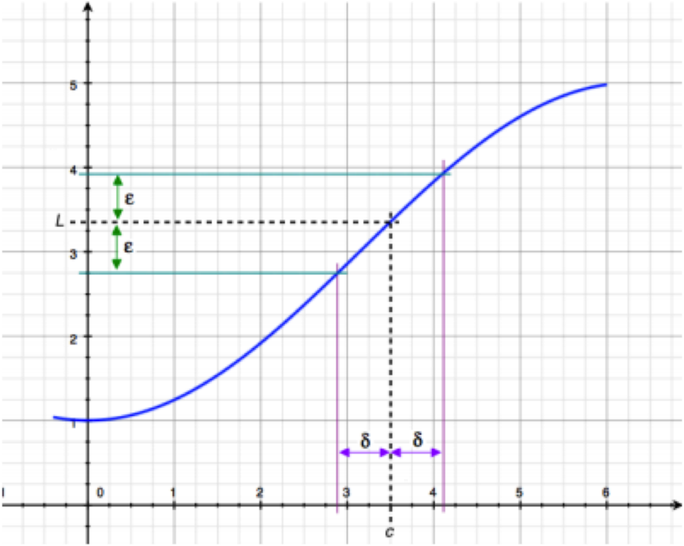
$$0 < |x - c| < \delta$$

then

$$|f(x) - L| < \epsilon$$

Section 1.3 - Definition of Limit and Arithmetic Rules




5
4
L
3
2
1
0
1
2
3
4
5
6
ε
ε
δ
δ
c

Example: Show that $\lim_{x \to 2} (3x + 5) = 11$ using the definition of limit.

Example: Give the largest δ that works with $\epsilon = 0.1$ for the limit

$$\lim_{x \to -1} (1 - 2x) = 3$$

Example: Give the largest δ that works with $\epsilon = 0.1$ for the limit

$$\lim_{x \to 3}(4x - 5) = 7$$

You try: Give the largest δ that works with $\epsilon = 0.02$ for the limit

$$\lim_{x \to -1} (2x + 5) = 3$$

Formal definitions for Left-handed and Right-handed limits:

Let f be a function defined on the interval $(c-\delta, c)$, where $\delta > 0$.

$$\lim_{x\to c-} f(x) = L$$

if and only if for each $\epsilon > 0$, there exists a $\delta > 0$ such that if $c-\delta < x < c$ then $|f(x) - L| < \epsilon$.

Let f be a function defined on the interval $(c, c+\delta)$, where $\delta > 0$.

$$\lim_{x\to c+} f(x) = L$$

if and only if for each $\epsilon > 0$, there exists a $\delta > 0$ such that if $c < x < c+\delta$ then $|f(x) - L| < \epsilon$.

Recall:
Let $P(x)$ and $Q(x)$ be polynomial functions and let a be a real number. Then,

$$\lim_{x\to a}\frac{P(x)}{Q(x)}=\begin{cases}\frac{P(a)}{Q(a)} & \text{if } Q(a)\neq 0\\ \text{undefined} & \text{if } P(a)\neq 0 \text{ and } Q(a)=0\end{cases}$$

If $P(a)$ and $Q(a)$ both equal 0 then more work is required.

Techniques to evaluate limits:

- direct substitution
- cancellation
- rationalization
- algebraic simplification

Examples:

1. $\lim_{x \to 4} \sqrt{x^2 - 3} =$

2. $\lim_{x \to 2} \frac{x-2}{x^3 - 8} =$

3. $\lim_{x \to -1} \dfrac{x^2 + x}{x^2 - 1} =$

4. $\lim_{x \to 1} \frac{x^2 + x}{x^2 - 1} =$

5. $\lim_{x \to 3} \dfrac{\sqrt{x+1} - 2}{x - 3} =$

6. $\lim_{x \to 1} \dfrac{\sqrt{x^2+1} - \sqrt{2}}{x-1} =$

7. If $g(x) = \begin{cases} \frac{3x-6}{x-2} & x \neq 2 \\ 10 & x = 2 \end{cases}$, find $\lim_{x \to 2} g(x)$

8. $\lim_{x \to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x} =$

Limits as $x \to \infty$

- $\lim_{x \to \infty} \frac{1}{x} =$

- $\lim_{x \to \infty} \frac{1}{x^2} =$

- $\lim_{x \to \infty} \frac{1}{x^n} =$

Examples:

1. $\lim_{x\to\infty} \dfrac{2x^2 - 3x + 1}{4x - x^2} =$

2. $\lim_{x \to -\infty} \dfrac{2x^2 - x + 5}{x^3 + x^2 + 1} =$

3. $\lim_{x\to\infty} \dfrac{3x^2 - 2x + 4}{\sqrt{x^4 + 3x^2 + 8}} =$

4. $\lim_{x \to \infty} \arctan(x) =$

5. $\lim_{x \to -\infty} \arctan(x) =$

Section 1.6 - The Pinching Theorem; Trig Limits

Suppose $f(x)$, $g(x)$ and $h(x)$ are defined on an open interval containing $x = c$ (except possibly at $x = c$).

If $f(x) \leq g(x) \leq h(x)$ and $\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L$, then $\lim_{x \to c} g(x) = L$.

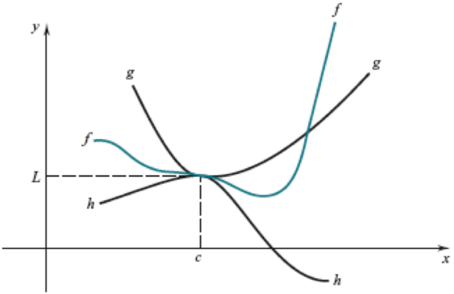


Figure 2.5.1

Note: Trigonometric functions are continuous on their domain:

$$\lim_{x \to c} \sin(x) = \sin(c) \qquad \lim_{x \to c} \cos(x) = \cos(c)$$

Also, recall:

$$\sin(0) = 0 \text{ and } \cos(0) = 1$$

In the posted video, I use the Pinching Theorem to show:

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

For any number $a \neq 0$, we have:

$$\lim_{x \to 0} \frac{\sin(ax)}{ax} = 1 \quad \text{and} \quad \lim_{x \to 0} \frac{1 - \cos(ax)}{ax} = 0$$

Examples:

1. $\lim\limits_{x\to 0} \dfrac{\sin(5x)}{5x} =$

2. $\lim\limits_{x\to 0} \dfrac{\sin(5x)}{x} =$

3. $\lim\limits_{x\to 0} \dfrac{\sin(5x)}{2x} =$

For any number $a \neq 0$, we have:

$$\lim_{x\to 0} \frac{\sin(ax)}{ax} = 1 \quad \text{and} \quad \lim_{x\to 0} \frac{1-\cos(ax)}{ax} = 0$$

More examples:

1. $\lim_{x \to 0} \dfrac{x}{\sin(x)} =$

2. $\lim_{x \to \pi/4} \dfrac{\sin(2x)}{x} =$

3. $\lim_{x \to 0} \dfrac{1 - \cos(3x)}{x} =$