Math 1431

Section 16679

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1. True or false? A circle is a function. (example: $x^2 + y^2 = 4$)

Section 4.1 - Inverses

What determines whether the graph of a function is invertible (has an inverse that is also a function)?

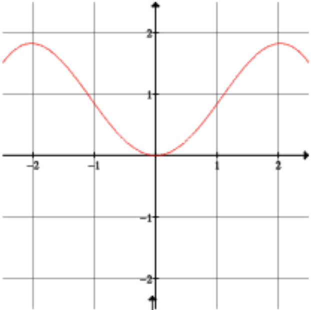
Definition: A function f is one-to-one if $f(x_1) = f(x_2)$ then $x_1 = x_2$.

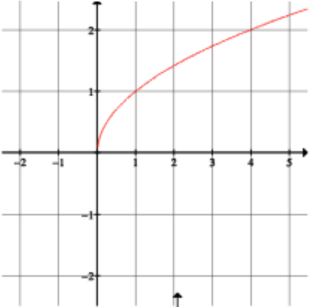
In other words, two different x values cannot have the same y values.

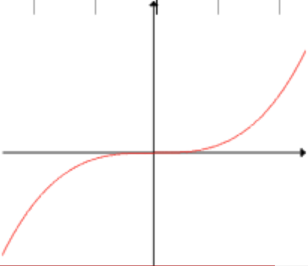
If a function is one-to-one, then it has an inverse.

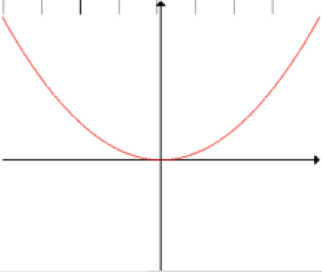
(Remember, domain of f equals the range of f^{-1})

Which of the following functions are invertible?









Section 4.1 - Inverses

Theorem: If f is either an increasing function or a decreasing function, then f is an invertible function.

Example: Show that $f(x) = x^3 + 3x$ is invertible on the interval [0, 10].

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2. True or false? A parabola has an inverse. (example: $y = 4x^2$)

Theorem: If f is either an increasing function or a decreasing function, then f is an invertible function.

Example: Show that $f(x) = x^3 + 3x$ is invertible on the interval $[0, 10]$.

Example: Show that $f(x) = \sin(x)$ is invertible on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

How do we find the formula for the inverse of a function?

1. Start with $y = f(x)$.
2. Solve for x in terms of y. This will give something like $x = g(y)$.
3. Switch the x's and y's. This will give $y = g(x)$.
4. The function g is the inverse of f.

We can only do this for simple functions.

We will use the notation $f^{-1}(x)$ to denote the inverse of $f(x)$.

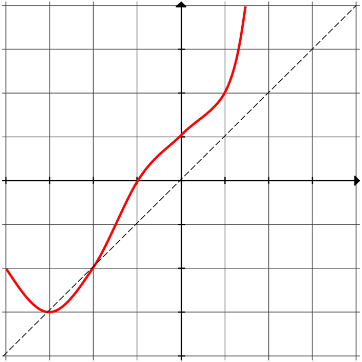
Example: Is $f(x) = 2x - 3$ invertible? If so, find its inverse.

Example: Find the inverse of $y = \dfrac{x+2}{x-3}$ if possible.

How are functions related to their inverses?

Algebraically:

Geometrically:



Theorem: If $f(x)$ is continuous and invertible then $f^{-1}(x)$ is continuous.

Theorem: If $f(x)$ is differentiable and invertible, and $f'(x)$ is nonzero, then $f^{-1}(x)$ is differentiable.

Also, if $f(a) = b$ and $f'(a) \neq 0$, then $\left(f^{-1}\right)'(b) = \dfrac{1}{f'\left(f^{-1}(b)\right)} = \dfrac{1}{f'(a)}$.

We found that $f(x) = x^3 + 3x$ was invertible on [0, 10]. Find $\left(f^{-1}\right)'(4)$.

Given $f(x) = 2x + \sin(x)$, find $\left(f^{-1}\right)'(2\pi)$ (if possible).

Let $f(x) = x^5 + 2x^3 + 2x$. Give an equation of the tangent line to the graph of $f^{-1}(x)$ at the point $(-5, -1)$.

Given $f(x) = x^5 + 1$, find $\left(f^{-1}\right)'(33)$ if possible.

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3. Is $f(x) = x^3 + 2x - 3$ invertible?

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4. Find $\left(f^{-1}\right)'(2)$ if
$f(2) = 3, f(4) = 2, f(3) = -2, f'(2) = 7, f'(3) = 5, f'(4) = 10.$

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5.

To Do

Read 4.1.

Take quiz 15.

Email questions if you have any.