Math 1431

Section 16679

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11/12/19

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Questions?

Definition of the **Definite Integral**:

A function f defined on an interval $[a, b]$ is integrable on $[a, b]$ if there is one and only one number I that satisfies the inequality

$$L_f(P) \leq I \leq U_f(P) \text{ for all partitions } P \text{ of } [a, b]$$

This unique number I is called the definite integral of f from a to b and is denoted by $\displaystyle\int_a^b f(x)dx$. So, we have

$$L_f(P) \leq \int_a^b f(x)dx \leq U_f(P) \text{ for all partitions } P \text{ of } [a, b]$$

Properties of the definite integral:

- $\int_a^b f(x)dx = -\int_b^a f(x)dx$
- $\int_a^a f(x)dx = 0$
- $\int_a^b k dx = kb - ka$
- $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$
- $\int_a^b kf(x)dx = k\int_a^b f(x)dx$

- if $f(x) \geq 0$ on $[a,b]$, $\displaystyle\int_a^b f(x)dx \geq 0$
- if $f(x) \geq g(x)$ on $[a,b]$, $\displaystyle\int_a^b f(x)dx \geq \int_a^b g(x)dx$
- if $m \leq f(x) \leq M$ on $[a,b]$, $m(b-a) \leq \displaystyle\int_a^b f(x)dx \leq M(b-a)$
- $\displaystyle\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$
- if $f(x)$ is an odd function, $\displaystyle\int_{-a}^a f(x)dx = 0$
- if $f(x)$ is an even function, $\displaystyle\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$

Section 6.1 -The Definite Integral

Thm: If f is continuous on $[a, b]$ and if $a < c < b$, then

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

Example: Given $\int_0^1 f(x)dx = 3$, $\int_0^3 f(x)dx = 5$, $\int_3^6 f(x)dx = 9$, find:

$$\int_0^6 f(x)dx$$

$$\int_1^6 f(x)dx$$

$$\int_6^1 2 \cdot f(x)dx$$

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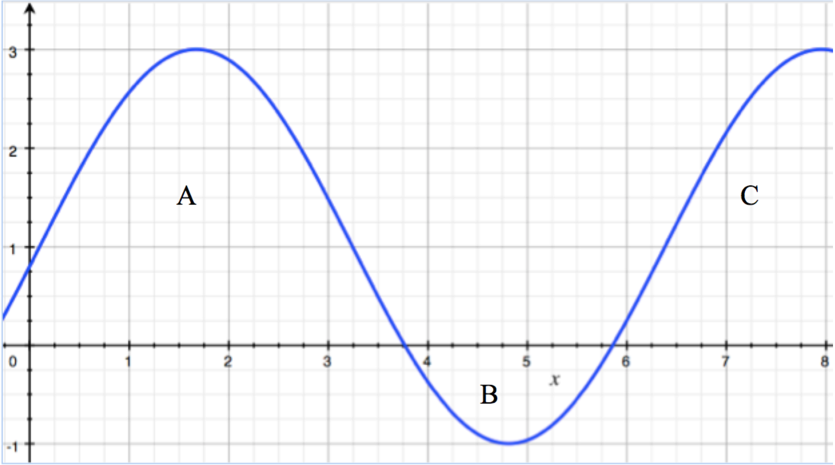
1. Choose the correct statement given that $\int_0^7 f(x)dx = 8$, $\int_1^7 f(x)dx = -3$.



The area of region A is 6, region B is 7/8 and of region C is 3. Find $\int_0^8 f(x)dx$

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2. What is the actual area between f and the x-axis between $x = 0$ and $x = 8$?

Section 6.2 - The Fundamental Theorem of Calculus

Let $F(x) = \int_a^x f(t)dt$.

F is the antiderivative of f and $\int_a^b f(x)dx = F(b) - F(a)$.

Also, $F'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$.

If we consider an integral as an "accumulation of area", then the derivative of the integral is a "rate of change" of an "accumulation of an area". Therefore, if $F(x) = \int_a^x f(t)dt$, then $F'(x) = f(x)$.

Examples:

1. $\frac{d}{dx}\int_3^x (t^2 - \sqrt{t})dt =$

2. $\frac{d}{dx}\int_1^x \frac{1}{t^2+4}dt =$

3. $\frac{d}{dx}\int_x^0 \sqrt{3s+1}ds =$

Use the chain rule when necessary $\frac{d}{dx}\int_{a}^{g(x)} f(t)dt = f(g(x)) \cdot g'(x)$

4. $\frac{d}{dx}\int_{\pi}^{4x} e^{2t+1}dt =$

5. $\frac{d}{dx}\int_{1}^{x^2} \sin(2t)dt =$

6. $\frac{d}{dx}\int_{\cos(x)}^{0} \sqrt{w^3+1}dw =$

One more:

7. $\dfrac{d}{dx}\displaystyle\int_{3x}^{x^2} \tan(t)dt =$

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3. Let $A(x) = \int_{-2}^{x} f(t)dt$. Find $A(-2)$

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4. Let $G(x) = \int_{x^2}^{0} \frac{1}{t+1} dt$. Find $G'(x)$

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5. Given $\int_0^1 f(x)dx = 3$, $\int_0^3 f(x)dx = 5$, $\int_3^6 f(x)dx = 9$, find $\int_6^1 2 \cdot f(x)dx$

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6. Given $\int_1^4 f(x)dx = -5$, $\int_3^4 f(x)dx = 8$, find $\int_1^3 \cdot f(x)dx$

How do we find the antiderivative?

Examples: Determine a function whose derivative is:

1. $f(x) = 5$

2. $f(x) = 5x$

3. $f(x) = x^2$

4. $f(x) = x^2 + 5x$

5. $f(x) = \sqrt{x}$

Some Antiderivatives:

Function	AN Antiderivative
$x^p \;\; p \neq -1$	
$\sin(x)$	
$\cos(x)$	
$\sec^2(x)$	
$\sec(x)\tan(x)$	
$\csc^2(x)$	
$\csc(x)\cot(x)$	

Evaluate each definite integral. Recall: $\int_a^b f(x)dx = F(b) - F(a)$

1. $\int_1^4 x dx$

2. $\int_{-2}^2 (2x-3)dx$

3. $\int_{\pi/2}^{\pi} \sin(x)dx$

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7. Let $F(x) = \int_0^{x^2} \sin(t)dt$. Find $F'(x)$

To Do

Read sections 6.1-6.3.

Work quizzes 22 & 23.

Email me questions to put in the notes.