1. Given $a_n = \left(\frac{(n+2)^{25}}{2^{n+3}}\right)\left(\frac{2^n}{(n+5)^{25}}\right)$, find $\lim\limits_{n\to\infty} a_n$.
2. Determine if the sequence $\{a_n\}$ converges when $a_n = \frac{3n^2(2n-1)!}{(2n+1)!}$. If it converges, find the limit.
3. Determine if the sequence $\{a_n\}$ converges when $a_n = \frac{12n^2}{3n+2} - \frac{4n^2+1}{n+2}$. If it converges, find the limit.
4. Determine if the sequence $\{a_n\}$ converges when $a_n = \frac{\ln(3n)}{\ln(5n)}$. If it converges, find the limit.
5. Determine if the sequence $\{a_n\}$ converges when $a_n = \frac{n^{4n}}{(n-2)^{4n}}$. If it converges, find the limit.
6. Determine if the sequence $\{a_n\}$ converges when $a_n = (n^9)^{\frac{1}{5n}}$. If it converges, find the limit.
7. Determine if the sequence $\{a_n\}$ converges when $a_n = \sqrt{n^3} - \sqrt{n^2}$. If it converges, find the limit.
8. Donny decides he is out of shape so he starts to run every day. His goal is to jog 4% more miles than the day before. How long will it take Donny to be able to run a total of 50 miles (for all days)? (Assume he runs one mile on the first day)
9. Determine if the infinite series $\sum_{n=1}^{\infty} \ln\left(\frac{2n}{3n+1}\right)$ converges or diverges. If it converges, find the sum.
10. Determine if the infinite series $\sum_{n=1}^{\infty} (\cos^2\theta)^n, \quad 0 \le \theta < 2\pi$, converges or diverges. If it converges, find the sum.
11. Find the rational representation of the repeating decimal $1.838383\overline{83}...$ using series.
12. Find the interval of all x for which the series $\sum_{n=1}^{\infty} 2^n x^n$ converges.
13. If the nth partial sum of an infinite series, $\sum_{n=1}^{\infty} a_n$, is $S_n = \frac{2n}{n+1}$, find a_n.
14. Determine if the infinite series $\sum_{n=1}^{\infty} \frac{3(n+1)^2}{n(n+4)}$ converges or diverges. If it converges, find the sum.

15. Determine if the infinite series $4-2+1-\frac{1}{2}+\frac{1}{4}...$ converges or diverges. If it converges, find the sum.

16. When applying the root test to an infinite series $\sum a_n$, we find the value of $\lim_{n\to\infty} |a_n|^{1/n} = \rho$. Compute the value of ρ for $\sum_{n=1}^{\infty} 2^{3n}\left(\frac{n-2}{n}\right)^{n^2}$.

17. When applying the root test to an infinite series $\sum a_n$, we find the value of $\lim_{n\to\infty} |a_n|^{1/n} = \rho$. Compute the value of ρ for $\sum_{n=1}^{\infty}\left(\frac{4\arctan n}{5}\right)^{n}$.

18. When applying the ratio test to an infinite series $\sum a_n$, we find the value of $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \rho$. Compute the value of ρ for $\sum_{n=1}^{\infty}\frac{\sin(1/n)}{6n+11}$.

19. When applying the ratio test to an infinite series $\sum a_n$, we find the value of $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \rho$. Compute the value of ρ for $\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}\left(\frac{2}{7}\right)^{n}$.

20. Use the integral test to determine if $\sum_{n=1}^{\infty}\frac{4}{n^2+1}$ converges or diverges.

21. If the improper integral $\int_1^{\infty}\frac{1}{x^p}dx$ converges, which of the following are always true?

   a. $\sum \frac{1}{n^p}$ converges
   b. $\sum \frac{1}{n^{p-1}}$ converges
   c. $\sum \frac{1}{n^{p+1}}$ diverges
   d. $\sum \frac{1}{n^p}$ diverges
   e. $\sum \frac{1}{n^{p-1}}$ diverges
   f. $\sum \frac{1}{n^{p+1}}$ converges

22. Determine if the following series converge or diverge:

a. $\sum_{n=0}^{\infty} \frac{4^n}{(n+2)^n}$

b. $\sum_{n=1}^{\infty} \left(\frac{3n}{4n+1}\right)^n \left(\frac{5}{4}\right)^n$

c. $\sum_{n=1}^{\infty} n!\left(\frac{3}{n}\right)^n$

d. $\sum_{n=1}^{\infty} \frac{n^6+1}{n+2}\left(\frac{2}{9}\right)^n$

e. $\sum_{n=1}^{\infty} \left(\frac{3n+7}{n^2+9}\right)^n$

f. $\sum_{n=1}^{\infty} \frac{\sqrt{n}-1}{\sqrt{n}+6}\left(\frac{3}{2}\right)^n$

g. $\sum_{n=1}^{\infty} \frac{(2n)^n}{n!}$

h. $\sum_{n=1}^{\infty} \frac{3^n}{(n+3)^n}$

i. $\sum_{k=1}^{\infty} \frac{2^k k^2}{k!}$

j. $\sum_{k=1}^{\infty} \frac{1}{(\sqrt[k]{3}-1)^k}$

k. $\sum_{k=2}^{\infty} \frac{5}{k(\ln k)^5}$

23. Determine if the following series converge absolutely, converge conditionally or diverge:

a. $\sum_{n=0}^{\infty} \frac{(-3)^n}{(2n)!}$

b. $\sum_{n=1}^{\infty} \frac{n(-2)^n}{4^{n-1}}$

c. $\sum_{n=1}^{\infty} \frac{(-1)^n}{(n+3)\ln n}$

d. $\sum_{n=1}^{\infty} \frac{4}{8n+5}$

e. $\sum_{n=1}^{\infty} \frac{4(-n)^n}{(n+5)^n}$

f. $\sum_{n=1}^{\infty}\left(-\frac{6}{5}\right)^n$

g. $\sum_{n=1}^{\infty}(-1)^n \frac{9}{3n\ln(n)+1}$

h. $\sum_{n=1}^{\infty}\cos(\pi n)\left(\frac{2n}{3n+7}\right)^n$

i. $\sum_{k=1}^{\infty}(-1)^k \sin\left(\frac{1}{k}\right)$

j. $\sum_{k=2}^{\infty}\frac{(-1)^{k-1}\sin^2(k)}{4^k}$

k. $\sum_{k=1}^{\infty}\frac{(-1)^{k-1}e^{1/k}}{4k}$

24. Which of the following statements are true:

a. If $0 \le a_n \le b_n$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

b. If $\lim_{n\to\infty} a_n = 0$ then $\sum a_n$ converges.

c. The ratio test can be used to show that $\sum \frac{1}{n^3}$ converges.

25. Determine the radius and interval of convergence for the following:

a. $\sum_{n=1}^{\infty}\frac{x^n}{(n+4)!}$

b. $\sum_{n=1}^{\infty}n^3(x-6)^n$

c. $\sum_{n=1}^{\infty}\sqrt{n}(x-3)^n$

d. $\sum_{n=1}^{\infty}\frac{(-1)^n(x+2)^n}{n5^n}$

e. $\sum_{n=1}^{\infty}n!(3x-1)^n$

f. $\sum_{n=1}^{\infty}\frac{(2x+1)^n n}{2^n}$

26. If the series $\sum_{n=0}^{\infty}c_n x^n$ converges when $x=-3$ and diverges when $x=5$. Which of the following series will converge without further restrictions on c_n?

a. $\sum_{n=0}^{\infty}c_n 7^n$ b. $\sum_{n=0}^{\infty}c_n(-2)^n$ c. $\sum_{n=0}^{\infty}c_n 3^n$

27. If the radius of convergence for the power series $\sum_{n=0}^{\infty} c_n x^n$ is 36, what is the radius of convergence for $\sum_{n=0}^{\infty} c_n x^{2n}$?

28. Find a power series representation for the given function centered at the origin:

a. $f(x) = \dfrac{1}{4+25x^2}$

b. $f(x) = \ln(3-x)$

c. $f(y) = \dfrac{y^3}{(1-y)^2}$

d. $f(x) = \tan^{-1}\left(\dfrac{x}{4}\right)$

e. $f(y) = \ln\left(\sqrt{\dfrac{1+2y}{1-2y}}\right)$

f. $f(x) = \dfrac{2+x}{1-x}$

29. Find a function *f* whose power series representation is $\sum_{n=2}^{\infty} n(n-1)x^{n+6}$ on $(-1,1)$.

30. Evaluate the integral $f(x) = \int_0^x \dfrac{t}{1-t^4}\,dt$ as a power series.