Math 1432
Exam 3 Review

1. Integrate:
   a. $\int \frac{3x^2+3x+3}{x^2+1}\,dx$
   b. $\int \frac{x^2}{(x+1)(x-1)^2}\,dx$
   c. $\int \frac{x^2+5x+2}{(x+1)(x^2+1)}\,dx$
   d. $\int \frac{2x^2}{\sqrt{9-x^2}}\,dx$
   e. $\int \frac{2}{x\sqrt{9+x^2}}\,dx$
   f. $\int \frac{5}{36+(x-1)^2}\,dx$
   g. $\int \frac{1}{\sqrt{4+x^2}}\,dx$
   h. $\int \frac{5x+14}{(x+1)(x^2-4)}\,dx$
   i. $\int_0^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}}\,dx$
   j. $\int \cos^4 x \sin^3 x\,dx$
   k. $\int \cos^5 x \sin^2 x\,dx$
   l. $\int \cot^3 x\,dx$
   m. $\int x\ln(2x)\,dx$
   n. $\int 2x\sin(3x)\,dx$
   o. $\int \frac{5}{36+(x-1)^2}\,dx$
   p. $\int \tan^4(x)\,dx$
   q. $\int 2x\sec\left(4x^2\right)dx$
   r. $\int \sec^4(x)\,dx$

2. Write an expression for the nth term of the sequence:
   a. 1, 4, 7, 10, …
   b. 2, -1, $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, \ldots$
3. Determine if the following sequences are monotonic. Also indicate if the sequence is bounded and if it is give the least upper bound and/or greatest lower bound.
   a. $a_n = \frac{2n}{1+n}$
   b. $a_n = \frac{\cos n}{n}$
4. Determine if the following sequences converge or diverge. If they converge, give the limit.
   a. $\left\{(-1)^n\left(\frac{n}{n+1}\right)\right\}$
   b. $\left\{\frac{6n^2-2n+1}{4n^2-1}\right\}$
   c. $\left\{\frac{(n+2)!}{n!}\right\}$
   d. $\left\{\frac{3}{e^n}\right\}$
   e. $\left\{\frac{4n+1}{n^2-3n}\right\}$
   f. $\left\{\frac{e^n}{n^3}\right\}$
5. Determine the values of *n* which guarantee a theoretical error less than ε if the integral is estimated by the trapezoidal rule and then by Simpson's rule if ε = 0.01.
   a. $\int_1^3\left[\frac{1}{4}x^2+3x-2\right]dx$
   b. $\int_1^3 \cos(5x)dx$
6. The series $4-3+\frac{9}{4}-\frac{27}{16}+\ldots$ is a geometric series. Find the general term, a_n, and write the sum in sigma notation. Does this series converge? If so, what is the sum?
7. Find the sum of the following (if possible):
   a. $\sum_{k=0}^{\infty}\left(-\frac{3}{4}\right)^k$

b. $\sum_{k=2}^{\infty}\left(\frac{2}{3}\right)^{k}$

c. $\sum_{k=0}^{\infty}\left(\frac{5}{4}\right)^{k-1}$

d. $\sum_{n=2}^{\infty}\left(\frac{1}{n}-\frac{1}{n+2}\right)$

e. $\sum_{k=0}^{\infty}\frac{6^{k+1}}{7^{k-2}}$

8. Determine whether the given series converges or diverges; state which test you are using to determine convergence/divergence and show all work.

a. $\sum\frac{k^2 2^k}{(k+1)!}$

b. $\sum\frac{3^{k+1}}{(k+1)^2 e^k}$

c. $\sum\frac{\ln n}{n}$

d. $\sum\frac{2n+1}{\sqrt{n^5+3n^3+1}}$

e. $\sum\frac{4n^2+1}{n^3-n}$

f. $\sum\frac{4n^2+1}{n^5-n}$

g. $\sum\left(1+\frac{1}{n}\right)^{n}$

h. $\sum\frac{n^3}{3^n}$

i. $\sum\frac{1}{\sqrt[4]{n^3}}$