Math 1432 Classwork for Week 12 Work in groups of 3-4 and be prepared to present your answers.

1. Let f be a function that has derivatives of all orders on the interval (–1, 1). Assume
$$f(0)=1,\ f'(0)=\frac{1}{2},\ f''(0)=-\frac{1}{4},\ f'''(0)=\frac{3}{8},\ \text{and}\ \left|f^{(4)}(x)\right|\le 6 \text{ for all x in the interval (0, 1).}$$
   a) Find the third-degree Taylor polynomial about x = 0 for the function f.
   b) Use your answer to part (a) to estimate the value of $f(0.5)$.
   c) What is the maximum possible error for the approximation made in part (b)?
2. Estimate the error that results when *sin* x is replaced by $x-\frac{1}{6}x^3$ for $|x|<0.2$. Show your reasoning.
3. Which term is truncated if we want to approximate the sum of $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n^3-1}$ with an error of less than $\frac{1}{1000}$?
4. Give the first four terms of the Taylor Polynomial for $f(x)=x\cos\left(x^3\right)$
5. Find the radius and interval of convergence for:
   a) $\sum_{n=0}^{\infty}\frac{(-1)^n(x-2)^n}{3^n n^2}$ b) $\sum_{n=0}^{\infty}(2n)!(x-5)^n$
6. What is the coefficient of x^6 in the Taylor series expansion about $x=0$ for $f(x)=\sin\left(x^2\right)$?