Math 1432
Final Exam Review

1. Find the **net area** bounded by the graph of $f(x) = x^3 - x^2$ and the x-axis on the interval [0,2].

2. Find the **area** bounded by the graph of $f(x) = x^3 - x^2$ and the x-axis on the interval [0,2].

3. Find the area of the region bounded by the line $x = 4$ and the graph of $f(x) = x^2$.

4. Revolve the region in problem 3 about the x-axis, and **give the integral resulting from using the method of washers** to find its volume. Do not compute the integral!

5. Revolve the region in problem 3 about the y-axis, and **give the integral resulting from using the method of cylindrical shells** to find its volume. Do not compute the integral!

6. R is the region bounded by the given graphs and the given axis. Sketch each graph then find the area of R, the volume when R is revolved about the x-axis and the volume when R is revolved about the y-axis.

   a. $y = x^2,\ y = 6 - x,\ \ x - axis$

   b. $y = x^2,\ y = 6 - x,\ \ y - axis$

7. Given F(x) for each problem, graph the function and shade the area between F(x) and the x-axis, find the x-coordinate of the centroid of the shaded region and find the y-coordinate of the centroid of the shaded region.

   a. $F(x) = x^2 - x$

   b. $F(x) = x^2 + 4x$

8. Given F(x) and the interval [a, b], graph F(x) over the interval, find the average value of F(x) on that interval and find the value of *c* that verifies the conclusion of the mean value theorem for integrals for the function *F* over the interval [a, b].

   a. $F(x) = x^2 - x \quad [0,1]$

   b. $F(x) = x^2 + 3x \quad [-3,0]$

   c. $F(x) = x^2 - 4 \quad [-2,2]$

9. Give an equation relating x and y for the curve given parametrically by

   a. $x(t) = -1 + 3\tan t \quad y(t) = 1 + 2\sec t$

   b. $x(t) = 2e^{t} \quad y(t) = 1 - 3e^{-2t}$

10. Integrate:

   a. $\int \frac{\csc^2 x}{\sqrt{\cot x}}\,dx$

   b. $\int_{-8}^{0} \frac{1}{\sqrt{1-x}}\,dx$

   c. $\int \sin^3 3x \cos 3x\,dx$

   d. $\int_{2}^{7} x\sqrt{x^2+2}\,dx$

   e. $\int (x^2-2)\cos(x^3-6x)\,dx$

   f. $\int \frac{2x}{\sqrt{9-x^2}}\,dx$

   g. $\int_{0}^{1} \frac{2x}{(x^2+3)^4}\,dx$

   h. $\int \sec^2(2x)\,dx$

   i. $\int \csc^2(3x)\,dx$

   j. $\int \sec(2x)\tan(2x)\,dx$

   k. $\int \sqrt{x+1}\,dx$

   l. $\int x(x^2+1)^4\,dx$

   m. $\int (\cosh(3x) + \sinh(2x))\,dx$

   n. $\int e^{3x}\,dx$

   o. $\int \frac{\ln(x^3)}{x}\,dx$

   p. $\int (e^{7x} - \sinh(5x))\,dx$

   q. $\int \frac{\sin(3x)}{16+\cos^2(3x)}\,dx$

   r. $\int \frac{6x}{4+x^4}\,dx$

   s. $\int \tan(3x)\,dx$

t. $\int \frac{\arctan(3x)}{1+9x^2}\,dx$

u. $\int \frac{1}{\sqrt{4+x^2}}\,dx$

v. $\int \sqrt{9-x^2}\,dx$

w. $\int 3\ln(4x)dx$

x. $\int x^2 e^x\,dx$

y. $\int \frac{5x+14}{(x+1)(x^2-4)}\,dx$

z. $\int \frac{x^2+5x+2}{(x+1)(x^2+1)}\,dx$

aa. $\int \frac{2x^2}{\sqrt{9-x^2}}\,dx$

bb. $\int 2\arctan(10x)dx$

cc. $\int 3x\cos(2x)dx$

11. Determine if the following sequences converge or diverge. If they converge, give the limit.

a. $\left\{\left(\frac{2n}{n+1}\right)\right\}$

b. $\left\{\frac{6n^2-2n+1}{\sqrt{4n^3-1}}\right\}$

c. $\left\{\frac{n!}{(n+2)!}\right\}$

d. $\left\{\frac{3^n}{e^n}\right\}$

e. $\left\{\frac{4n^2+1}{n^2-3n}\right\}$

12. Determine if the following series (A) converge absolutely, (B) converge conditionally or (C) diverge.

a. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+3}$

b. $\sum_{n=1}^{\infty} \frac{\cos \pi n}{n^2}$

c. $\sum_{n=0}^{\infty} \frac{4n(-1)^n}{3n^2+2n+1}$

d. $\sum_{n=0}^{\infty} \frac{3(-1)^n}{\sqrt{3n^2+2n+1}}$

e. $\sum_{n=0}^{\infty} \frac{3n(-1)^n}{\sqrt{3n^2+2n+1}}$

f. $\sum_{n=0}^{\infty} \left(4(-1)^n \left(\frac{n}{n+3} \right)^n \right)$

g. $\sum_{n=0}^{\infty} \left(\frac{2(-1)^n \arctan n}{3+n^2+n^3} \right)$

h. $\sum_{n=0}^{\infty} \left(\frac{(-1)^n 3^n}{4^n+3n} \right)$

i. $\sum_{n=0}^{\infty} \left(\frac{(-1)^n 3}{(n+2)\ln(n+2)} \right)$

13. Find the sum of the following convergent series:

a. $\sum_{n=0}^{\infty} 2\left(-\frac{4}{9}\right)^n$

b. $\sum_{n=0}^{\infty} \left(\frac{1}{3^n} - \frac{5}{6^n} \right)$

14. Give the derivative of each power series below:

a. $\sum_{n=0}^{\infty} \frac{(n^2+1)x^n}{\sqrt{n^5+3n}}$

b. $\sum_{n=0}^{\infty} \frac{(2n+1)x^n}{n^3}$

15. For each of the problems in number 14, give the antiderivative F of the power series so that F(0)=0.

16. Evaluate each improper integral:

a. $\int_1^9 (x-1)^{-2/3} dx$

b. $\int_0^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

c. $\int_0^{\infty} \frac{2}{1+x^2} dx$

17. Find the formula for the area of $r = 1 + 2\sin\theta$
    a. Inside inner loop
    b. Inside outer loop but outside inner loop
    c. Inside outer loop and below x-axis

18. Find the smallest value of n so that the nth degree Taylor Polynomial for $f(x) = \ln(1+x)$ centered at x = 0 approximates $\ln(2)$ with an error of no more than 0.001 (also be able to do this with some of the other Taylor Polynomials)

19. Find the radius of convergence and interval of convergence for the following Power series:
    a. $\sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{(n+1)3^{n+1}}$
    b. $\sum_{n=0}^{\infty} \frac{1}{3^n}(x-1)^n$
    c. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{4^n}$

20. Determine the convergence or divergence for each series with the given general term:

Series	Converge or Diverge?	Test used
$\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$		
$\sum_{n=1}^{\infty} \frac{2^n}{n^3}$		
$\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n}\right)$		
$\sum_{n=1}^{\infty} \frac{3^{2n}}{n!}$		
$\sum_{n=1}^{\infty} \cos(\pi n)$		
$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$		
$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{3n^3+1}$		

$\sum_{n=0}^{\infty} 3\left(-\frac{1}{2}\right)^n$		
$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$		
$\sum_{n=1}^{\infty} ne^{-n^3}$		
$\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^n$		
$\sum_{n=1}^{\infty} \frac{1}{n^3+1}$		
$\sum_{n=0}^{\infty} \left(\frac{2}{9}\right)^n$		
$\sum_{n=1}^{\infty} \frac{n^2}{2^n}$		
$\sum_{n=1}^{\infty} (0.34)^n$		
$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$		
$\sum_{n=1}^{\infty} \frac{1}{2n+1}$		