Final exam review.

There will be an online final exam review on 5/7 5/4 at 10am

110 minutes 18-20 questions

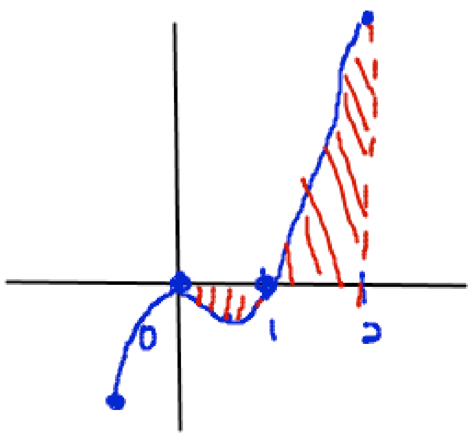
Review Sheet (problems not done in class)

1. Find the **net area** bounded by the graph of $f(x) = x^3 - x^2$ and the x-axis on the interval $[0,2]$.

2. Find the **area** bounded by the graph of $f(x) = x^3 - x^2$ and the x-axis on the interval $[0,2]$.

actual area $\Rightarrow$ positive

$$x^3 - x^2 = 0$$
$$x^2(x-1) = 0$$
$$x = 0, 1$$



x	y
-1	$-1-1=-2$
0	0
$1/2$	$1/8 - 1/4 = -1/8$
2	$8-4=4$

① Net area $\int_0^2 (x^3 - x^2)\,dx = \frac{x^4}{4} - \frac{x^3}{3}\Big|_0^2 = \left(4 - 8/3\right) - 0 = 4/3$

② Area: $\left|\int_0^1 (x^3 - x^2)\,dx\right| + \int_1^2 (x^3 - x^2)\,dx = \frac{1}{12} + \frac{17}{12} = \frac{3}{2}$

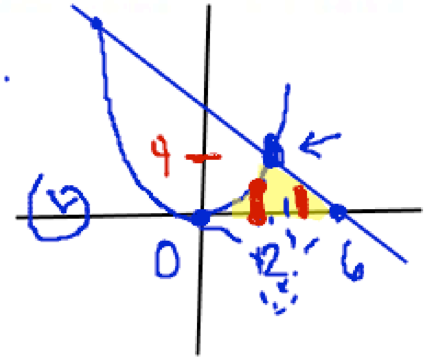
6. R is the region bounded by the given graphs and the given axis. Sketch each graph then find the area of R, the volume when R is revolved about the x-axis and the volume when R is revolved about the y-axis.

a. $y = x^2$, $y = 6 - x$, $x - axis$

b. $y = x^2$, $y = 6 - x$, $y - axis$

1st quad.

a.



$x^2 = 6 - x$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3, 2$

$$A = \int_0^2 x^2\,dx + \int_2^6 (6-x)\,dx$$

$$\frac{8}{3} + 8 = \frac{32}{3}$$

two integrals for disc/washer method

$$V = \int_0^2 \pi (x^2)^2\,dx + \int_2^6 \pi (6-x)^2\,dx$$

shell method:

$$V = \int_0^4 2\pi y\,[\underbrace{6-y-\sqrt{y}}_{\text{Right - left}}]\,dy$$

Rev. around x.

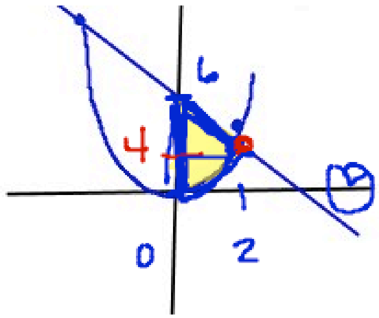
$$V_y = \int_0^4 \pi[(6-y)^2 - (\sqrt{y})^2]\,dy$$

washer

$$V_y = \int_0^2 2\pi x\,(x^2-0)\,dx + \int_2^6 2\pi x\,(6-x-0)\,dx$$

Shell

b.



$y = 6 - x,\ y = x^2,\ y\text{-axis}$

$$A = \int_0^2 (6 - x - x^2)\,dx$$

$$V_x = \int_0^2 \pi \left[(6-x)^2 - (x^2)^2 \right] dx$$

$$V_x = \int_0^4 2\pi y\,(\sqrt{y} - 0)\,dy + \int_4^6 2\pi y\,(6 - y - 0)\,dy$$

$$V_y = \int_0^4 \pi\,(\sqrt{y})^2\,dy + \int_4^6 \pi (6-y)^2\,dy$$

$$V_y = \int_0^2 2\pi x\,(6 - x - x^2)\,dx$$

Area: $\int_a^b (\text{top} - \text{bottom})\, dx$

V (washer): $\int_a^b \pi \left(R^2 - r^2\right) dx$ (x-axis)

$\uparrow$

y (y-axis)

V (shell): x-axis $\int_c^d 2\pi y\,(\text{Right} - \text{left})\, dy$

y-axis $\int_a^b 2\pi x\,(\text{top} - \text{bottom})\, dx$

8. Given F(x) and the interval [a, b], graph F(x) over the interval, find the average value of F(x) on that interval and find the value of c that verifies the conclusion of the mean value theorem for integrals for the function F over the interval [a, b].

a. $F(x) = x^2 - x \quad [0,1]$

b. $F(x) = x^2 + 3x \quad [-3,0]$

$$Av = \frac{1}{b-a}\int_a^b f(x)\,dx$$

a. $\frac{1}{1-0}\int_0^1 (x^2 - x)\,dx = \left.\frac{x^3}{3} - \frac{x^2}{2}\right|_0^1 = \frac{-1}{6}$ ← avg value

to find c: $x^2 - x = \frac{-1}{6}$

$6x^2 - 6x = -1$

$6x^2 - 6x + 1 = 0$

$x = \frac{6 \pm \sqrt{36 - 24}}{12}$

$\boxed{c = \frac{1}{2} \pm \frac{\sqrt{3}}{6}}$

b. $\frac{1}{0 - -3}\int_{-3}^{0} (x^2 + 3x)\,dx = \frac{1}{3}\left(\frac{x^3}{3} + \frac{3x^2}{2}\right)_{-3}^{0}$

$\frac{1}{3}\left[0 - \left(-9 + \frac{27}{2}\right)\right] = \frac{3}{2}$

$$x^2 + 3x = 3/2$$

$$2x^2 + 6x - 3 = 0$$

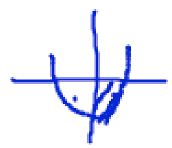
$\sqrt{60} \sim 8$

$$x = \frac{-6 \pm \sqrt{36+24}}{4} = \frac{-3 \pm 2\sqrt{15}}{4}$$

$[-3, 0]$ $\qquad \dfrac{-3+2\sqrt{15}}{4}, \dfrac{-3-2\sqrt{15}}{4}$

$+.$

$$c = \frac{-3-2\sqrt{15}}{4}$$



c. $F(x) = x^2 - 4 \quad [-2, 2]$

$$AV = \frac{1}{2 - -2}\int_{-2}^{2} (x^2 - 4)\,dx = \frac{1}{4}\left(\frac{-32}{3}\right) = -8/3$$

$c:$ $\quad x^2 - 4 = -8/3$

$$3x^2 - 12 = -8$$

$$3x^2 = 4$$

$$x^2 = 4/3 \qquad x = \pm 2/\sqrt{3}$$

$$\boxed{c = \pm 2/\sqrt{3}}$$

10. (q, r, v, w: done on Monday)

a. $\int \frac{\csc^2 x}{\sqrt{\cot x}}\,dx$

b. $\int_{-8}^{0} \frac{1}{\sqrt{1-x}}\,dx$

c. $\int \sin^3 3x \cos 3x\,dx$

d. $\int_{2}^{7} x\sqrt{x^2+2}\,dx$

e. $\int (x^2-2)\cos(x^3-6x)\,dx$

f. $\int \frac{2x}{\sqrt{9-x^2}}\,dx$

g. $\int_{0}^{1} \frac{2x}{(x^2+3)^4}\,dx$

h. $\int \sec^2(2x)\,dx$

i. $\int \csc^2(3x)\,dx$

j. $\int \sec(2x)\tan(2x)\,dx$

k. $\int \sqrt{x+1}\,dx$

l. $\int x(x^2+1)^4\,dx$

m. $\int (\cosh(3x)+\sinh(2x))\,dx$

n. $\int e^{3x}\,dx$

o. $\int \frac{\ln(x^3)}{x}\,dx$

p. $\int (e^{7x}-\sinh(5x))\,dx$

a. $u = \cot x$
$du = -\csc^2 x\,dx$

$\int \frac{-1}{\sqrt{u}}\,du = -2\sqrt{u} + C$

$\boxed{-2\sqrt{\cot x} + C}$

b. $u = 1-x$
$du = -\,dx$

$\int_9^1 \frac{-1}{\sqrt{u}}\,du = -2\sqrt{u}\Big|_9^1 = -2 - {-6} = \boxed{4}$

c. $u = \sin(3x)$
$du = 3\cos(3x)\,dx$

$\frac{1}{3}\int u^3\,du$

$\boxed{\frac{\sin^4(3x)}{12} + C}$

d. $u = x^2 + 2$
$du = 2x\,dx$

$\int_6^{51} \frac{1}{2}\sqrt{u}\,du$

$\frac{1}{3}u^{3/2}\Big|_6^{51} = \frac{1}{3}\left(51^{3/2} - 6^{3/2}\right)$

e. $u = x^3 - 6x$
$du = (3x^2-6)\,dx = 3(x^2-2)\,dx$

o. $\int \frac{3\ln x}{x}\,dx$ $\quad u = \ln x \quad du = \frac{1}{x}\,dx$

$3\int u\,du = 3\cdot u^2/2 + C$

$\boxed{\frac{3}{2}(\ln x)^2 + C}$

s. $\int \tan(3x)\,dx$ ← Know formula!

t. $\int \frac{\arctan(3x)}{1+9x^2}\,dx$

u. $\int \frac{1}{\sqrt{4+x^2}}\,dx$

t. $u = \arctan(3x)$

$du = \frac{3}{1+9x^2}\,dx$

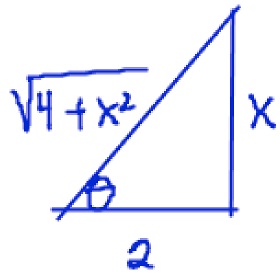
$\frac{1}{3}\int u\,du$

$\frac{1}{6}(\arctan(3x))^2 + C$

trig sub

$\Rightarrow x = 2\tan\theta$

$\frac{x}{2} = \tan\theta$ ←



$\frac{\sqrt{4+x^2}}{2} = \sec\theta$ ←

$\sqrt{4+x^2} = 2\sec\theta$

$dx = 2\sec^2\theta\,d\theta$

$\int \frac{1}{2\sec\theta}(2\sec^2\theta\,d\theta)$

$= \int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C$

$$\boxed{\ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C}$$

x. $\int x^2 e^x\,dx$ ← IBP

y. $\int \frac{5x+14}{(x+1)(x^2-4)}\,dx$

PFD ↗ ↑ $(x-2)(x+2)$

x. $u = x^2 \quad dv = e^x dx$

$du = 2x\,dx \quad v = e^x$

$x^2e^x - \underbrace{\int e^x \cdot 2x\,dx}$ → $u = 2x \quad dv = e^x dx$; $du = 2\,dx \quad v = e^x$

$x^2 e^x - \left[2xe^x - \int 2e^x\,dx\right]$

$$\boxed{x^2e^x - 2xe^x + 2e^x + C}$$

y. form: $\frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+2}$

$A(x-2)(x+2) + B(x+1)(x+2) + C(x+1)(x-2) = 5x + 14$

$x = 2$: $\quad A(0) + B(3)(4) + C(0) = 10 + 14$

$12B = 24 \rightarrow B = 2$

$x = -2$: $\quad A(0) + B(0) + C(-1)(-4) = -10 + 14 \rightarrow C = 1$

$x = -1$: $\quad A(-3)(1) + B(0) + C(0) = -5 + 14 \quad A = -3$

$$\int\left(\frac{-3}{x+1} + \frac{2}{x-2} + \frac{1}{x+2}\right)dx = -3\ln|x+1| + 2\ln|x-2| + \ln|x+2| + C$$

z. $\int \frac{x^2+5x+2}{(x+1)(x^2+1)}\,dx$

form: $\frac{A}{x+1} + \frac{Bx+C}{x^2+1}$

$A(x^2+1) + (Bx+C)(x+1) = x^2+5x+2$

$x=-1$: $2A = 1-5+2$ $A=-1$

$x=0$ $A + C = 2$

$-1 + C = 2$ $C=3$

$x=1$: $2A + 2B + 2C = 8$

$-2 + 2B + 6 = 8$

$2B = 4$ $B=2$

$$\int\left(\frac{-1}{x+1} + \frac{2x}{x^2+1} + \frac{3}{x^2+1}\right)dx$$

$$\boxed{-\ln|x+1| + \ln(x^2+1) + 3\arctan(x) + C}$$

aa. $\int \frac{2x^2}{\sqrt{9-x^2}}\,dx$

bb. $\int 2\arctan(10x)\,dx$ ← IBP

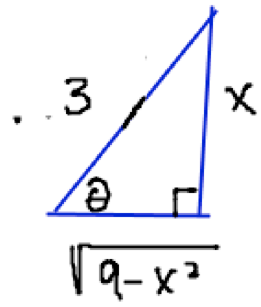
$u = 2\arctan(10x) \qquad dv = dx$

$du = \frac{20}{1+100x^2}\,dx \qquad v = x$

$2x\arctan(10x) - \int \frac{20x}{1+100x^2}\,dx$

$\boxed{2x\arctan(10x) - \frac{1}{10}\ln(1+100x^2) + C}$

aa. (trig sub)



$x = 3\sin\theta$

$\frac{x}{3} = \sin\theta$ ← *

$dx = 3\cos\theta\,d\theta$

$\frac{\sqrt{9-x^2}}{3} = \cos\theta$ ←

$\sqrt{9-x^2} = 3\cos\theta$

$$\int \frac{2(3\sin\theta)^2}{3\cos\theta}\,3\cos\theta\,d\theta = 18\int \sin^2\theta\,d\theta = 18\left(\tfrac{1}{2}\theta - \tfrac{1}{2}\sin\theta\cos\theta\right) + C$$

$= 9\theta - 9\sin\theta\cos\theta + C$

$9\sin^{-1}\left(\frac{x}{3}\right) - \cancel{9}\left(\frac{x}{\cancel{3}}\right)\left(\frac{\sqrt{9-x^2}}{\cancel{3}}\right) + C$

cc. $\int 3x\cos(2x)\,dx$ IBP

$u = 3x \quad dv = \cos(2x)\,dx$

$du = 3\,dx \quad v = \frac{1}{2}\sin(2x)$

$\frac{3}{2}x\sin(2x) - \int \frac{3}{2}\sin(2x)\,dx$

$\frac{3}{2}x\sin(2x) + \frac{3}{4}\cos(2x) + C$

11. Determine if the following sequences converge or diverge. If they converge, give the limit.

a. $\left\{\left(\frac{2n}{n+1}\right)\right\}$

b. $\left\{\frac{6n^2-2n+1}{\sqrt{4n^3-1}}\right\}$

c. $\left\{\frac{n!}{(n+2)!}\right\}$

d. $\left\{\frac{3^n}{e^n}\right\}$

e. $\left\{\frac{4n^2+1}{n^2-3n}\right\}$

if it has a limit

$3 > e$

d. $\lim_{n\to\infty} \frac{3^n}{e^n} \to \infty$

diverges

e. $\lim_{n\to\infty} \frac{4n^2+1}{n^2-3n} = 4$

converges to 4

a. $\lim_{n\to\infty} \frac{2n}{n+1} = 2$

converges to 2

b. $\lim_{n\to\infty} \frac{6n^2-2n+1}{\sqrt{4n^3 \cancel{+} 1}}$ $\leftarrow n^2$ (bigger) $\leftarrow n^{3/2}$

diverges

12. Determine if the following series (A) converge absolutely, (B) converge conditionally or (C) diverge.

a non alternating series cannot converge conditionally.

alternates

a. $\sum \frac{(-1)^{n+1}\sqrt{n}}{n+3}$ $\qquad \frac{\sqrt{n}}{n+3} \to 0 \quad \Rightarrow$ conv.

abs convergence??

$$\sum \left| \frac{(-1)^{n+1}\sqrt{n}}{n+3} \right| = \sum \frac{\sqrt{n}}{n+3} \leftarrow \text{conv ??}$$

No

$\Rightarrow$ $\boxed{\text{conditionally convergent}}$

b. $\sum \frac{\cos(\pi n)}{n^2} \leftarrow = (-1)^n$ $\quad = \sum \frac{(-1)^n}{n^2}$ $\quad \boxed{\text{conv. absolutely}}$

abs? $\sum \frac{1}{n^2}$ conv.

c. $\displaystyle\sum_{n=0}^{\infty} \frac{4n(-1)^n}{3n^2+2n+1}$ conditionally conv.

d. $\displaystyle\sum_{n=0}^{\infty} \frac{3(-1)^n}{\sqrt{3n^2+2n+1}}$ conditionally conv.

e. $\displaystyle\sum_{n=0}^{\infty} \frac{3n(-1)^n}{\sqrt{3n^2+2n+1}}$ diverges. $a_n \not\to 0 \Rightarrow$ div. by BDT

f. $\displaystyle\sum_{n=0}^{\infty} \left(4(-1)^n\left(\frac{n}{n+3}\right)^n\right)$ diverges by BDT

g. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{2(-1)^n \arctan n}{3+n^2+n^3}\right)$ absolutely conv.

h. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{(-1)^n 3^n}{4^n+3n}\right)$ abs. conv.

$$\sum \frac{3^n}{4^n + 3n} < \sum \left(\frac{3}{4}\right)^n$$

geom $|r|<1 \Rightarrow$ conv.

$$\left(\frac{n}{n+3}\right)^n = \left(\frac{n+3}{n}\right)^{-n} = \left(1+\frac{3}{n}\right)^{-n} \to \underline{\underline{e^{-3}}} \neq 0$$

13. Find the sum of the following convergent series:

a. $\sum_{n=0}^{\infty} 2\left(-\frac{4}{9}\right)^n$

$$\frac{a_1 \leftarrow 1^{st}\text{ term}}{1-r}$$

$$2\sum_{n=0}^{\infty}\left(-\frac{4}{9}\right)^n$$

$$2\left(\frac{(-4/9)^0}{1-{}^-4/9}\right) = 2\left(\frac{1}{13/9}\right) = \frac{18}{13}$$

14. Give the derivative of each power series below:

a. $\sum_{n=0}^{\infty} \frac{(n^2+1)x^n}{\sqrt{n^5+3n}}$

b. $\sum_{n=0}^{\infty} \frac{(2n+1)x^n}{n^3}$

15. For each of the problems in number 14, give the antiderivative F of the power series so that F(0)=0.

*

14 a. $$\frac{d}{dx} \sum \frac{(n^2+1)x^n}{\sqrt{n^5+3n}} = \sum \frac{(n^2+1)\, n\, x^{n-1}}{\sqrt{n^5+3n}}$$

15 a. $$\int \sum \frac{(n^2+1)x^n}{\sqrt{n^5+3n}}\,dx = \boxed{\sum \frac{(n^2+1)\, x^{n+1}}{\sqrt{n^5+3n}\,(n+1)}} + C$$

$$\underbrace{\sum \frac{(\;)\,0}{\sqrt{\;\;}}}_{=0} + C = 0 \Rightarrow C = 0$$

14 b. $$\sum \frac{(2n+1)\cancel{n}\, x^{n-1}}{n^{\cancel{3}\,2}}$$

15 b. $$\sum \frac{(2n+1)\, x^{n+1}}{n^3(n+1)}$$

16. Evaluate each improper integral:

a. $\int_1^9 (x-1)^{-2/3}\,dx$ $= \int_1^9 \frac{1}{(x-1)^{2/3}}\,dx$

b. $\int_0^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ ← Monday

c. $\int_0^\infty \frac{2}{1+x^2}\,dx$

a. $\lim_{a\to 1^+} \int_a^9 (x-1)^{-2/3}\,dx = \lim_{a\to 1^+} 3(x-1)^{1/3}\Big|_a^9$

$= \lim_{a\to 1^+} 3\sqrt[3]{9-1} - 3\sqrt[3]{a-1} = 6 - 0 = \boxed{6}$

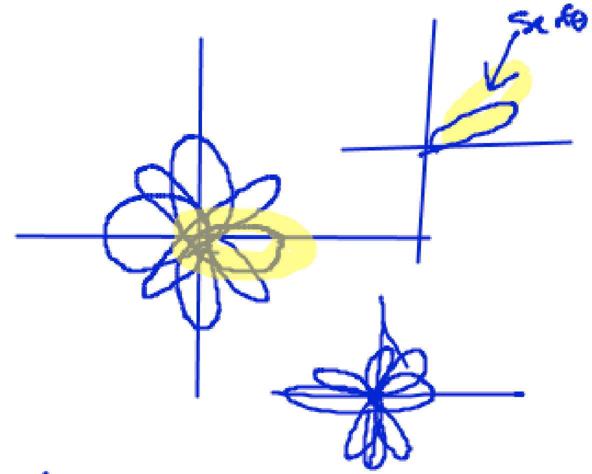
c. $\lim_{b\to\infty} \int_0^b \frac{2}{1+x^2}\,dx = \lim_{b\to\infty} 2\arctan x\Big|_0^b = \lim_{b\to\infty}(2\tan^{-1} b - 0)$

$= 2(\pi/2) = \boxed{\pi}$

17. done Monday

Here's more polar:

$r = \_\cos(4\theta)$ area of 1 petal:



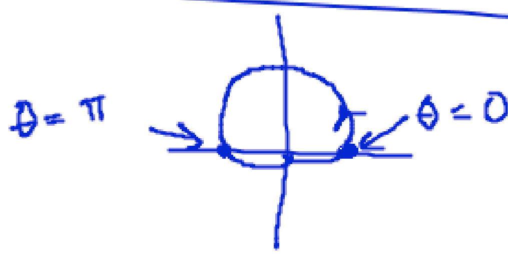
$\cos(4\theta) = 0$

$4\theta = \pi/2,\ 3\pi/2,\ \dots$

$\theta = \pi/8,\ 3\pi/8$

$$A = \int_{\pi/8}^{3\pi/8} \frac{1}{2}\left(\cos(4\theta)\right)^2 d\theta$$

$r = 3 + 3\sin\theta$



above x-axis

$\theta = 0 \rightarrow r = 3$

$$\int_0^{\pi} \frac{1}{2}(3 + 3\sin\theta)^2 d\theta$$

below x-axis:

$$\int_{\pi}^{2\pi} \frac{1}{2}(3 + 3\sin\theta)^2 d\theta$$

18. Find the smallest value of n so that the nth degree Taylor Polynomial for $f(x) = \ln(1+x)$ centered at x = 0 approximates ln(2) with an error of no more than 0.001 (also be able to do this with some of the other Taylor Polynomials)

$x=1$

$x=1$

$$\frac{\max|f^{(n+1)}(c)|}{(n+1)!}x^{n+1} \le .001$$

$$\frac{n!}{(n+1)!} \le \frac{1}{1000}$$

$$\frac{1}{n+1} \le \frac{1}{1000}$$

$$n+1 \ge 1000$$

$$\boxed{n = 999}$$

$$f'(x) = \frac{1}{1+x}$$

$$f''(x) = \frac{-1}{(1+x)^2}$$

$$f'''(x) = \frac{2}{(1+x)^3}$$

$$f^{(4)}(x) = \frac{-2\cdot 3}{(1+x)^4}$$

$$f^{(5)}(x) = \frac{2\cdot 3\cdot 4}{(1+x)^5}$$

$$f^{(6)}(x) = \frac{-2\cdot 3\cdot 4\cdot 5}{(1+x)^6} = \frac{(n-1)!}{(1+x)^n}$$

$$f^{(n+1)}(x) = \frac{(n+1-1)!}{(1+x)^n}$$

$x=0$

$$\max f^{(n+1)}(x) = n!$$

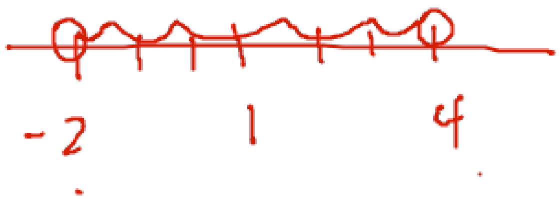
b. $\sum_{n=0}^{\infty} \frac{1}{3^n}(x-1)^n$

c. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{4^n}$ $\leftarrow$

Root: $\sum \left| \frac{(-1)^{n+1} x^n}{4^n} \right| = \sum \frac{|x|^n}{4^n}$ $R = 4$

or geom. $(-4, 4)$

b. $\sum \left(\frac{x-1}{3} \right)^n$ $\left| \frac{x-1}{3} \right| < 1$



$|x-1| < 3$ $R = 3$

$x = -2$: $\sum \left(\frac{-3}{3} \right)^n = \sum (-1)^n$ div.

$\boxed{(-2, 4)}$

$x = 4$: $\sum \left(\frac{3}{3} \right)^n = \sum (1)^n$ div.

20. Determine the convergence or divergence for each series with the given general term:

Series	Converge or Diverge?	Test used
$\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$	div.	p-series $p = 3/4$
$\sum_{n=1}^{\infty} \frac{2^n}{n^3}$	div.	Root test or BDT
$\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n}\right)$	conv.	telescoping
$\sum_{n=1}^{\infty} \frac{3^{2n}}{n!}$	conv.	Ratio

$$\frac{3^{2(n+1)}}{(n+1)\cancel{!}} \cdot \frac{\cancel{n!}}{\cancel{3^{2n}}} = \frac{3^2}{n+1} \to 0 < 1$$

$\sum_{n=1}^{\infty} \cos(\pi n)$	div.	BDT
$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$	div.	p series $p = 1/2$
$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{3n^3 + 1}$	conv.	AST
$\sum_{n=0}^{\infty} 3\left(-\frac{1}{2}\right)^n$	conv.	geometric

$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$	conv.	Integral test
$\sum_{n=1}^{\infty} ne^{-n^3}$	conv.	Root
$\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^n$	div.	BDT
$\sum_{n=1}^{\infty} \frac{1}{n^3+1}$	conv.	comp to p series
$\sum_{n=0}^{\infty} \left(\frac{2}{9}\right)^n$	conv.	geom.
$\sum_{n=1}^{\infty} \frac{n^2}{2^n}$	conv.	Root
$\sum_{n=1}^{\infty} (0.34)^n$	conv.	geom
$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$	conv.	pseries
$\sum_{n=1}^{\infty} \frac{1}{2n+1}$	div.	(limit) comp to harmonic

* differential eq: ex. solve dy/dx = something

13. Find the general solution for each:

a. $\frac{dy}{dx} = (y+5)(x+2)$

b. $y' = \frac{e^x}{y}$

c. $y' = xy - y$

d. $\frac{dy}{dx} = x^2 \sec(y)$

e. $y' = e^{2x}(1+y^2)$

$\int \cos y \, dy = \int x^2 dx$

$\sin y = \frac{x^3}{3} + C$

14. Find the specific solution given the initial condition: $\frac{dy}{dx} = y - 2 \quad y(0) = 6$

a. $dy = (y+5)(x+2)\,dx$

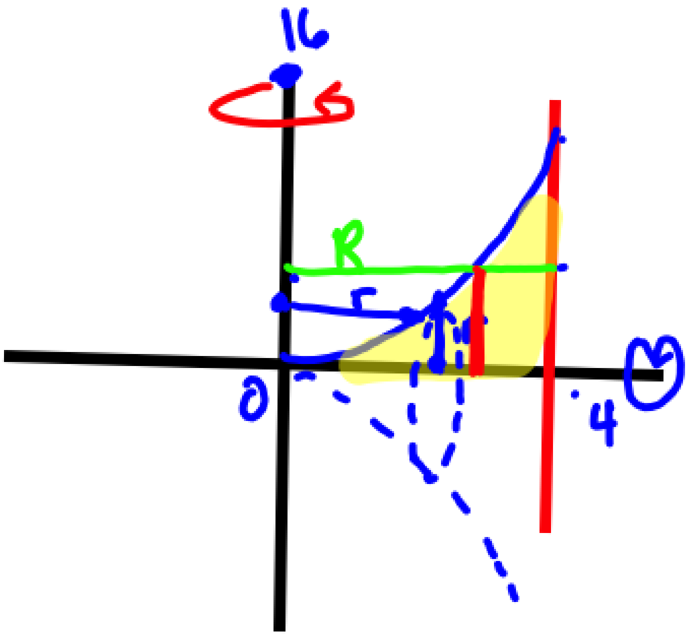
$\int \frac{1}{y+5} dy = \int (x+2)\,dx$

$\ln|y+5| = \frac{x^2}{2} + 2x + C$

3. Find the area of the region bounded by the line $x = 4$ and the graph of $f(x) = x^2$. + x-axis

4. Revolve the region in problem 3 about the x-axis, and **give the integral resulting from using the method of washers** to find its volume. Do not compute the integral!

5. Revolve the region in problem 3 about the y-axis, and **give the integral resulting from using the method of cylindrical shells** to find its volume. Do not compute the integral!



$$3.\ A = \int_0^4 x^2\,dx = \frac{x^3}{3}\Big|_0^4 = \frac{64}{3}$$

$$4.\ V_x = \int_0^4 \pi (x^2)^2\,dx = \int_0^4 \pi x^4\,dx$$

$$\pi \frac{x^5}{5}\Big|_0^4 = \frac{1024\pi}{5}$$

$y = x^2 \Leftrightarrow x = \sqrt{y}$

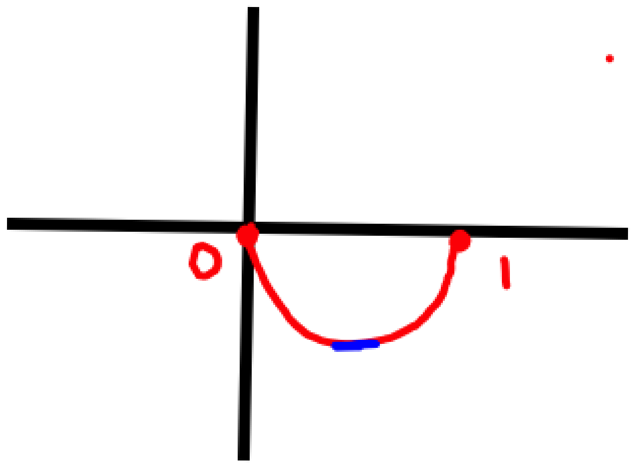
5. Washer: $r = \sqrt{y}$ $R = 4$

$$V_y = \int_0^{16} \pi\left[4^2 - \sqrt{y}^{\,2}\right] dy$$

Shell: $V_y = \int_0^4 2\pi x\,[x^2]\,dx$

7. Given F(x) for each problem, graph the function and shade the area between F(x) and the x-axis, find the x-coordinate of the centroid of the shaded region and find the y-coordinate of the centroid of the shaded region.

a. $F(x) = x^2 - x$



$$A = \int_0^1 \left(0 - (x^2 - x)\right) dx$$

$$-\frac{x^3}{3} + \frac{x^2}{2} \Big|_0^1 = \frac{-1}{3} + \frac{1}{2} = \frac{1}{6}$$

$$\bar{x} = \int_0^1 x\left(0 - (x^2 - x)\right) dx \Big/ A$$

$$= \int_0^1 (-x^3 + x^2)\, dx \Big/ A$$

$$= \frac{-x^4/4 + x^3/3 \Big|_0^1}{A} = \left(-\frac{1}{4} + \frac{1}{3}\right)6 = \frac{1}{2}$$

$$\bar{y} = \int_0^1 (x^2 - x)\left(0 - (x^2 - x)\right) dx \Big/ A$$

9. Give an equation relating x and y for the curve given parametrically by

a. $x(t) = -1 + 3\tan t \quad y(t) = 1 + 2\sec t$

b. $x(t) = 2e^{t} \quad y(t) = 1 - 3e^{-2t}$

$$e^{-2t} = (e^{t})^{-2}$$

a. $1 + \tan^2 t = \sec^2 t$

$$\frac{x+1}{3} = \tan t \qquad \frac{y-1}{2} = \sec t$$

$$1 + \left(\frac{x+1}{3}\right)^2 = \left(\frac{y-1}{2}\right)^2$$

b. $e^{t} = x/2 \rightarrow y = 1 - 3(x/2)^{-2}$

q. $-\frac{1}{3}\int \frac{-3\sin(3x)}{16+\cos^2(3x)}\,dx$ $\rightarrow$

$u = \cos(3x)$

$du = -3\sin(3x)\,dx$

$$-\frac{1}{3}\int \frac{du}{4^2+u^2}$$

r. $\int \frac{6x}{4+x^4}\,dx$

$$\int \frac{3(2x\,dx)}{2^2+(x^2)^2}$$

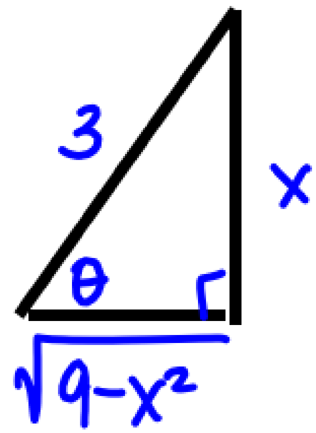
$$-\frac{1}{3}\left(\frac{1}{4}\arctan\left(\frac{u}{4}\right)\right)+C$$

$u = x^2$ $\quad du = 2x\,dx$

$$\int \frac{3\,du}{2^2+u^2} = 3\cdot\frac{1}{2}\arctan\left(\frac{u}{2}\right)+C$$

v. $\int \sqrt{9 - x^2}\, dx$

$$\int 3\cos\theta \cdot 3\cos\theta \, d\theta$$



$$\frac{x}{3} = \sin\theta \Leftarrow *$$

$$x = 3\sin\theta$$

$$dx = 3\cos\theta \, d\theta$$

$$\frac{\sqrt{9-x^2}}{3} = \cos\theta$$

$$\sqrt{9-x^2} = 3\cos\theta$$

$$9\int \cos^2\theta \, d\theta$$

$$9\left(\frac{1}{2}\underset{*}{\underline{\underline{\theta}}} + \frac{1}{2}\sin\theta\cos\theta\right) + C$$

$$\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + \frac{\cancel{9}}{2}\left(\frac{x}{\cancel{3}}\right)\left(\frac{\sqrt{9-x^2}}{\cancel{3}}\right) + C$$

w. $\int 3\ln(4x)dx$

$u = 3\ln(4x)$ $\quad dv = dx$

$\left(du = 3\cdot\frac{4}{4x}dx \quad v = \int dv = \int dx \right)$

$du = \frac{3}{x}\cdot dx \quad v = x$

$uv - \int v\,du$

$3\ln(4x)\cdot x - \int \cancel{x}\cdot\frac{3}{\cancel{x}}\,dx$

$$\boxed{3x\ln(4x) - 3x + C}$$

c. $\left\{\frac{n!}{(n+2)!}\right\} = \left\{\frac{n!}{(n+2)(n+1)n!}\right\} = \left\{\frac{1}{(n+2)(n+1)}\right\} \to 0$

Converges to 0

i. $\sum_{n=0}^{\infty}\left(\frac{(-1)^n 3}{(n+2)\ln(n+2)}\right)$

alt + $\frac{3}{(n+2)(\ln(n+2))} \to 0$

Conditionally Converges

abs? NO

$\sum \frac{3}{(n+2)\ln(n+2)}$

$\int_0^{\infty} \frac{3}{(x+2)\ln(x+2)}\,dx$

$u = \ln(x+2)$

$du = \frac{1}{x+2}$

$= \lim_{b\to\infty} \int_0^b \frac{3}{(x+2)\ln(x+2)}\,dx$

$\int \frac{3}{u}\,du$

$3\ln u$

$= \lim_{b\to\infty} 3\ln(\ln(x+2))\Big|_0^b \to \infty$ div.

$\sum \frac{1}{\ln n}$ div. by Basic comp to $\sum \frac{1}{n}$

$\frac{1}{\ln n} > \frac{1}{n}$

$\sum \frac{1}{n \ln n}$ use integral test

$\sum \frac{1}{n(\ln n)^2}$

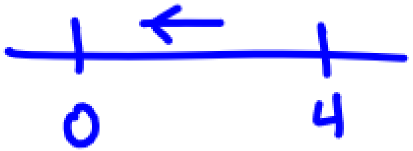
b. $\sum_{n=0}^{\infty}\left(\frac{1}{3^n}-\frac{5}{6^n}\right)$

$S = \frac{a_1}{1-r}$

$\sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n - 5\sum_{n=0}^{\infty}\left(\frac{1}{6}\right)^n$

$\frac{1}{1-1/3} - 5\left(\frac{1}{1-1/6}\right)$

b. $\displaystyle\int_0^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$



$$\lim_{a\to 0^+} 2\int_a^4 \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx$$

$u=\sqrt{x}$

$du=\dfrac{1}{2\sqrt{x}}\,dx$

$2\int e^u\,du = 2e^u$

$$\lim_{a\to 0^+} 2e^{\sqrt{x}}\Big|_a^4 = \lim_{a\to 0^+}\left(2e^2 - 2e^{\sqrt{a}}\right)$$

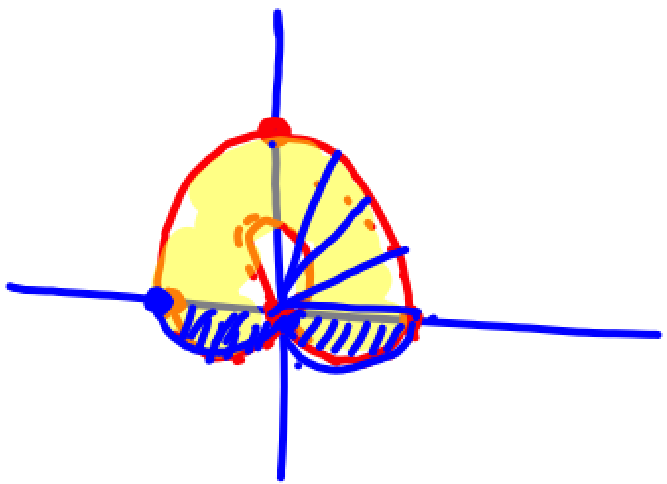
$$= \boxed{2e^2-2}$$

17. Find the formula for the area of $r = 1 + 2\sin\theta$

a. Inside inner loop $\int_{7\pi/6}^{11\pi/6} \frac{1}{2}(1+2\sin\theta)^2\,d\theta$

b. Inside outer loop but outside inner loop

c. Inside outer loop and below x-axis $2\int_{\pi}^{7\pi/6} \frac{1}{2}(1+2\sin\theta)^2\,d\theta$



θ	r
0	1
$\frac{\pi}{2}$	$1+2=3$
π	1
$\frac{7\pi}{6}$	0
$\frac{11\pi}{6}$	0
2π	1

$0 = 1 + 2\sin\theta$

$\sin\theta = -\frac{1}{2}$

b. $\int_0^{2\pi} \frac{1}{2}(1+2\sin\theta)^2\,d\theta$

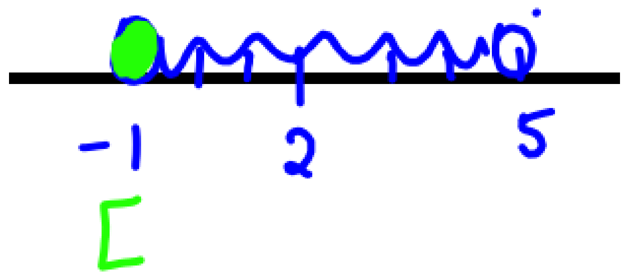
$- 2\int_{7\pi/6}^{11\pi/6} \frac{1}{2}(1+2\sin\theta)^2\,d\theta$

$[-1, 5)$

* a. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{(n+1)3^{n+1}}$

Ratio:

$$\frac{|x-2|^{\cancel{n+2}}}{(n+2)\,3^{\cancel{n+2}}} \cdot \frac{(n+1)\cancel{3^{n+1}}}{\cancel{|x-2|^{n+1}}}$$



$$\lim_{n\to\infty} \frac{|x-2|}{3}\left(\frac{n+1}{n+2}\right) = \frac{|x-2|}{3} < 1$$

$\frac{n+1}{n+2} \to 1$

$|x-2| < \underline{3} = R$

$x = -1$:

$$\sum \frac{(-1-2)^{n+1}}{(n+1)\,3^{n+1}} = \sum \frac{(-1)^{n+1}\,\cancel{3^{n+1}}}{(n+1)\,\cancel{3^{n+1}}}$$

Conv

$x = 5$:

$$\sum \frac{(5-2)^{n+1}}{(n+1)\,3^{n+1}} = \sum \frac{1}{n+1} \quad \text{div}$$

$$(-a)^n = (-1)^n a^n$$