Math 1432

Exam 2 Review

1. Find the average value of each function over the given interval:
   a. $f(x) = 3x^2 - x, \quad [-1,4]$
   b. $f(x) = \sqrt{x}, \quad [1,9]$
   c. $f(x)$ is positive with an area of $\frac{19}{3}$ for the region encloses between $f(x)$, the x - axis between $x = 1$ and $x = 5$.
   d. $f(x)$ is an odd function where $\int_{-2}^{0} f(x)dx = -4$ on $[-2,2]$
2. Given F(x) and the interval [a, b], graph F(x) over the interval, find the average value of F(x) on that interval and find the value of *c* that verifies the conclusion of the mean value theorem for integrals for the function *F* over the interval [a, b].
   a. $F(x) = x^2 - x \quad [0,1]$
   b. $F(x) = x^2 + 3x \quad [-3,0]$
   c. $F(x) = x^2 - 4 \quad [-2,2]$
3. Find the area for each region:
   a. between $f(x) = x^3 - x^2$ and the *x*-axis on the interval [0,2].
   b. between $y = x^2 - 1$ and $y = 3$
   c. between $f(x) = \sqrt{x}$ and $g(x) = \frac{x}{a}$
   d. between $x^2 + y^2 = 9, x = y^2$ and the x-axis in the first quadrant (set up the formula only)
4. Find the coordinates of the centroid for each region given in problem 3 (a-c only).
5. Find the area of the region bounded by x – axis, the line $x = 4$ and the graph of $f(x) = x^2$.
6. Revolve the region in problem 5 about the x-axis, and **give the integral resulting from using the method of washers** to find its volume. Do not compute the integral!
7. Revolve the region in problem 5 about the y-axis, and **give the integral resulting from using the method of cylindrical shells** to find its volume. Do not compute the integral!
8. Derive the formula $V = \frac{4}{3}\pi r^3$ for the volume of a sphere of radius *r* by revolving the region bounded by a circle of radius *r*, centered at the origin, around either the *x* axis or the *y* axis.
9. R is the region bounded by the given graphs and the given axis. Sketch each graph then find the area of R, the volume when R is revolved about the x-axis and the volume when R is revolved about the y-axis
   a. $y = x^2, y = 6 - x, \ x - axis$
   b. $y = x^2, y = 6 - x, \ y - axis$
10. Give the formula to find the length of the curve, f, over the given interval.
    a. $f(x) = \frac{2}{3}(x-1)^{3/2}, \quad x \in [1,2]$
    b. $f(x) = \cosh 3x, \quad x \in [0, \ln 2]$
    c. $f(x) = \arcsin x, \quad x \in [0, 1/2]$
11. Give the formula for the surface area for each part in #10 when revolved about the *x* –axis.

12. The base of a solid is the region bounded by $y = 2x^2$ and $y = 8$. Find the volume of the solid given that the cross sections perpendicular to the $x-$axis are:
    a. Squares
    b. Semicircles
    c. Right triangles with leg on the xy-plane.
13. Find the general solution for each:
    a. $\frac{dy}{dx} = (y+5)(x+2)$
    b. $y' = \frac{e^x}{y}$
    c. $y' = xy - y$
    d. $\frac{dy}{dx} = x^2 \sec(y)$
    e. $y' = e^{2x}(1+y^2)$
14. Find the specific solution given the initial condition: $\frac{dy}{dx} = y - 2 \quad y(0) = 6$
15. The number N of bacteria in a culture is given by $N = 200e^{kt}$. If N = 300 when t = 4 hours, find k (to the nearest tenth) and then determine approximately how long it will take for the number of bacteria to triple in size.
16. Suppose that the population of Zeegers grows at a rate proportional to itself, doubling every 12500 years. When the Zeeger population has reached 93 percent more than their current population, they plan to invade Earth. How many years will it be before the Zeegers attack Earth?
17. At what rate *r* of continuous compounding does a sum of money double in 15 years?
18. A population P of insects increases at a rate proportional to the current population. After 1 month, the population has increased by 30%. If there are 500 insects initially, write an expression for the number of insects at any time *t*.
19. Identify which of the following are improper. If the integral is improper, re-write it in proper limit notation and evaluate.
    a. $\int_0^{27} x^{-2/3}\, dx$
    b. $\int_0^1 \frac{1}{e^x}\, dx$
    c. $\int_0^4 \frac{1}{\sqrt{4-x}}\, dx$
    d. $\int_e^{\infty} \frac{\ln x}{x}\, dx$
    e. $\int_{-1}^0 \frac{1}{1+x^2}\, dx$
    f. $\int_0^{\infty} e^{-2t}\, dt$