# Math 1432
## Exam 3 Review **KEY**

1. a. $\int \frac{3x^2+3x+3}{x^2+1}\,dx = 3x + \frac{3}{2}\ln(x^2+1) + C$

   b. $\int \frac{x^2}{(x+1)(x-1)^2}\,dx = \frac{1}{4}\ln|x+1| + \frac{3}{4}\ln|x-1| - \frac{1}{2(x-1)} + C$

   c. $\int \frac{x^2+5x+2}{(x+1)(x^2+1)}\,dx = -\ln|x+1| + \ln(x^2+1) + 3\arctan(x) + C$

   d. $\int \frac{2x^2}{\sqrt{9-x^2}}\,dx = 9\arcsin\left(\frac{x}{3}\right) - x\sqrt{9-x^2} + C$

   e. $\int \frac{2}{x\sqrt{9+x^2}}\,dx = \frac{2}{3}\ln\left|\frac{\sqrt{9+x^2}}{x} - \frac{3}{x}\right| + C$

   f. $\int \frac{5}{36+(x-1)^2}\,dx = \frac{5}{6}\arctan\left(\frac{x-1}{6}\right) + C$

   g. $\int \frac{1}{\sqrt{4+x^2}}\,dx = \ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C$

   h. $\int \frac{5x+14}{(x+1)(x^2-4)}\,dx = 2\ln|x-2| - 3\ln|x+1| + \ln|x+2| + C$

   i. $\int_0^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}}\,dx = \frac{\pi}{3}$

   j. $\int \cos^4 x \sin^3 x\,dx = -\frac{1}{5}\cos^5 x + \frac{1}{7}\cos^7 x + C$

   k. $\int \cos^5 x \sin^2 x\,dx = \frac{1}{3}\sin^3 x - \frac{2}{5}\sin^5 x + \frac{1}{7}\sin^7 x + C$

   l. $\int \cot^3 x\,dx = -\frac{1}{2}\cot^2 x - \ln|\sin x| + C$

   m. $\int x\ln(2x)\,dx = \frac{1}{2}x^2\ln(2x) - \frac{1}{4}x^2 + C$

   n. $\int 2x\sin(3x)\,dx = -\frac{2}{3}x\cos(3x) + \frac{2}{9}\sin(3x) + C$

   o. $\int \frac{5}{36+(x-1)^2}\,dx = \frac{5}{6}\arctan\left(\frac{x-1}{6}\right) + C$

   p. $\int \tan^4(x)\,dx = \frac{1}{3}\tan^3 x - \tan x + x + C$

   q. $\int 2x\sec\left(4x^2\right)dx = \frac{1}{4}\ln\left|\sec(4x^2) + \tan(4x^2)\right| + C$

   r. $\int \sec^4(x)\,dx = \frac{1}{3}\tan^3 x + \tan x + C$

2. Write an expression for the nth term of the sequence:

   a. 1, 4, 7, 10, ... 3n-2

   b. 2, -1, $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, \ldots$ $(-1)^{n+1}(2)^{2-n}$

3. Determine if the following sequences are monotonic. Also indicate if the sequence is bounded and if it is give the least upper bound and/or greatest lower bound.

   Incr. ⇒ monotonic

   a. $a_n = \frac{2n}{1+n}$ glb = 1 lub = 2

   b. $a_n = \frac{\cos n}{n}$ not monotonic glb = $\frac{\cos(3)}{3}$ lub = cos(1)

4. Determine if the following sequences converge or diverge. If they converge, give the limit.

   a. $\left\{(-1)^n\left(\frac{n}{n+1}\right)\right\}$ diverges

   b. $\left\{\frac{6n^2-2n+1}{4n^2-1}\right\}$ converges to 3/2

   c. $\left\{\frac{(n+2)!}{n!}\right\}$ diverges

   d. $\left\{\frac{3}{e^n}\right\}$ converges to 0

   e. $\left\{\frac{4n+1}{n^2-3n}\right\}$ converges to 0

   f. $\left\{\frac{e^n}{n^3}\right\}$ diverges

5. Determine the values of *n* which guarantee a theoretical error less than ε if the integral is estimated by the trapezoidal rule if ε = 0.01.

   a. $\int_1^3 (\frac{1}{4}x^2 + 3x - 2)dx$ : $f(x) = \frac{1}{4}x^2 + 3x - 2 \Rightarrow f''(x) = \frac{1}{2} \Rightarrow M = \frac{1}{2}$

$$|E_n^T| = \frac{(b-a)^3}{12n^2}M$$

$$0.01 > \frac{(3-1)^3}{12n^2}\left(\frac{1}{2}\right)$$

$$n^2 > \frac{100}{3}$$

$$n > 5.77$$

$$n \geq 6$$

$f(x)=\frac{1}{4}x^2+3x-2 \Rightarrow f^{(4)}(x)=0 \Rightarrow |E_n^S|=0$ for all $n$

$n \ge 2$

b. $\int_1^3 \cos(5x)dx$ : $f(x)=\cos(5x) \Rightarrow f''(x)=-25\cos(5x) \Rightarrow M=25$

$$|E_n^T| = \frac{(b-a)^3}{12n^2}M$$

$$0.01 > \frac{(3-1)^3}{12n^2}(25)$$

$$n^2 > 1666.67$$

$$n > 40.82$$

$$n \ge 41$$

$$f(x)=\cos(5x) \Rightarrow f^{(4)}(x)=625\cos(5x) \Rightarrow M=625$$

$$|E_n^S| = \frac{(b-a)^5}{180n^4}M$$

$$0.01 > \frac{(3-1)^5}{180n^4}(625)$$

$$n^4 > 11111.11$$

$$n \ge 12$$

** Also have a general understanding of the different methods described in section 8.5

6. The series $4-3+\frac{9}{4}-\frac{27}{16}+\ldots$ is a geometric series. Find the general term, $a_n$, and write the sum in sigma notation. $\sum_{k=0}^{\infty}\frac{(-1)^k 3^k}{4^{k-1}}$ Does this series converge? yes If so, what is the sum? $\frac{16}{7}$

7. Find the sum of the following (if possible):

a. $\sum_{k=0}^{\infty}\left(-\frac{3}{4}\right)^k$ $\frac{4}{7}$

b. $\sum_{k=2}^{\infty}\left(\frac{2}{3}\right)^k$ $\frac{4}{3}$

c. $\sum_{k=0}^{\infty}\left(\frac{5}{4}\right)^{k-1}$ diverges

d. $\sum_{n=2}^{\infty}\left(\frac{1}{n}-\frac{1}{n+2}\right)$ $\frac{5}{6}$

e. $\sum_{k=0}^{\infty} \frac{6^{k+1}}{7^{k-2}}$ 2058

8. Determine whether the given series converges or diverges; state which test you are using to determine convergence/divergence and show all work.

a. $\sum \frac{k^2 2^k}{(k+1)!}$ converge, ratio test

b. $\sum \frac{3^{k+1}}{(k+1)^2 e^k}$ diverges; root or ratio test.

c. $\sum \frac{\ln n}{n}$ diverges; basic comparison test or integral test

d. $\sum \frac{2n+1}{\sqrt{n^5+3n^3+1}}$ converges; limit comparison to $\sum \frac{1}{n^{3/2}}$

e. $\sum \frac{4n^2+1}{n^3-n}$ diverges; limit comparison to $\sum \frac{1}{n}$

f. $\sum \frac{4n^2+1}{n^5-n}$ converges; limit comparison to $\sum \frac{1}{n^3}$

g. $\sum \left(1+\frac{1}{n}\right)^n$ diverges; BDT

h. $\sum \frac{n^3}{3^n}$ converges; root or ratio test

i. $\sum \frac{1}{\sqrt[4]{n^3}}$ diverges; p-series