# Math 1432
## Exam 4 Review

1. Determine whether the given series converges or diverges; state which test you are using to determine convergence/divergence and show all work.
   a. $\sum \frac{k^2 2^k}{(k+1)!}$
   b. $\sum \frac{3^{k+1}}{(k+1)^2 e^k}$
   c. $\sum \frac{\ln n}{n}$
   d. $\sum \frac{2n+1}{\sqrt{n^5+3n^3+1}}$
   e. $\sum \frac{4n^2+1}{n^3-n}$
   f. $\sum \frac{4n^2+1}{n^5-n}$
   g. $\sum \left(1+\frac{1}{n}\right)^n$
   h. $\sum \frac{n^3}{3^n}$
   i. $\sum \frac{1}{\sqrt[4]{n^3}}$
2. Determine if the following series (A) converge absolutely, (B) converge conditionally or (C) diverge.
   a. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+3}$
   b. $\sum_{n=1}^{\infty} \frac{\cos \pi n}{n^2}$
   c. $\sum_{n=0}^{\infty} \frac{4n(-1)^n}{3n^2+2n+1}$
   d. $\sum_{n=0}^{\infty} \frac{3(-1)^n}{\sqrt{3n^2+2n+1}}$
   e. $\sum_{n=0}^{\infty} \frac{3n(-1)^n}{\sqrt{3n^2+2n+1}}$

3. Find the radius of convergence and interval of convergence for the following Power series:
   a. $\sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{(n+1)3^{n+1}}$
   b. $\sum_{n=0}^{\infty} \frac{1}{3^n}(x-1)^n$
   c. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{4^n}$
   d. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n n!}{n^n}$
4. Give the derivative of each power series below:
   a. $\sum_{n=0}^{\infty} \frac{(n+1)x^n}{n^2+2}$
   b. $\sum_{n=0}^{\infty} \frac{x^n}{2n+1}$
5. For each of the problems in number 4, give the antiderivative F of the power series so that F(0)=0.
6. Use the Taylor series expansion (in powers of x) for $f(x)=e^x$ to find the Taylor series expansion $f(x)=\cosh x$.
7. Determine the Taylor polynomial in powers of $x$ of degree 8 for the function $f(x)=x-\cos(x^2)$.
8. Determine the Taylor polynomial in powers of $x$ of degree 5 for the function $f(x)=\frac{1-e^x}{x}$
9. Determine the Taylor polynomial in powers of $x$-$\pi$ of degree 4 for the function $f(x)=\sin(2x)$.
10. Assume that $f$ is a function such that $|f^{(n)}(x)| \le 2$ for all $n$ and $x$.
    a. Estimate the maximum possible error if $P_4(0.5)$ is used to approximate $f(0.5)$
    b. Find the least integer $n$ for which $P_n(0.5)$ approximates $f(0.5)$ with an error less than $10^{-3}$.
11. Use the values in the table below and the formula for Taylor polynomials to give the 5th degree Taylor polynomial for $f$ centered at $x = 0$.

| $f(0)$ | $f'(0)$ | $f''(0)$ | $f'''(0)$ | $f^{(4)}(0)$ | $f^{(5)}(0)$ |
|---|---|---|---|---|---|
| 1 | 0 | -2 | 3 | -4 | 1 |

12. Write the equation in polar coordinates:
    a. $x^2 + y^2 = 4$
    b. $x^2 + y^2 = 4x$
    c. $\left(x^2 + y^2\right)^2 = 4xy$
    d. $x = 4y$

13. Write the given equations in rectangular coordinates:
    a. $r = -2\sin\theta$
    b. $r\cos\theta = 5$

14. Recognize all types of polar graphs.

15. Given $r = 4 - 8\cos\theta$, give the formula (only) for the area inside the inner loop.

16. Given $r = 2\sin(3\theta)$, give the formula (only) for the area of one petal.

17. Find the arc length for $r = 2\sec(\theta), \quad \theta \in \left[0, \frac{\pi}{4}\right]$

18. Do the following problems from section 10.3: #7,9,11,15,43,49