Math 1432

Exam 4 Review - KEY

1. Determine whether the given series converges or diverges; state which test you are using to determine convergence/divergence and show all work.
   a. $\sum \frac{k^2 2^k}{(k+1)!}$ converge, ratio test

   b. $\sum \frac{3^{k+1}}{(k+1)^2 e^k}$ diverges; root or ratio test.

   c. $\sum \frac{\ln n}{n}$ diverges; basic comparison test or integral test

   d. $\sum \frac{2n+1}{\sqrt{n^5+3n^3+1}}$ converges; limit comparison to $\sum \frac{1}{n^{3/2}}$

   e. $\sum \frac{4n^2+1}{n^3-n}$ diverges; limit comparison to $\sum \frac{1}{n}$

   f. $\sum \frac{4n^2+1}{n^5-n}$ converges; limit comparison to $\sum \frac{1}{n^3}$

   g. $\sum \left(1+\frac{1}{n}\right)^n$ diverges; BDT

   h. $\sum \frac{n^3}{3^n}$ converges; root or ratio test

   i. $\sum \frac{1}{\sqrt[4]{n^3}}$ diverges; p-series

2. Determine if the following series (A) converge absolutely, (B) converge conditionally or (C) diverge.
   a. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+3}$ B

   b. $\sum_{n=1}^{\infty} \frac{\cos \pi n}{n^2}$ A

   c. $\sum_{n=0}^{\infty} \frac{4n(-1)^n}{3n^2+2n+1}$ B

   d. $\sum_{n=0}^{\infty} \frac{3(-1)^n}{\sqrt{3n^2+2n+1}}$ B

   e. $\sum_{n=0}^{\infty} \frac{3n(-1)^n}{\sqrt{3n^2+2n+1}}$ C

3. Find the radius of convergence and interval of convergence for the following Power series:

   a. $\sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{(n+1)3^{n+1}}$ $R=3,[-1,5)$

   b. $\sum_{n=0}^{\infty} \frac{1}{3^n}(x-1)^n$ $R=3,(-2,4)$

   c. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{4^n}$ $R=4,(-4,4)$

   d. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n n!}{n^n}$ $R=e,(-e,e)$

4. Give the derivative of each power series below:

   a. $\sum_{n=0}^{\infty} \frac{(n+1)x^n}{n^2+2}$ $deriv=\sum_{n=0}^{\infty} \frac{n(n+1)x^{n-1}}{n^2+2}$

   b. $\sum_{n=0}^{\infty} \frac{x^n}{2n+1}$ $deriv=\sum_{n=0}^{\infty} \frac{nx^{n-1}}{2n+1}$

5. For each of the problems in number 4, give the antiderivative F of the power series so that F(0)=0.

   a. $\sum_{n=0}^{\infty} \frac{x^{n+1}}{n^2+2}$

   b. $\sum_{n=0}^{\infty} \frac{x^{n+1}}{(2n+1)(n+1)}$

6. Use the Taylor series expansion (in powers of x) for $f(x)=e^x$ to find the Taylor series expansion $f(x)=\cosh x$. $1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\ldots=\sum_{k=0}^{n} \frac{x^{2k}}{(2k)!}$

7. Determine the Taylor polynomial in powers of x of degree 8 for the function $f(x)=x-\cos(x^2)$. $-1+x+\frac{x^4}{2!}-\frac{x^8}{4!}$

8. Determine the Taylor polynomial in powers of x of degree 5 for the function $f(x)=\frac{1-e^x}{x}$ $-1-\frac{x}{2!}-\frac{x^2}{3!}-\frac{x^3}{4!}-\frac{x^4}{5!}-\frac{x^5}{6!}$

9. Determine the Taylor polynomial in powers of $x-\pi$ of degree 4 for the function $f(x)=\sin(2x)$. $2(x-\pi)-\frac{4}{3}(x-\pi)^3$

10. Assume that f is a function such that $|f^{(n)}(x)|\le 2$ for all n and x.

    a. Estimate the maximum possible error if $P_4(0.5)$ is used to approximate $f(0.5)$ $\frac{2}{5!}(.5)^5 \approx 0.00052$

b. Find the least integer n for which $P_n(0.5)$ approximates $f(0.5)$ with an error less than 10^{-3}. $n=4$

11. Use the values in the table below and the formula for Taylor polynomials to give the 5th degree Taylor polynomial for f centered at $x = 0$.

$f(0)$	$f'(0)$	$f''(0)$	$f'''(0)$	$f^{(4)}(0)$	$f^{(5)}(0)$
1	0	-2	3	-4	1

$$P_5(x)=1-\frac{2}{2!}x^2+\frac{3}{3!}x^3-\frac{4}{4!}x^4+\frac{1}{5!}x^5=1-x^2+\frac{1}{2}x^3-\frac{1}{6}x^4+\frac{1}{120}x^5$$

12. a. $r=2$ b. $r=4\cos\theta$ c. $r^2=4\sin\theta\cos\theta$ *or* $r^2=2\sin(2\theta)$ d. $\cot\theta=4$

13. a. $x^2+y^2=-2y$ b. $x=5$

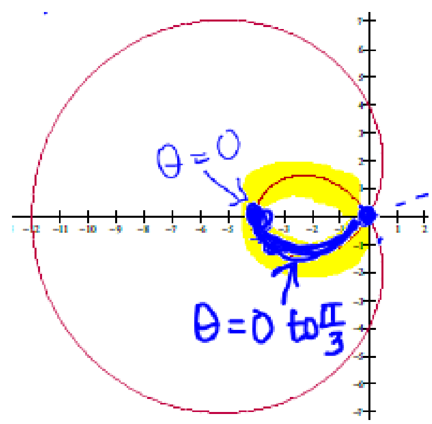
14.

15. Given $r=4-8\cos\theta$, give the formula (only) for the area inside the inner loop.

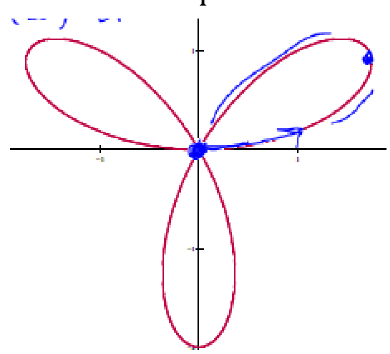
$$2\int_0^{\pi/3}\tfrac{1}{2}(4-8\cos\theta)^2\,d\theta$$

or

$$\int_0^{\pi/3}\tfrac{1}{2}(4-8\cos\theta)^2\,d\theta+\int_{5\pi/3}^{2\pi}\tfrac{1}{2}(4-8\cos\theta)^2\,d\theta$$



16. Given $r=2\sin(3\theta)$, give the formula (only) for the area of one petal.



$$A=\int_0^{\pi/3}\frac{1}{2}\left(2\sin(3\theta)\right)^2 d\theta$$

17. Find the arc length for $r = 2\sec(\theta), \quad \theta \in \left[0, \frac{\pi}{4}\right]$

$$\int_0^{\pi/4} \sqrt{(2\sec\theta)^2 + (2\sec\theta\tan\theta)^2}\, d\theta = 2$$