Math 1432

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Office Hours (starting Monday):

Mondays 1-2pm,
Fridays noon-1pm
(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

Reminders:

- Access Code (before this weekend)
- Poppers (start next week)
- Practice Test 1
- Test 1
- Quizzes

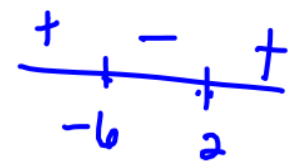
Section 7.3 - Area

So, we know that if $f(x)$ is on or above the x-axis over a region from $x = a$ to $x = b$ then

$$A = \int_a^b f(x)\,dx$$

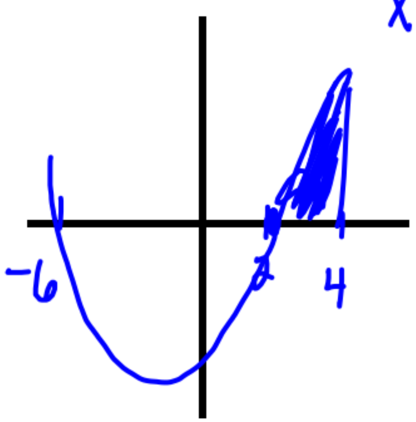
Example:

Find the area of the region: $f(x) = x^2 + 4x - 12, \quad x \in [2,4]$



$x^2+4x-12 = (x+6)(x-2) = 0$

$x = -6, 2$

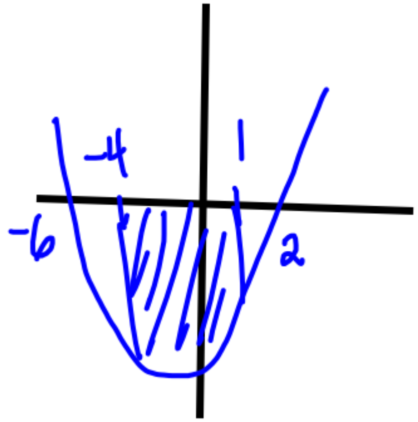


$$\int_2^4 (x^2+4x-12)\,dx = \frac{x^3}{3} + 2x^2 - 12x \Big|_2^4$$

$$= \left(\frac{64}{3} + 32 - 48\right) - \left(\frac{8}{3} + 8 - 24\right)$$

$$\frac{64}{3} - \frac{8}{3} + 32 - 48 - 8 + 24 = \boxed{\frac{56}{3}}$$

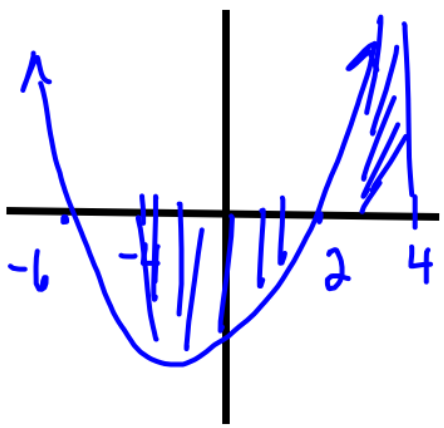
What about when $f(x)$ is below the x-axis?



Same $f(x)$ but $[-4, 1]$

$$\left| \int_{-4}^{1} (x^2+4x-12)\, dx \right| = A$$

Or part above and part below?

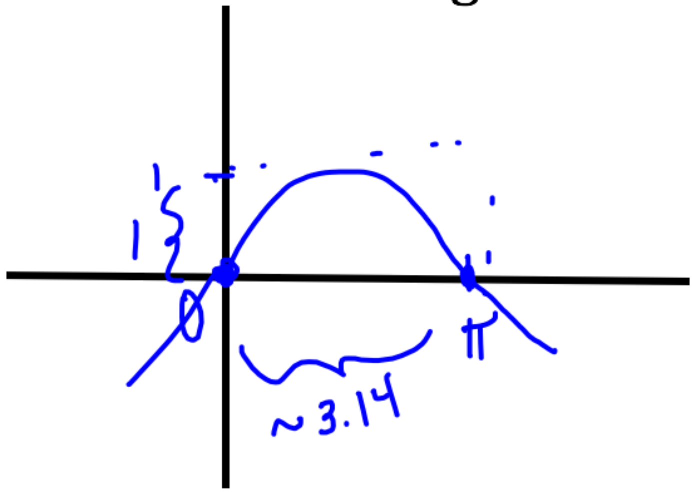


Same $f(x)$ but $[-4, 4]$

$$A = \left| \int_{-4}^{2} (x^2+4x-12)\, dx \right| + \int_{2}^{4} (x^2+4x-12)\, dx$$

Example:

Find the area of the region bounded by the graph of $f(x) = \sin x, x \in [0, \pi]$



$$\int_0^{\pi} \sin x \, dx$$

$$= -\cos x \Big|_0^{\pi}$$

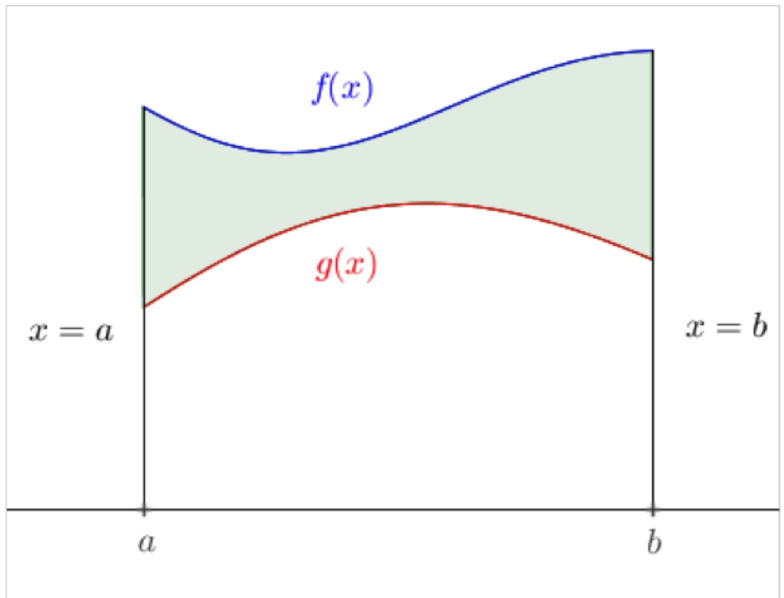
$$= -\cos \pi - {-\cos 0}$$

$$= -(-1) + 1$$

$$= \boxed{2}$$

The area between two curves is found by integrating the difference of the larger function minus the smaller function over the region.

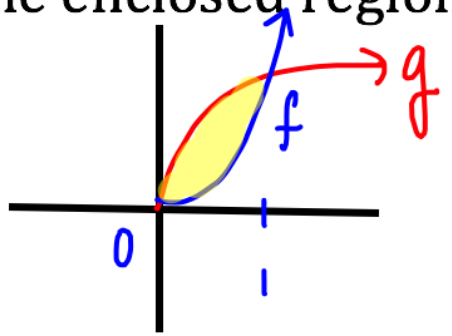
$$A=\int_a^b\left[\overset{\text{top}}{f(x)}-\overset{\text{bottom}}{g(x)}\right]dx.$$



← y=0

Sketch the region bounded by the graphs of the equations and determine the area of the enclosed region.

$f(x) = x^2$ and $g(x) = \sqrt{x}$



$$x^2 = \sqrt{x}$$

$$x^2 - x^{1/2} = 0$$

$$x^{1/2} \cdot (x^{3/2} - 1) = 0$$

$$x^{1/2} = 0 \qquad x^{3/2} = 1$$

$$\underline{x = 0} \qquad \underline{x = 1}$$

$$f(1/4) = 1/16$$

$$g(1/4) = 1/2 \leftarrow \text{top}$$

$$A = \int_0^1 (\sqrt{x} - x^2)\,dx = \int_0^1 (x^{1/2} - x^2)\,dx$$

$$= \frac{2}{3}x^{3/2} - \frac{x^3}{3} \Big|_0^1$$

$$= \left(2/3 - 1/3\right) - (0 - 0)$$

$$= \boxed{1/3}$$

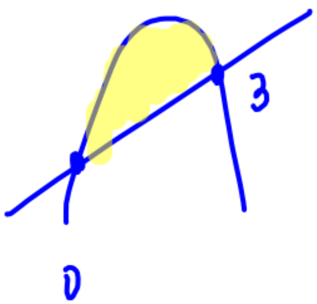
$$f(x) = -x^2 + 4x + 2, \quad g(x) = x + 2$$

$$-x^2 + 4x + 2 = x + 2$$

$$-x^2 + 3x = 0$$

$$x(-x + 3) = 0$$

$$x = 0, 3$$



$$A = \int_0^3 \left[(-x^2 + 4x + 2) - (x + 2) \right] dx$$

$$= \int_0^3 (-x^2 + 3x)\, dx$$

$$= -\frac{x^3}{3} + \frac{3x^2}{2} \Bigg|_0^3$$

$$-18/2$$

$$= \left(-9 + \frac{27}{2} \right) - (0 + 0)$$

$$= \boxed{9/2}$$

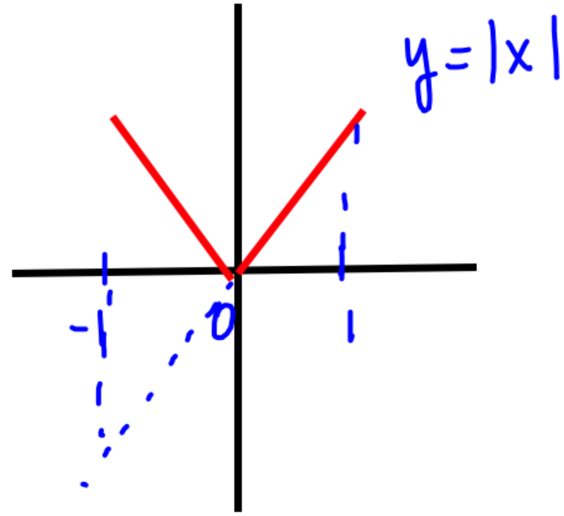
Examples with absolute value

$$\int_{-1}^{1} |x|\, dx = \int_{-1}^{0} -x\, dx + \int_{0}^{1} x\, dx$$

$$\left.\frac{-x^2}{2}\right|_{-1}^{0} + \left.\frac{x^2}{2}\right|_{0}^{1}$$

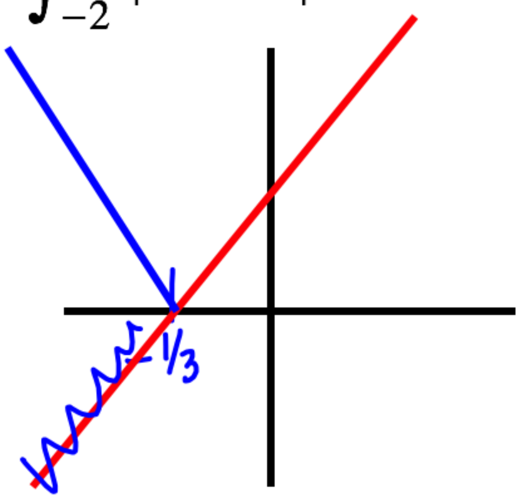
$$0 - \left(-\tfrac{1}{2}\right) + \left(\tfrac{1}{2} - 0\right)$$

$$\tfrac{1}{2} + \tfrac{1}{2} = \boxed{1}$$



$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$\int_{-2}^{2} |3x+1|\, dx =$$

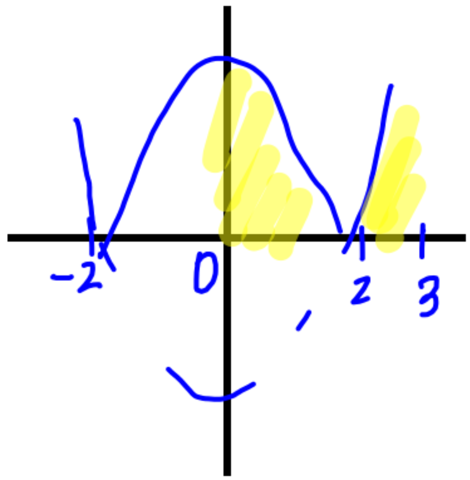


$$3x+1=0$$
$$x = -1/3$$

x int

$$\int_{-2}^{-1/3} (-3x-1)\, dx + \int_{-1/3}^{2} (3x+1)\, dx$$

$$\int_0^3 \left| x^2 - 4 \right| dx =$$

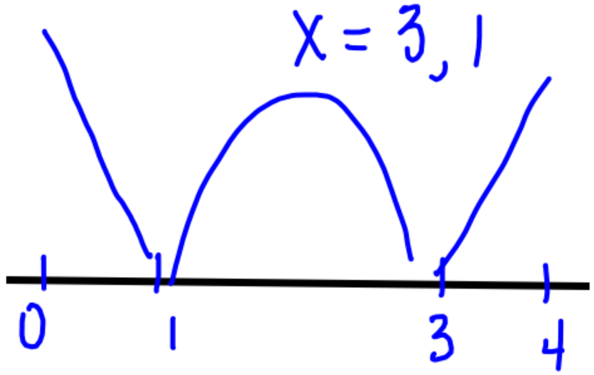


$$\int_0^2 -(x^2-4)\,dx + \int_2^3 (x^2-4)\,dx$$

$$\int_0^4 \left| x^2 - 4x + 3 \right| dx =$$

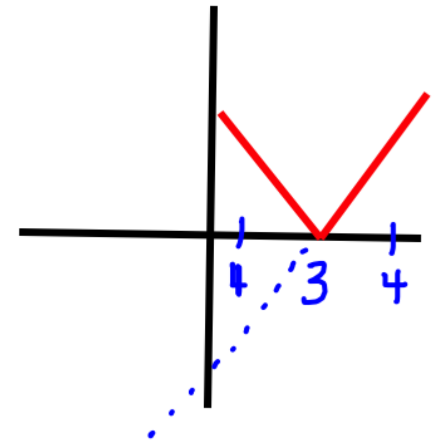
$$x^2 - 4x + 3 = (x-3)(x-1) = 0$$

$$x = 3, 1$$



$$\int_0^1 (x^2-4x+3)\,dx + \int_1^3 -(x^2-4x+3)\,dx + \int_3^4 (x^2-4x+3)\,dx$$

$$\int_1^4 (3-|x-3|)\,dx = \underbrace{\int_1^4 3\,dx}_{} - \underbrace{\int_1^4 |x-3|\,dx}_{}$$



$$3x\Big|_1^4$$

$$12-3=9$$

$$9-\left[\int_1^3 -(x-3)\,dx + \int_3^4 (x-3)\,dx\right]$$

$$-x+3$$

$$-\frac{x^2}{2}+3x\Big|_1^3 + \frac{x^2}{2}-3x\Big|_3^4$$

$$\left(-\frac{9}{2}+9\right)-\left(-\frac{1}{2}+3\right)+(8-12)-\left(\frac{9}{2}-9\right)$$

$$-\frac{9}{2}+9+\frac{1}{2}-3+8-12-\frac{9}{2}+9$$

$$-9$$

$$\boxed{5/2}$$

$$9-\frac{5}{2}$$

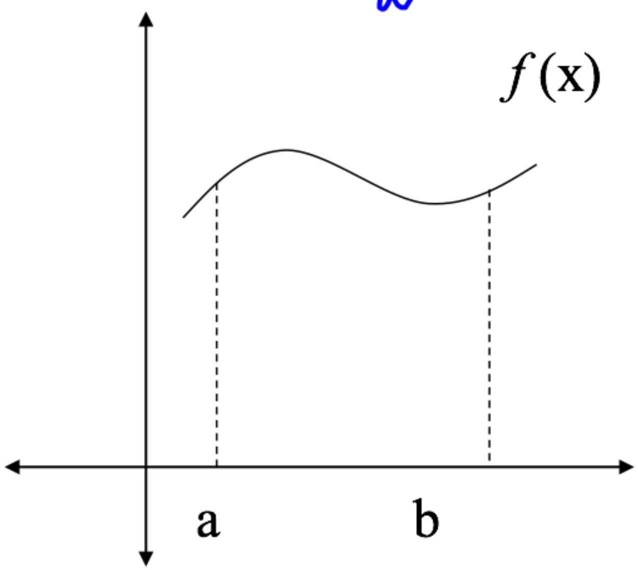
$$\boxed{\frac{13}{2}}$$

final

Summary for the area between two curves:

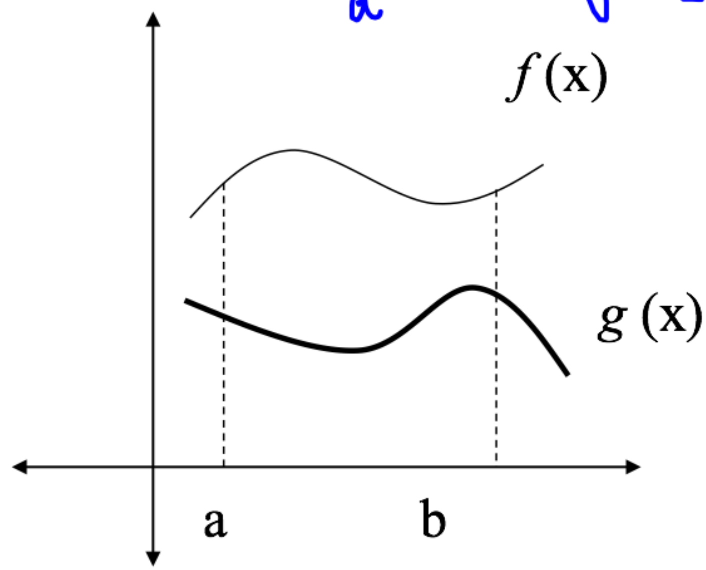
Basic.

$A = \int_a^b f(x)\,dx$



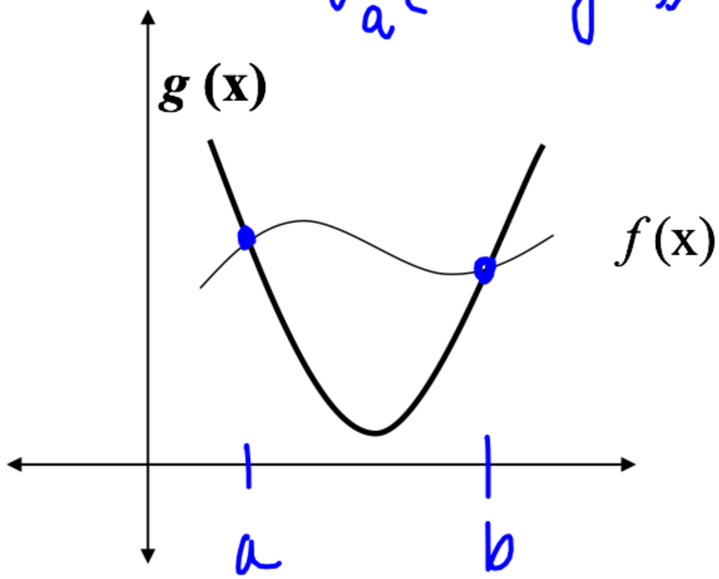
Two non-intersecting curves.

$A = \int_a^b [f(x) - g(x)]\,dx$

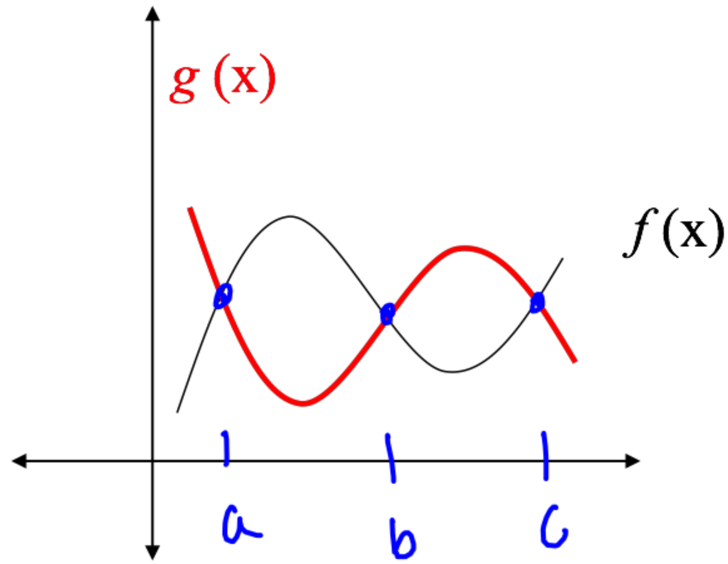


Two curves that intersect.

$$A = \int_a^b \left(f(x) - g(x)\right) dx$$



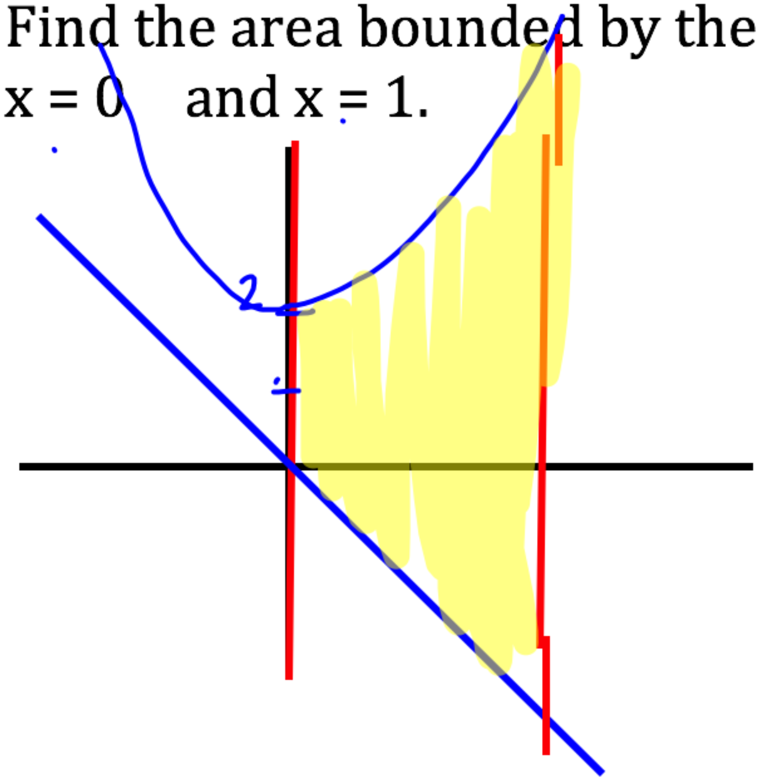
Two curves that intersect several times.



$$A = \int_a^b \left(f(x) - g(x)\right) dx + \int_b^c \left(g(x) - f(x)\right) dx$$

More Examples:

Find the area bounded by the graphs of $y = x^2 + 2$, $y = -x$, $x = 0$ and $x = 1$.



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$$\int_0^1 \left[(x^2+2) - (-x)\right] dx$$

$$\int_0^1 (x^2+2+x)\, dx$$

$$\frac{x^3}{3} + 2x + \frac{x^2}{2} \Big|_0^1$$

$$\frac{1}{3} + 2 + \frac{1}{2} - 0$$

$$\boxed{\frac{11}{6}}$$

Find the area of the region bounded by $f(x) = 2 - x^2$ and $g(x) = x$.

$$2 - x^2 = x$$

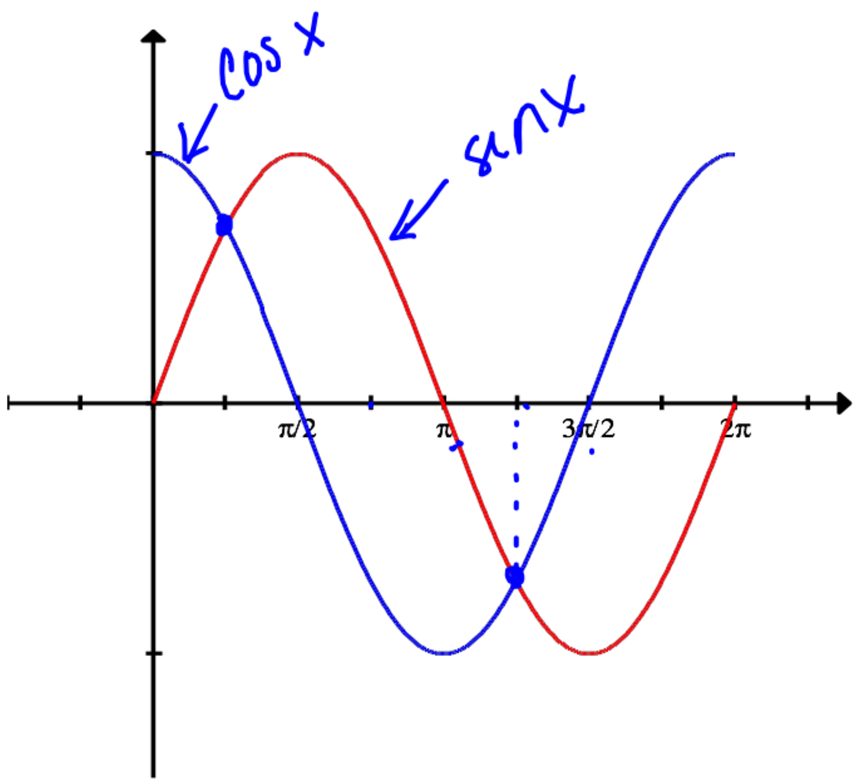
$$0 = x^2 + x - 2$$

$$0 = (x+2)(x-1)$$

$$x = -2, 1$$

$$\int_{-2}^{1} \left((2 - x^2) - x \right) dx$$

Find the area bounded by $f(x) = \sin x$ and $g(x) = \cos x$ for $x \in [0, 2\pi]$



$$\int_0^{\pi/4} (\cos x - \sin x)\,dx$$

$$+ \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\,dx$$

$$+ \int_{5\pi/4}^{2\pi} (\cos x - \sin x)\,dx$$

$$= \Big[\sin x + \cos x\Big]_0^{\pi/4} + \Big[-\cos x - \sin x\Big]_{\pi/4}^{5\pi/4} + \Big[\sin x + \cos x\Big]_{5\pi/4}^{2\pi}$$

$$\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - (0+1) + \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) + (0+1) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)$$

$$\sqrt{2} - 1 + \sqrt{2} + \sqrt{2} + 1 + \sqrt{2} = \boxed{4\sqrt{2}}$$