Math 1432

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Office Hours:

Mondays 1-2pm,
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(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

$$AV = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Find the average value of the function over the interval.

$y = 2x + 3e^x, \quad [1,4]$

$$AV = f(c) = \frac{1}{3}\int_1^4 2x + 3e^x \, dx$$

$$= \frac{1}{3}\left[x^2 + 3e^x\right]_1^4$$

$$= \frac{1}{3}\left[(16 + 3e^4) - (1 + 3e)\right]$$

$$= \frac{1}{3}\left(15 + 3e^4 - 3e\right]$$

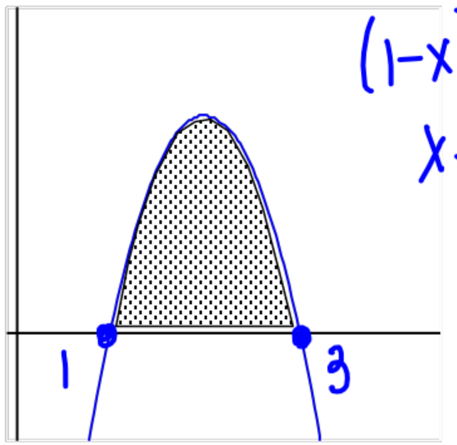
$$= \boxed{5 + e^4 - e}$$

Set up the definite integral(s) that gives the area of the shaded region.

$y = (1 - x)(x - 3)$

$(1-x)(x-3)=0$

$x=1,3$



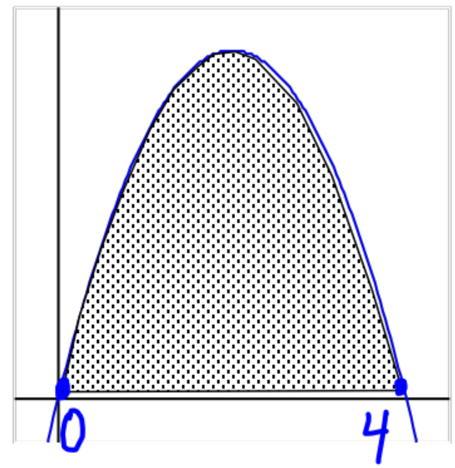
$A = \int_1^3 (1-x)(x-3)\,dx$

$= \int_1^3 (-3+4x-x^2)\,dx$

$y = 4x - x^2$

$x(4-x)=0$

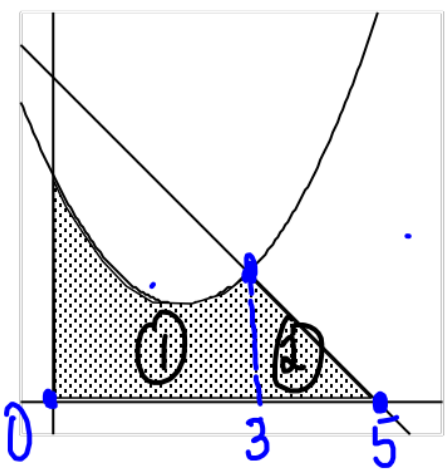
$x=0,4$



$A = \int_0^4 (4x-x^2)\,dx$

$y = x^2 - 4x + 7$

$y = 10 - 2x$



$x^2 - 4x + 7 = 10 - 2x$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3, -1$

$$A = \int_0^3 (x^2 - 4x + 7)\,dx + \int_3^5 (10 - 2x)\,dx$$

$y = 6 - x$

$y = \sqrt{x}$

$6 - x = \sqrt{x}$

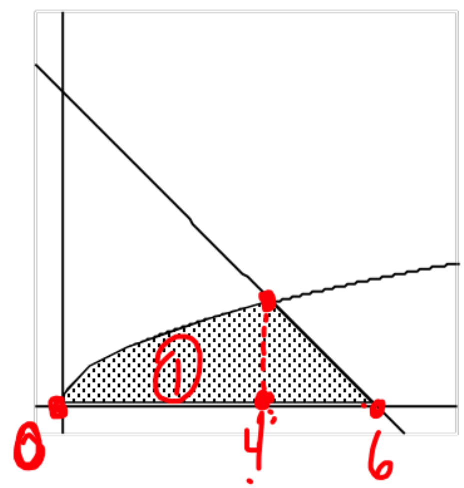
$(6-x)^2 = x$

$36 - 12x + x^2 = x$

$x^2 - 13x + 36 = 0$

$(x-4)(x-9) = 0$

$x = 4, \cancel{9}$



$$A = \int_0^4 \sqrt{x}\,dx + \int_4^6 (6-x)\,dx$$

Find the area bounded by $f(x) = 3x^3 - x^2 - 10x$ and $g(x) = -x^2 + 2x$.

First, find the intersection by setting the functions equal to each other.
We get $x = 0, -2, 2$.

1 2

-2 -1 0 1 2

$f(-1) = -3 - 1 + 10 = 6$ *

$g(-1) = -1 - 2 = -3$

$f(1) = 3 - 1 - 10 = -8$

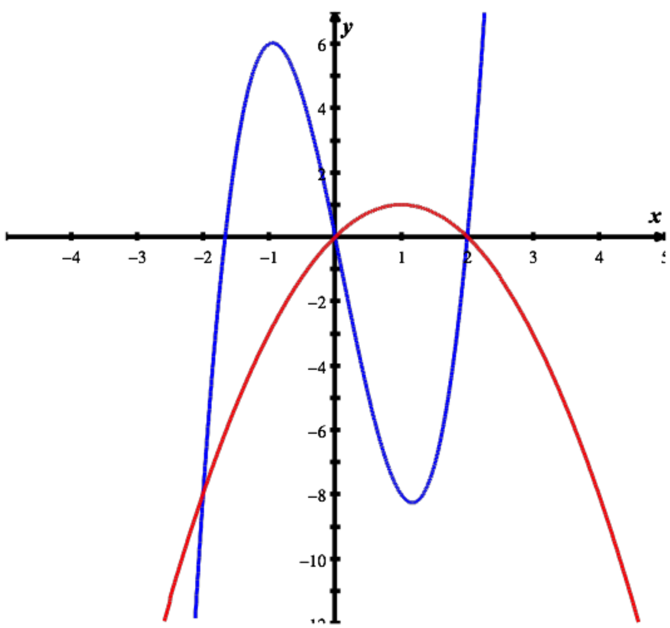
* $g(1) = -1 + 2 = 1$

Next, determine which function is larger on each interval. What do you do if you don't know how the functions graph?

Finally, set up the integrals.

$$\int_{-2}^{0} (3x^3 - x^2 - 10x) - (-x^2 + 2x)\, dx$$

$$+ \int_{0}^{2} (-x^2 + 2x) - (3x^3 - x^2 - 10x)\, dx$$



Now to change things up a bit.....

Right - left

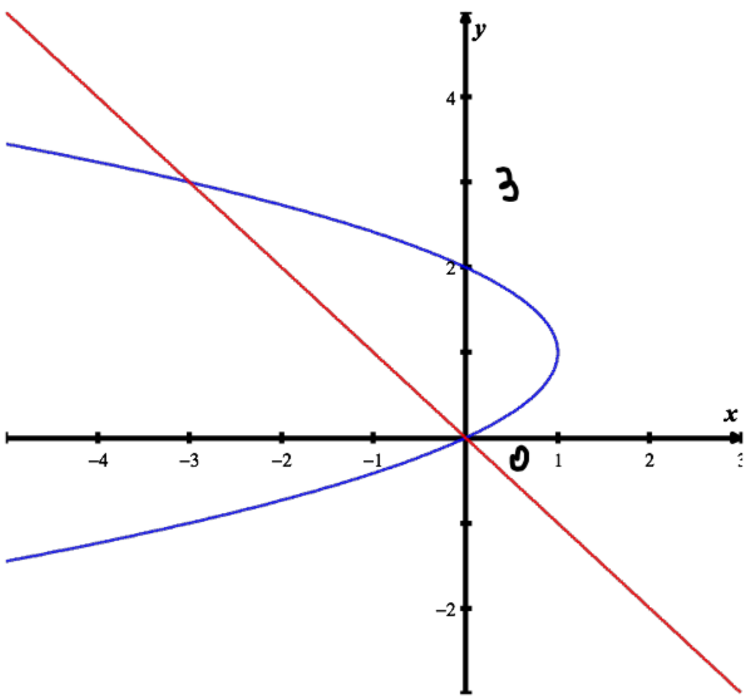
$f(y) = -(y-1)^2 + 1$ and $g(y) = -y$

$$-y^2 + 2y - 1 + 1 = -y$$
$$0 = y^2 - 3y = y(y-3)$$

First, find the intersection by setting the functions equal to each other. We get y = 0, 3.

Set up the integrals.

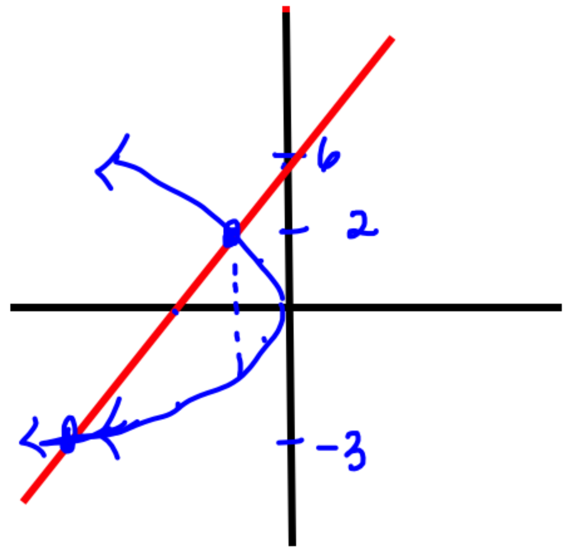
$$A = \int_0^3 \left(-(y-1)^2+1\right) - (-y)\, dy$$
$$= \int_0^3 -y^2 + 3y \, dy$$
$$\left. \frac{-y^3}{3} + \frac{3y^2}{2} \right|_0^3$$
$$\left(\frac{-27}{3} + \frac{27}{2}\right) - 0$$
$$-9 + \frac{27}{2} = \boxed{9/2}$$



Find the area between the graphs of $y = x + 6$ and $x = -y^2$

$x = y - 6$

$\cancel{x + 6 = -y^2}$



$y - 6 = -y^2$

$y^2 + y - 6 = 0$

$(y+3)(y-2) = 0$

$y = -3, 2$

$$\boxed{A = \frac{125}{6}}$$

$$A = \int_{-3}^{2} \left[-y^2 - (y-6)\right] dy = \int_{-3}^{2} \left(-y^2 - y + 6\right) dy$$

$$= \left[\frac{-y^3}{3} - \frac{y^2}{2} + 6y \right]_{-3}^{2} = \left(-\frac{8}{3} - 2 + 12\right) - \left(\cancel{\frac{27}{3}}^{\,9} - \frac{9}{2} - 18\right)$$

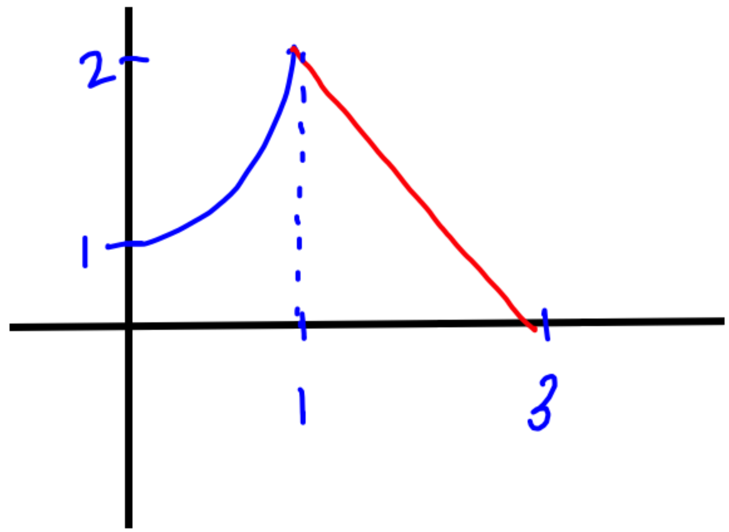
$$-\overset{-16/6}{8/3} + \underbrace{10 + 9}_{19} + \overset{27/6}{\frac{9}{2}}$$

$$\frac{11}{6} + \frac{114}{6}$$

$$\frac{125}{6}$$

$$\begin{array}{r} 19 \\ \times\, 6 \\ \hline 114 \end{array}$$

Find the area between the graph of $f(x)=\begin{cases} x^2+1 & 0\le x\le 1 \\ 3-x & 1<x\le 3 \end{cases}$ and the x-axis.



$$\int_0^1 (x^2+1)\,dx + \int_1^3 (3-x)\,dx$$

$$\left.\frac{x^3}{3}+x\right|_0^1 + \left.3x-\frac{x^2}{2}\right|_1^3$$

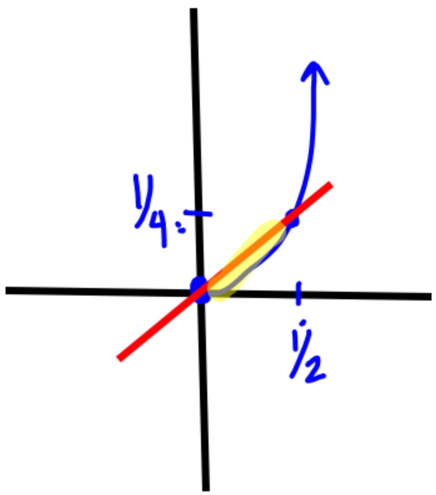
$$\frac{4}{3} + \left(9-\frac{9}{2}\right)-\left(3-\frac{1}{2}\right) = \frac{4}{3}+2 = \boxed{\frac{10}{3}}$$

$$6 \cancel{-\frac{9}{2}+\frac{1}{2}} - 4$$

Sketch the region bounded by the curves and find the area of that region.

$x = \sqrt{y},\ x - 2y = 0$

$x = 2y \Rightarrow y = \frac{1}{2}x$



$\sqrt{y} = 2y$

$y = 4y^2$

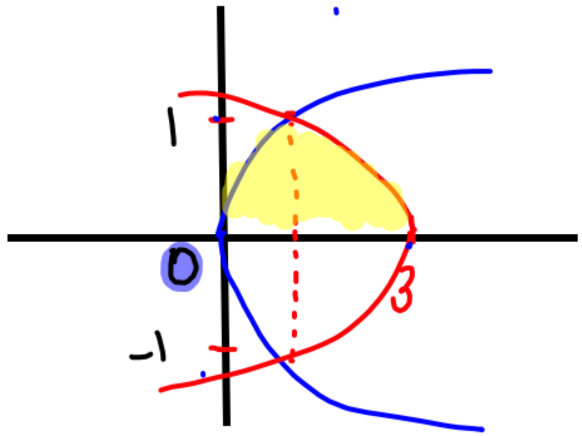
$0 = 4y^2 - y$

$0 = y(4y - 1)$

$y = 0, \frac{1}{4}$

$A = \int_0^{1/2} \left(\frac{1}{2}x - x^2\right) dx$ OR $A = \int_0^{1/4} \left(\sqrt{y} - 2y\right) dy$

$x = y^2, \; x = 3 - 2y^2$



$$y^2 = 3 - 2y^2$$

$$3y^2 - 3 = 0$$

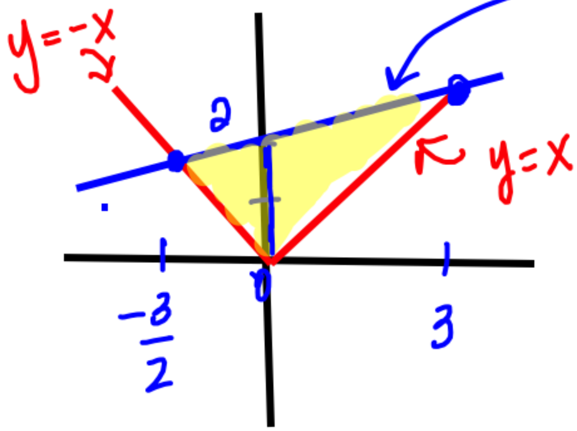
$$3(y^2 - 1) = 0$$

$$y = \pm 1$$

$$A = 2\int_0^1 (3 - 2y^2) - (y^2)\, dy$$

$$= 2\int_0^1 (3 - 3y^2)\, dy$$

$y=|x|,\ 3y-x=6 \rightarrow y=\frac{1}{3}x+2$



$$A=\int_{-3/2}^{0}\left(\tfrac{1}{3}x+2\right)-(-x)\,dx$$

$$+\int_{0}^{3}\left[\left(\tfrac{1}{3}x+2\right)-x\right]dx$$

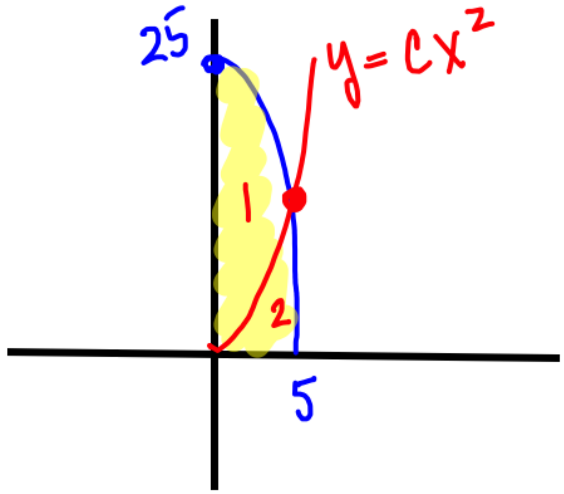
$$-x=\frac{1}{3}x+2 \qquad \frac{1}{3}x+2=x$$

$$-\frac{4}{3}x=2 \qquad 2=\frac{2}{3}x$$

$$x=2\left(\frac{-3}{4}\right)=\frac{-3}{2} \qquad 3=x$$

How would you solve this?

Let R be the region in the first quadrant bounded by the graph of $y = 25 - x^2$ and the coordinate axes. Determine the value of c such that $y = cx^2$ separates R into two regions of equal area.



Find $\int_0^5 (25 - x^2)\,dx$

take ½

answer $= \int_0^{?} \left[(25 - x^2) - cx^2\right] dx$

Use integration to find the area of the triangle whose vertices are $(0, 0)$, $(1, 3)$ and $(1, 5)$.

The function $f(x) = x^3 + x$ is invertible. What is the area between the graph of $y = f^{-1}(x)$ and the *y*-axis for $0 \le y \le 2$?