Math 1432

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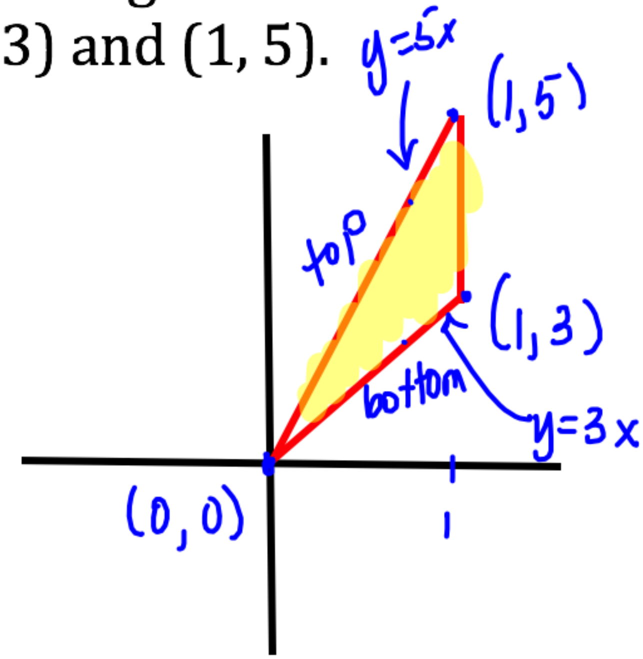
Office Hours:

Mondays 1-2pm,
Fridays noon-1pm
(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

Use integration to find the area of the triangle whose vertices are (0, 0), (1, 3) and (1, 5).



$$A = \int_0^1 (5x - 3x)\,dx$$

$$= \int_0^1 2x\,dx$$

$$= x^2 \Big|_0^1 = \boxed{1}$$

The function $f(x) = x^3 + x$ is invertible. What is the area between the graph of $y = f^{-1}(x)$ and the y-axis for $0 \le y \le 2$?

$$y = x^3 + x$$

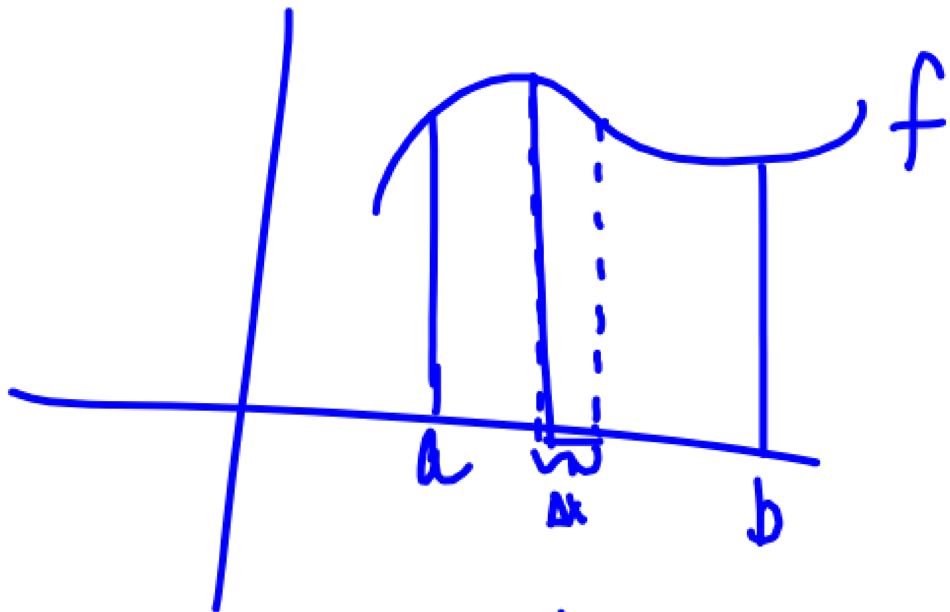
$$x = y^3 + y \quad \leftarrow \text{inverse of } f(x)$$

$$A = \int_0^2 (y^3 + y)\, dy = \frac{y^4}{4} + \frac{y^2}{2} \Big|_0^2$$

$$= 4 + 2$$

$$= \boxed{6}$$

Area



$$A = \int_a^b \underbrace{f(x)}_{\text{height}}\, dx$$

Volumes of Known Cross Sections

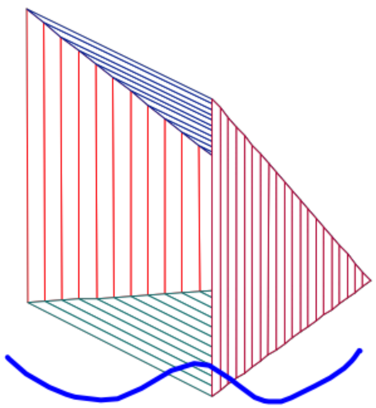
* If the cross section is perpendicular to the x-axis and its area is a function of x, say A(x), then the volume of the solid from *a* to *b* is given by $V = \int_a^b A(x)\,dx$

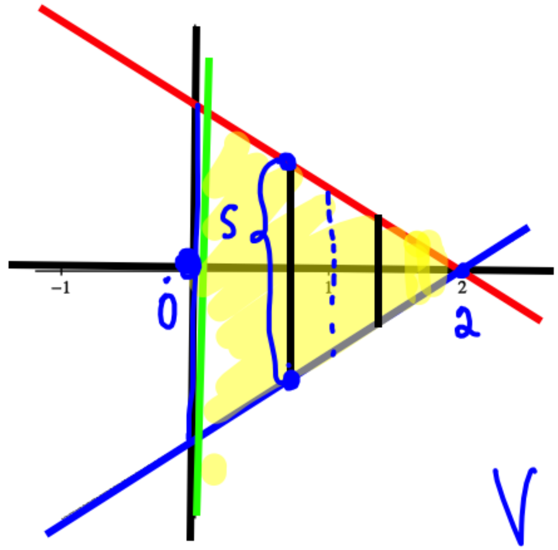
S = top - bottom

* If the cross section is perpendicular to the y-axis and its area is a function of y, say A(y), then the volume of the solid from *c* to *d* is given by $V = \int_c^d A(y)\,dy$

S = Rt - left

1. Find the volume of the solid whose base is bounded by $f(x) = 1 - \frac{1}{2}x$, $g(x) = -1 + \frac{1}{2}x$ and x = 0 if the solid is formed by squares perpendicular to the x-axis.





$$A = s^2$$

$$s = \left(1 - \tfrac{1}{2}x\right) - \left(-1 + \tfrac{1}{2}x\right)$$

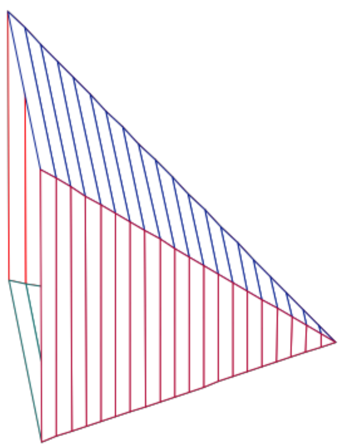
$$s = 2 - x$$

$$A = (2-x)^2$$

$$V = \int_0^2 (2-x)^2\,dx$$

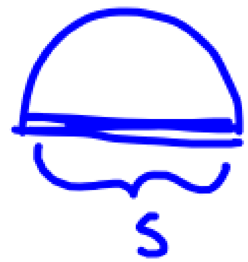
$$= \int_0^2 \left(4 - 4x + x^2\right) dx$$

$$= 4x - 2x^2 + \frac{x^3}{3}\Big|_0^2 = \frac{8}{3}$$



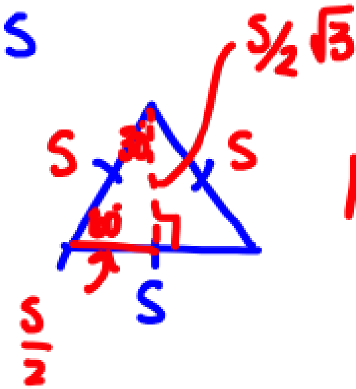
Squares: $A = s^2$

Semicircles: $A = \dfrac{\pi\left(\frac{s}{2}\right)^2}{2} = \dfrac{\pi s^2}{8}$



$r = \frac{s}{2}$

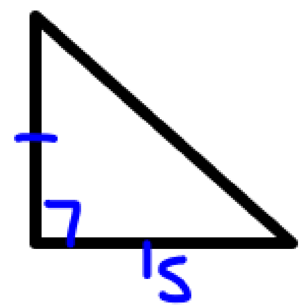
Equilateral Δ's: $A = \dfrac{s^2\sqrt{3}}{4}$



$A = \frac{1}{2}(s)\left(\frac{s}{2}\sqrt{3}\right)$

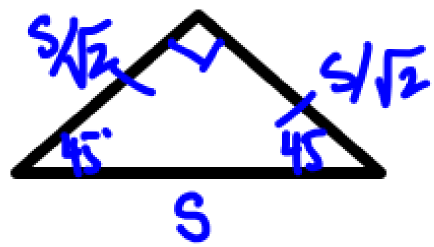
Isosc. Rt Δ's

s is leg: $A = \frac{1}{2}s^2$

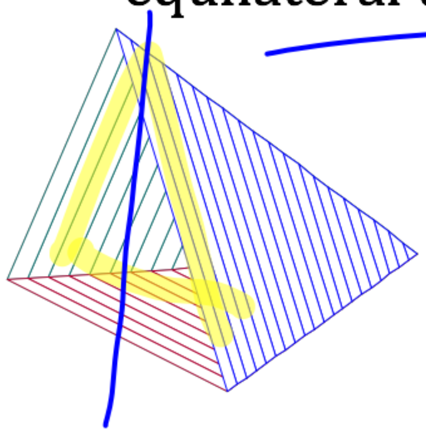


s is hyp.: $A = \frac{1}{2}\left(\frac{s}{\sqrt{2}}\right)^2$

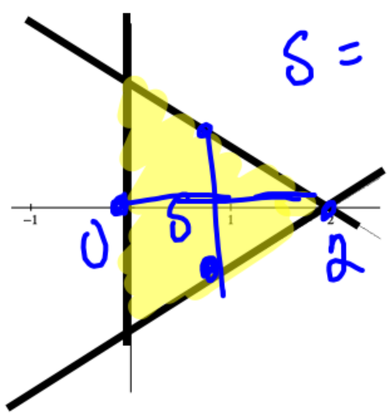
$= \dfrac{s^2}{4}$



2. Find the volume of the solid whose base is bounded by $f(x)=1-\frac{1}{2}x$, $g(x)=-1+\frac{1}{2}x$ and x = 0 if the solid is formed by equilateral triangles perpendicular to the x-axis.



$\hookrightarrow A = \frac{s^2\sqrt{3}}{4}$ $\qquad A(x) = \frac{(2-x)^2\sqrt{3}}{4}$



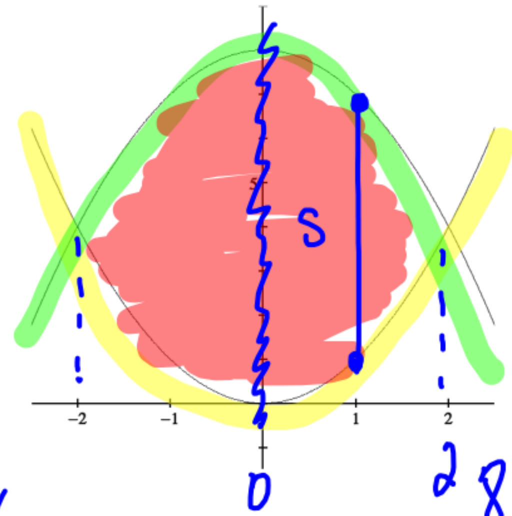
$s = 2-x$

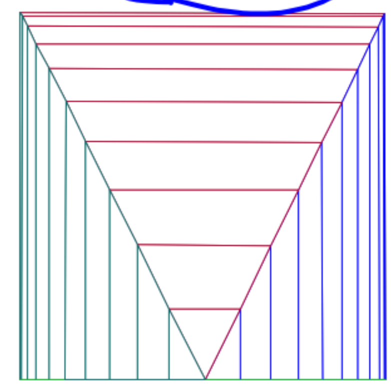
$$V = \int_0^2 \frac{\sqrt{3}}{4}\left(4-4x+x^2\right)dx$$

$$= \frac{\sqrt{3}}{4}\int_0^2 \left(4-4x+x^2\right)dx$$

$$= \frac{\sqrt{3}}{4}\left(\frac{8}{3}\right) = \boxed{\frac{2\sqrt{3}}{3}}$$

3. Find the volume of the solid whose base is bounded by $f(x) = x^2$, $g(x) = 8 - x^2$ and the solid is formed by squares perpendicular to the x-axis.





$$S = 8 - x^2 - x^2 = 8 - 2x^2$$

$$A = (8 - 2x^2)^2$$

$$8 - x^2 = x^2$$
$$8 = 2x^2$$
$$4 = x^2$$
$$x = \pm 2$$

$$V = \int_{-2}^{2} (64 - 32x^2 + 4x^4)\, dx$$

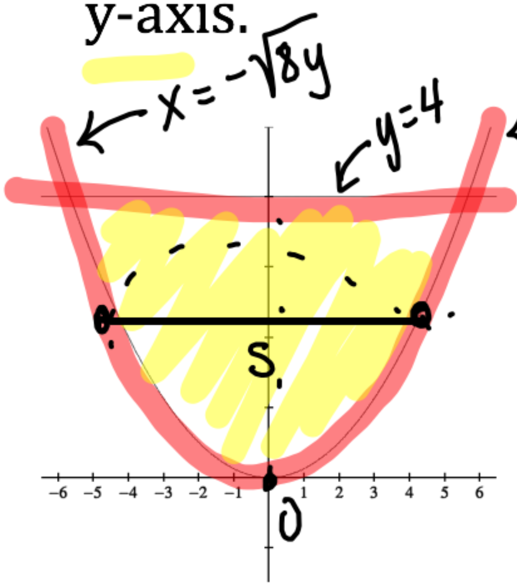
$$= 2\int_{0}^{2} (64 - 32x^2 + 4x^4)\, dx$$

$$= 2\left[64x - \frac{32x^3}{3} + \frac{4x^5}{5}\right]_0^2 = 2\left[128 - \frac{256}{3} + \frac{128}{5}\right]$$

$$= 2\left(\frac{8 \cdot 128}{15}\right) = \boxed{\frac{2048}{15}}$$

$$x = 128$$
$$x - \frac{2x}{3} + \frac{x}{5}$$
$$\frac{15x - 10x + 3x}{15} = \frac{8x}{15}$$

4. Find the volume of the solid whose base is bounded by $y = \frac{1}{8}x^2$ and y = 4 if the solid is formed by semicircles perpendicular to the y-axis.



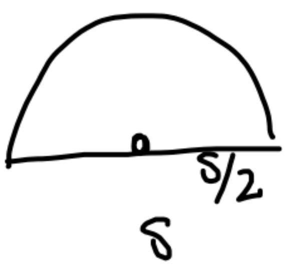
$$8y = x^2$$

$$\pm\sqrt{8y} = x$$

S = Rt - left

$$S = \sqrt{8y} - -\sqrt{8y} = 2\sqrt{8y}$$

$$A = \frac{\pi\left(\frac{2\sqrt{8y}}{2}\right)^2}{2} = \frac{8y\pi}{2} = 4y\pi$$

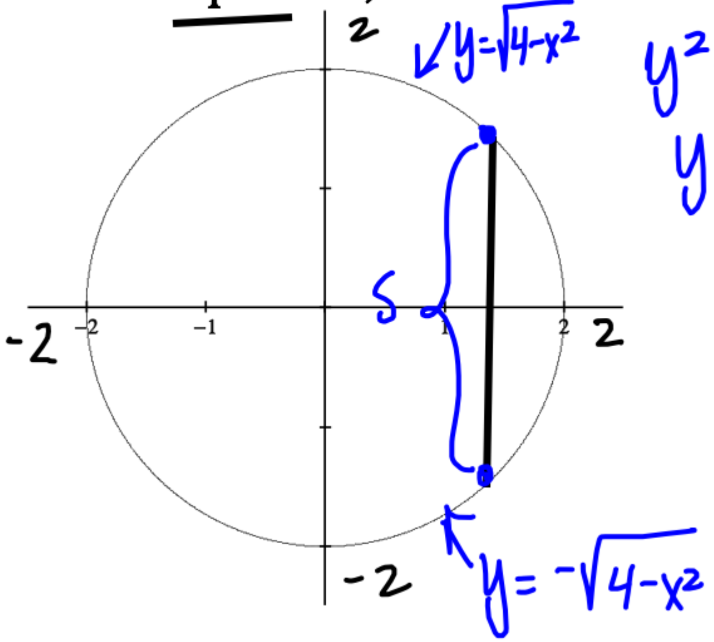


$$A = \frac{\pi\left(s/2\right)^2}{2}$$

$$V = \int_0^4 4y\pi\, dy = 2y^2\pi\Big|_0^4$$

$$\boxed{32\pi}$$

5. Consider a solid whose base is the region inside the circle $x^2 + y^2 = 4$. If cross sections taken perpendicular to the x-axis are squares, find the volume of this solid.



$$y^2 = 4 - x^2$$

$$y = \pm\sqrt{4-x^2}$$

$$S = \sqrt{4-x^2} - -\sqrt{4-x^2} = 2\sqrt{4-x^2}$$

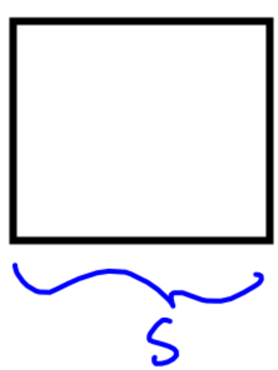
$$A = \left(2\sqrt{4-x^2}\right)^2 = 4(4-x^2)$$

$$V = \int_{-2}^{2} 4(4-x^2)\,dx$$

$$= 2\int_0^2 (16-4x^2)\,dx = 2\left[16x - \frac{4x^3}{3}\right]_0^2$$

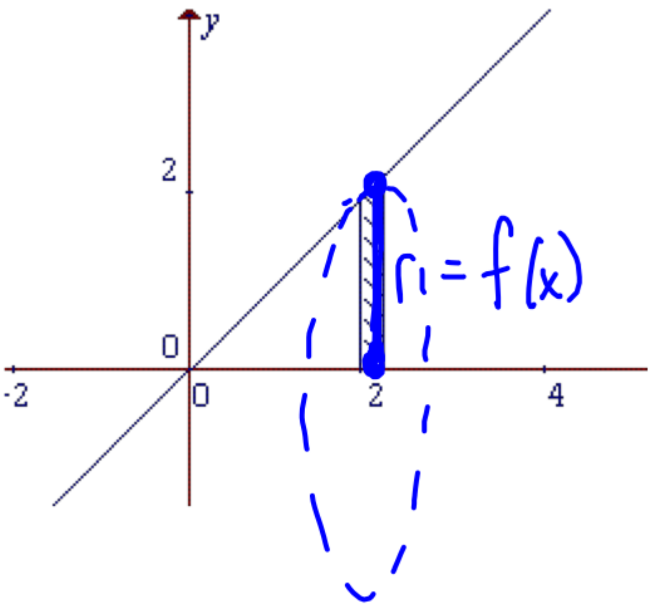
$$= 2\left[32 - \frac{32}{3}\right] = 2\left(\frac{64}{3}\right) = \boxed{\frac{128}{3}}$$

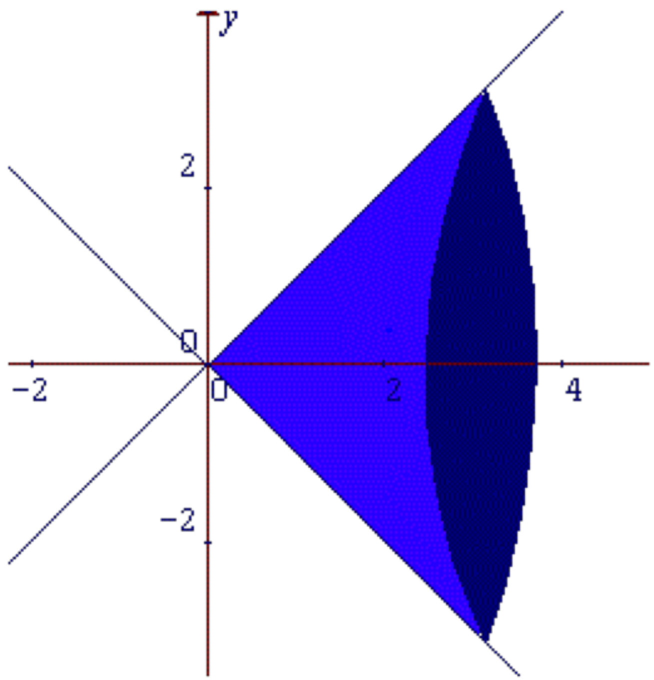
$$x - \frac{x}{3} = \frac{2x}{3}$$



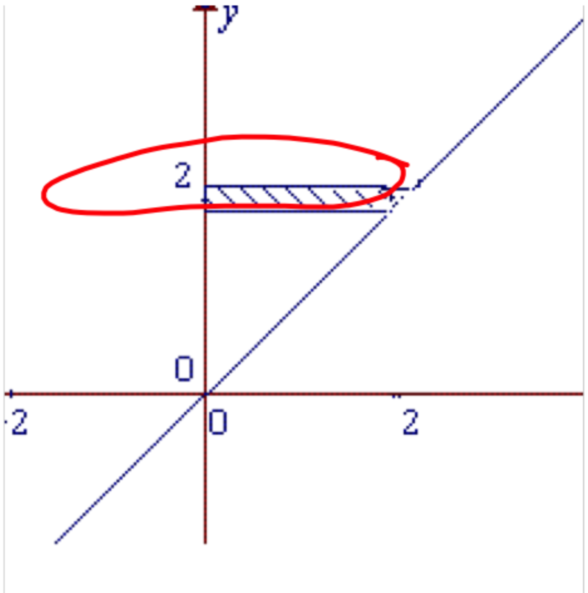
Volume with the Disc Method:

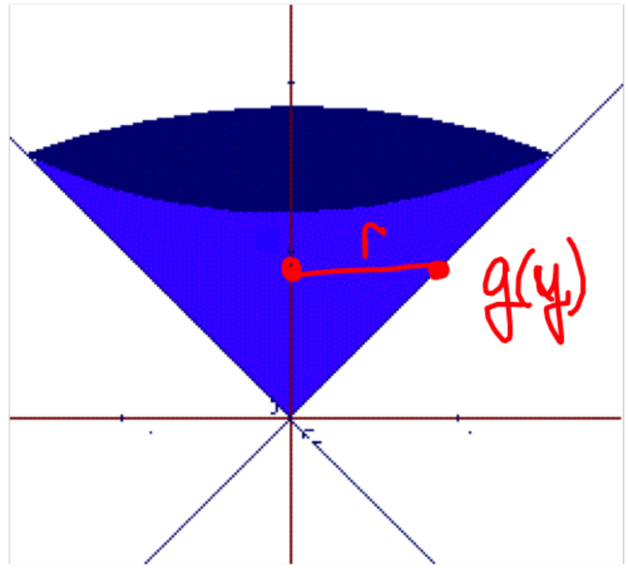
Revolving about the x-axis: $V = \int_a^b \pi \left[f(x) \right]^2 \, dx$



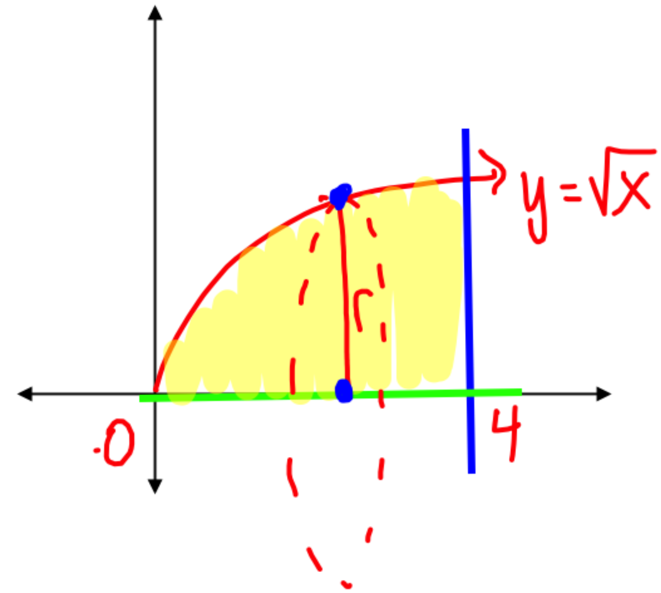


Revolving about the y-axis: $V = \int_c^d \pi \left[g(y) \right]^2 \, dy$





Let R be the region bounded by the x-axis and the graphs of $y = \sqrt{x}$ and x = 4. Sketch and shade the region R. Label points on the x and y-axis.



a. Give the formula the area of region R

$$A = \int_0^4 \sqrt{x}\, dx$$

b. Find the area of region R

$$= \int_0^4 x^{1/2}\, dx = \frac{2}{3} x^{3/2} \Big|_0^4 = \frac{16}{3}$$

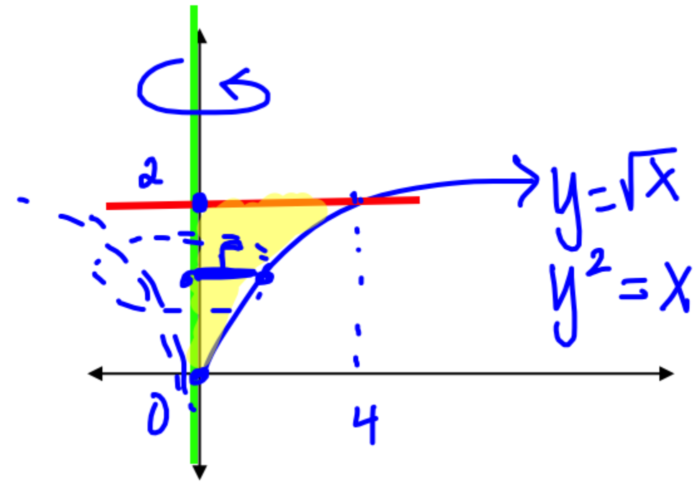
c. Give the formula the volume of the solid generated when the region R is rotated about the x-axis.

$$V = \int_0^4 \pi \left(\sqrt{x}\right)^2 dx$$

d. Find the volume for the solid in (c).

$$= \int_0^4 \pi x\, dx = \pi \, x^2/2 \Big|_0^4 = \boxed{8\pi}$$

Let R be the region bounded by the y-axis and the graphs of $y = \sqrt{x}$ and y = 2. Sketch and shade the region R. Label points on the x and y-axis.



Give the formula the volume of the solid generated when the region R is rotated about the y-axis.

$r = y^2$

$$V = \int_0^2 \pi (y^2)^2 \, dy$$

Find the volume for the solid.

$$= \pi \int_0^2 y^4 \, dy = \pi \frac{y^5}{5} \Big|_0^2 = \boxed{\frac{32\pi}{5}}$$