Math 1432

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Office Hours:

Mondays 1-2pm,
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(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

Popper 01

1. Find the average value of the function $f(x) = x^2$ over the interval $[0, 1]$.

a. 1/2

b. 1

c. 1/3

d. 1/4

e. None of the above

2. Compute $\int (x+1)^{1/3} x \, dx$.

a. $\frac{3}{7}(x+1)^{7/3}+C$

b. $\frac{3}{7}(x+1)^{7/3}-\frac{3}{4}(x+1)^{4/3}$

c. $(x+1)^{7/3}-\frac{3}{4}(x+1)^{4/3}$

d. $-\frac{3}{4}(x+1)^{4/3}+C$

e. $\frac{3}{7}(x+1)^{7/3}-\frac{3}{4}(x+1)^{4/3}+C$

$u = x+1 \rightarrow x = u-1$

$du = dx$

$\int u^{1/3}(u-1)\,du$

$\int \left(u^{4/3}-u^{1/3}\right)du$

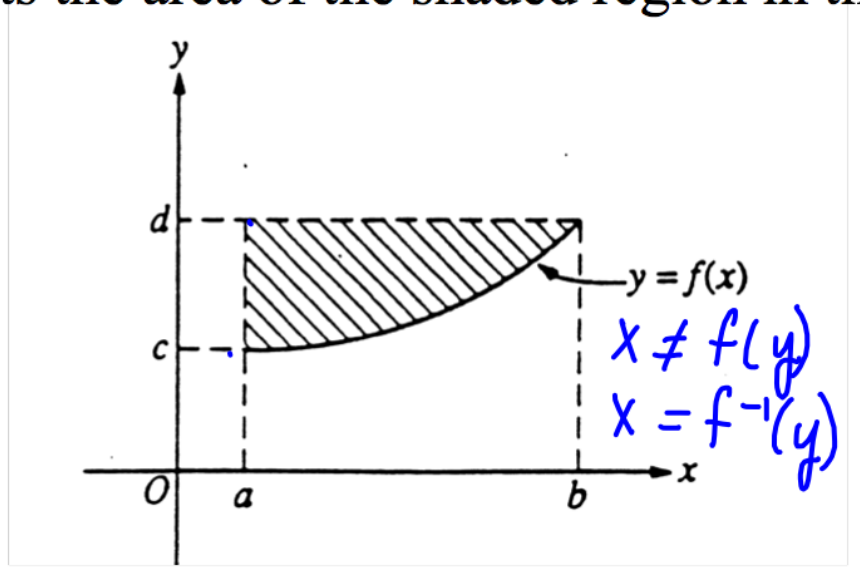
$$\int \frac{5 - e^x}{e^{2x}}\,dx = \int \frac{5}{e^{2x}} - \frac{e^x}{e^{2x}}\,dx$$

monomial

$$= \int 5e^{-2x} - e^{-x}\,dx$$

$$-\frac{5}{2}e^{-2x} + e^{-x} + C$$

3. Which of the following represents the area of the shaded region in the figure shown?



a. $\int_c^d f(y)\,dy$

b. $\int_a^b \big(d - f(x)\big)\,dx$

c. $f'(b) - f'(a)$.

d. $(b-a)\big[f(b) - f(a)\big]$

e. $(d-c)\big[f(b) - f(a)\big]$

4. The area of the region enclosed by the graphs of $y = x^2$ and $y = x$ is

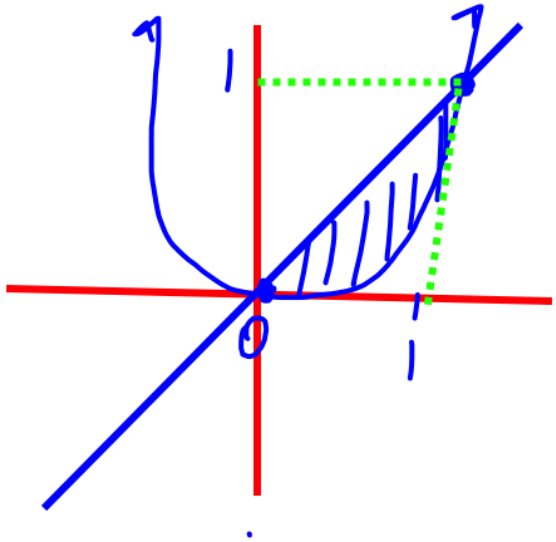
a. 1/6

b. 1/3

c. 1/2

d. 5/6

e. 1



$x^2 = x$

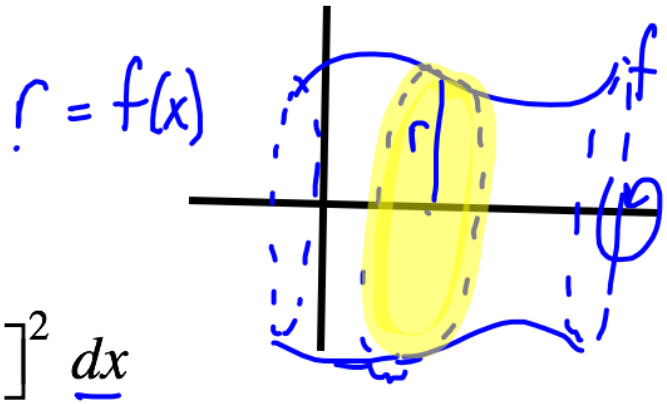
$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, 1$

More About Volume

Disc Method:



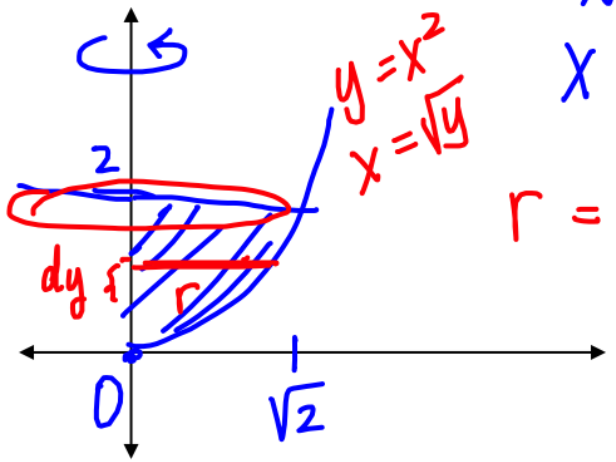
Revolving about the x-axis: $V = \int_a^b \pi \left[f(x) \right]^2 \, dx$

Revolving about the y-axis: $V = \int_c^d \pi \left[g(y) \right]^2 \, dy$

Let R be the region in the first quadrant bounded by the y-axis and the graphs of $y = x^2$ and $y = 2$. Sketch and shade the region R. Label points on the x and y-axis.

$x^2 = 2$

$x = \pm\sqrt{2}$



$r = \sqrt{y}$

Give the formula for the volume of the solid generated when the region R is rotated about the y-axis.

$$V = \int_0^2 \pi \left(\sqrt{y}\right)^2 dy$$

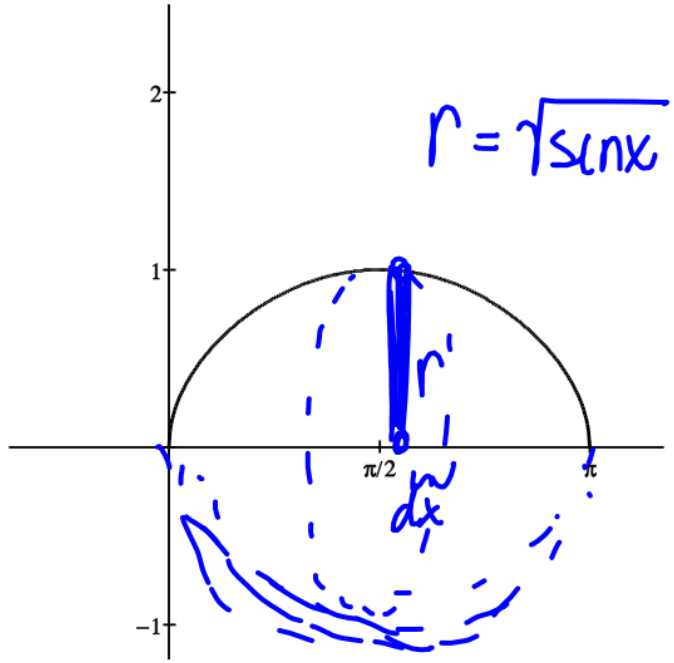
Rotate the region enclosed by $y = \sqrt{\sin x}$ $0 \le x \le \pi$ about the x-axis.
Determine the volume of the solid formed.

$$V = \int_0^{\pi} \pi \left(\sqrt{\sin x}\right)^2 \, dx$$

$$= \int_0^{\pi} \pi \cdot \sin x \, dx$$

$$= -\pi \cos x \Big|_0^{\pi}$$

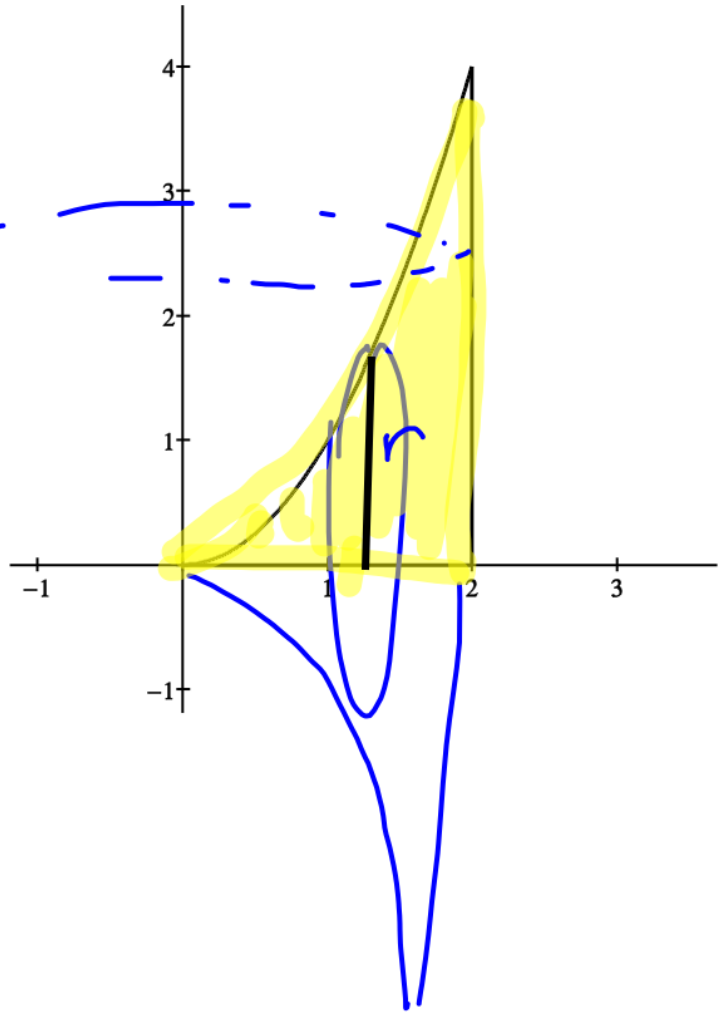
$$= -\pi \left(\cos \pi - \cos 0\right) = -\pi \left(-1 - 1\right) = \boxed{2\pi}$$



Rotate the region enclosed by $y = x^2$, $y = 0$, $x = 2$ about the x-axis. Give the formula for the volume of the solid formed.

$$V = \int_0^2 \pi (x^2)^2 \, dx$$

?

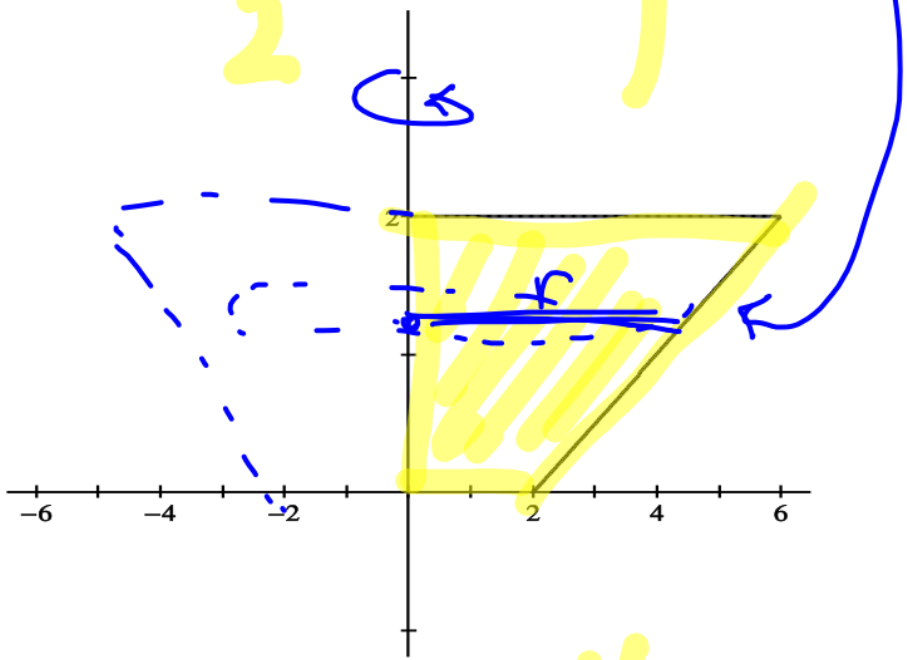


Consider the region in the first quadrant bounded by $y = \frac{1}{2}x - 1$, $y = 2$. Give the formula for the volume of the solid formed by revolving this region around the y-axis.

$y + 1 = \frac{1}{2}x$

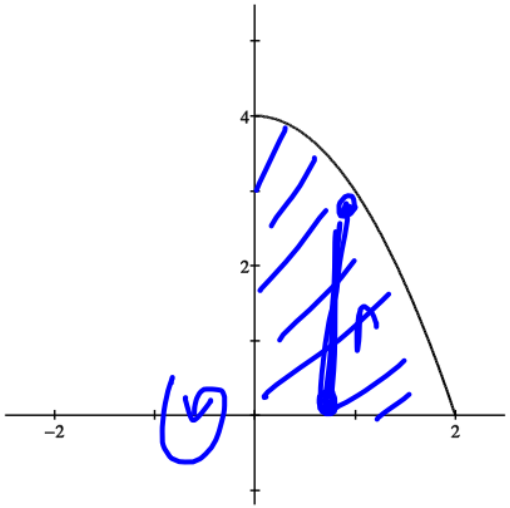
$2y + 2 = x$

$$\int_0^2 \pi (2y+2)^2 \, dy$$



Consider the region in the first quadrant enclosed by $y = 4 - x^2$. Give the formula for the volume of the solid formed by revolving this region about the *x*-axis.

$$V = \int_0^2 \pi (4-x^2)^2 \, dx$$

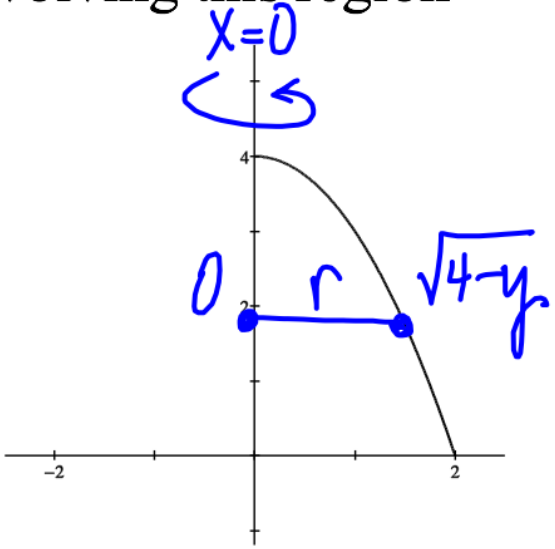


Now consider the region in the first quadrant enclosed by $y = 4 - x^2$. Give the formula for the volume of the solid formed by revolving this region about the y-axis.

$y = 4 - x^2$

$x^2 = 4 - y$

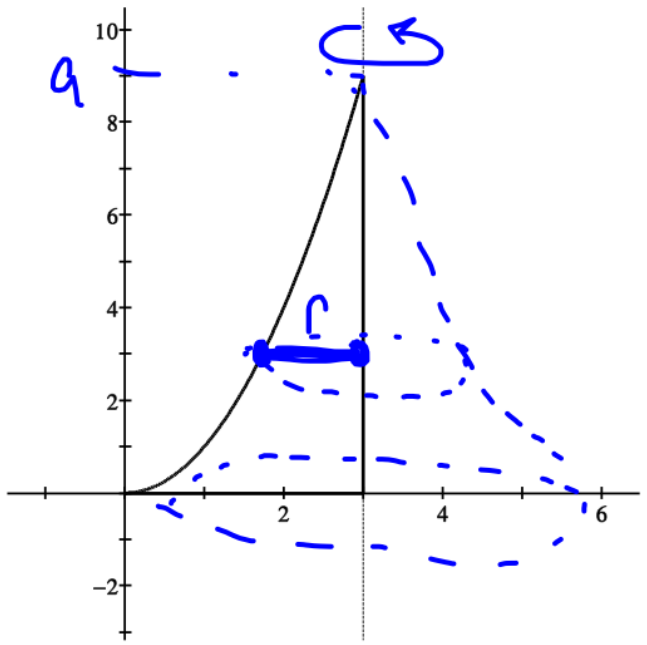
$x = \pm\sqrt{4-y}$



$$V = \int_0^4 \pi \left(\sqrt{4-y}\right)^2 dy$$

$\rightarrow x = \sqrt{y}$

Consider the region enclosed by $y = x^2$, $y = 0$, $x = 3$. Give the formula for the volume of the solid formed by revolving this region around the line $x = 3$.



$r = 3 - \sqrt{y}$

$$V = \int_0^9 \pi (3 - \sqrt{y})^2 \, dy$$

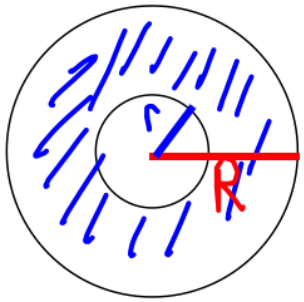
Vertical axis: r = right - left

horizontal axis: r = top - bottom

Washer Method

Revolving about the x-axis: $V = \int_a^b \pi\left(\left[f(x)\right]^2 - \left[g(x)\right]^2\right) dx$

Revolving about the y-axis: $V = \int_c^d \pi\left(\left[f(y)\right]^2 - \left[g(y)\right]^2\right) dy$



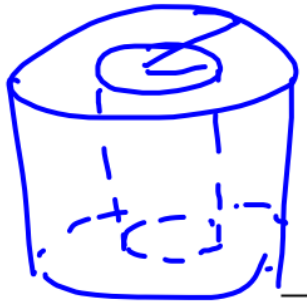
$A = \pi R^2 - \pi r^2 = \pi\left(R^2 - r^2\right)$

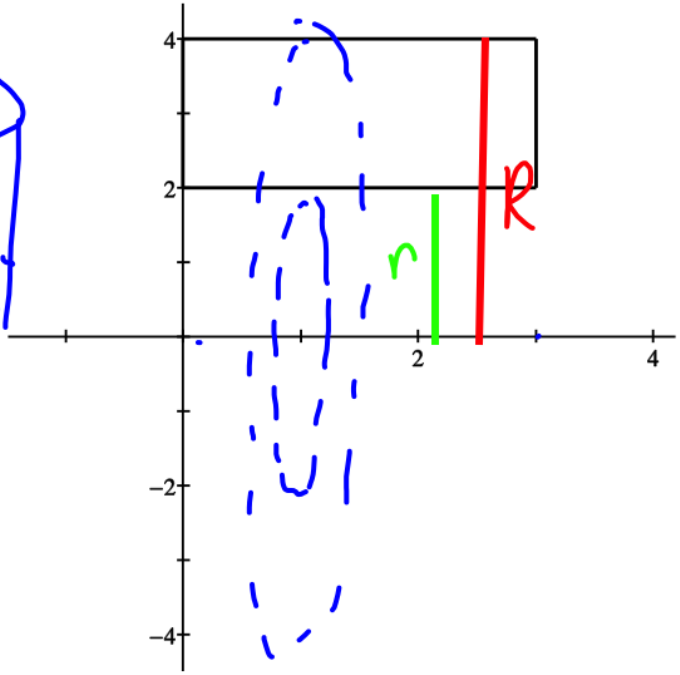
outer radius

inner radius

Consider the region enclosed by $y = 2$, $y = 4$, $x = 0$, $x = 3$. Find the volume of the solid formed by revolving this region around the x-axis.

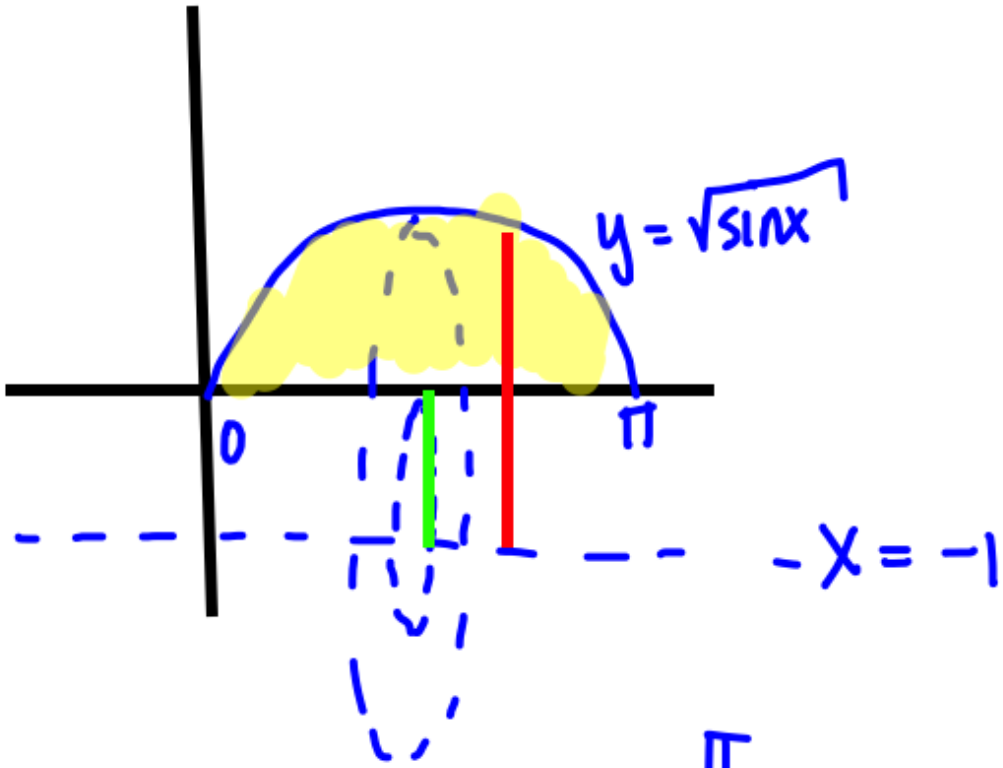
$$V = \int_0^3 \pi \left(4^2 - 2^2\right) dx$$





$$\int_0^3 12\pi \, dx = 12\pi x \Big|_0^3$$

$$= \boxed{36\pi}$$



$R = \sqrt{\sin x} - -1$

$r = 0 - -1$

$$V = \int_0^{\pi} \pi \left[(\sqrt{\sin x} + 1)^2 - (1)^2 \right] dx$$

(popper)

5. The region enclosed by the x-axis, the line x = 3, and the curve $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated?

a. 3π

b. $2\sqrt{3}\,\pi$

c. $\frac{9}{2}\pi$

d. 9π

e. $\frac{36\sqrt{3}}{5}\pi$