Math 1432

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Mondays 1-2pm,
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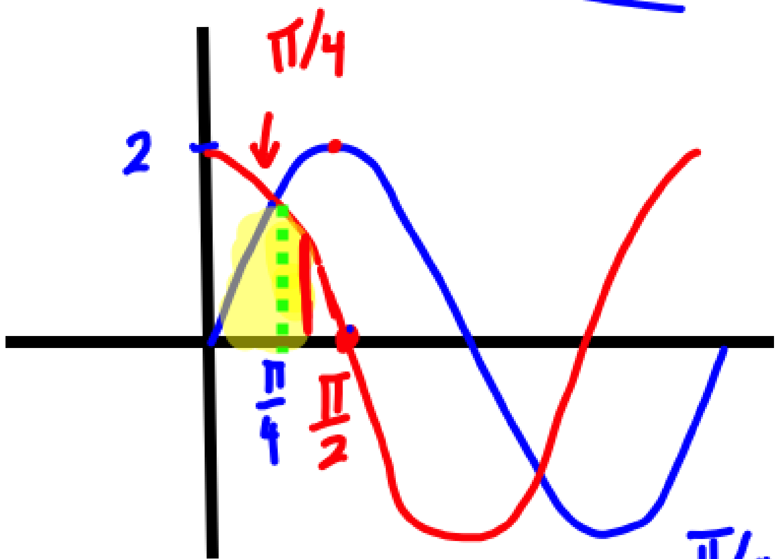
Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

Quiz 3 #11

Find the area of the region bounded by the x-axis and the curves $y = 2\,\sin(x)$ and $y = 2\,\cos(x)$ where $x \in \left[0, \frac{\pi}{3}\right]$



$$2\sin x = 2\cos x$$

$$\tan x = 1$$

$$x = \pi/4$$

$$\int_0^{\pi/4} 2\sin x\,dx + \int_{\pi/4}^{\pi/3} 2\cos x\,dx$$

$$-2\cos x\Big|_0^{\pi/4} + 2\sin x\Big|_{\pi/4}^{\pi/3}$$

$$-2\left(\frac{\sqrt{2}}{2}\right) - (-2) + 2\left(\frac{\sqrt{3}}{2}\right) - 2\left(\frac{\sqrt{2}}{2}\right)$$

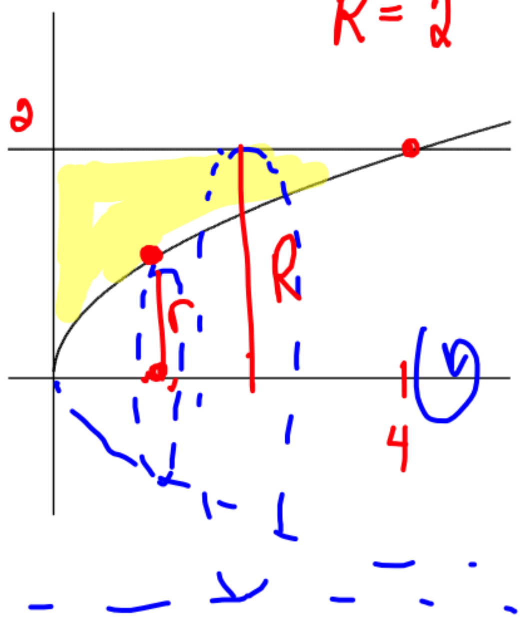
$$-\sqrt{2} + 2 + \sqrt{3} - \sqrt{2} = 2 + \sqrt{3} - 2\sqrt{2}$$

Disc/Washer Method

Find the volume of the region bounded by $y = \sqrt{x}$, $x = 0$, $y = 2$ being revolved about:

a. x – axis

$R = 2 \qquad r = \sqrt{x}$

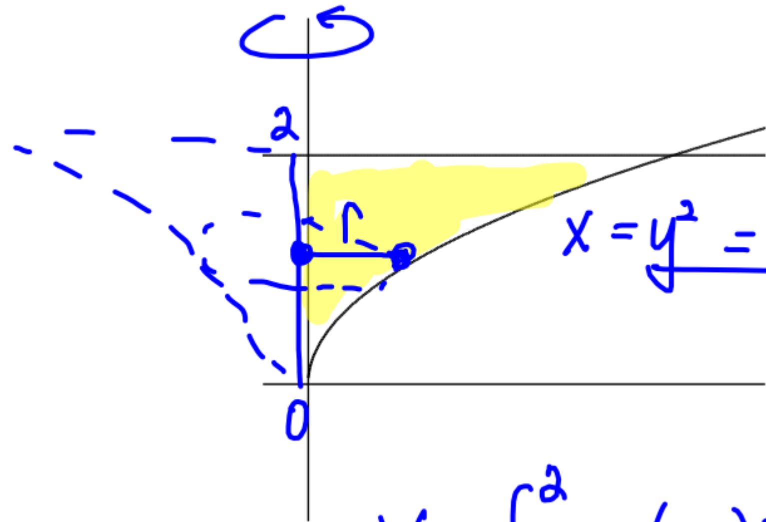


$$V = \int_0^4 \pi\left(2^2 - \sqrt{x}^2\right) dx$$

$$\pi \int_0^4 4 - x \, dx = \pi\left[4x - x^2/2\right]_0^4$$

$$= \pi(16 - 8) = 8\pi$$

b. y – axis



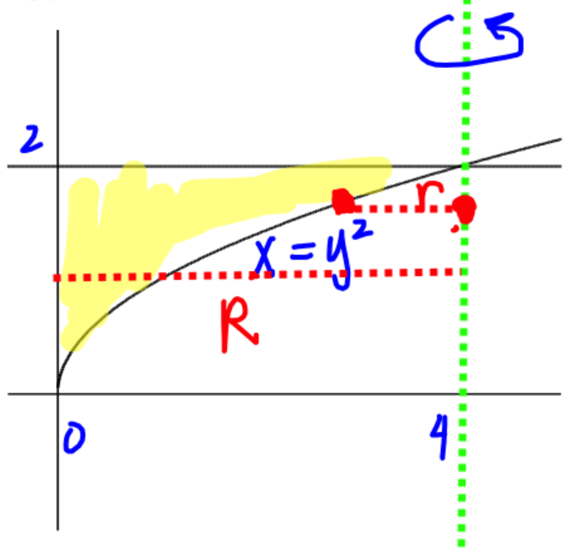
$x = y^2 = r$

$$V = \int_0^2 \pi\left(y^2\right)^2 dy$$

$$= \pi \int_0^2 y^4 \, dy = \pi \cdot \frac{y^5}{5}\Big|_0^2$$

$$= \frac{32\pi}{5}$$

c. $x = 4$



$R = 4$

$r = 4 - y^2$

$$V = \pi \int_0^2 \left(4^2 - \underbrace{(4-y^2)^2}_{}\right) dy$$

$$16 - 16 + 8y^2 - y^4$$

$$= \pi \int_0^2 8y^2 - y^4 \, dy = \pi \left[\frac{8y^3}{3} - \frac{y^5}{5} \right]_0^2$$

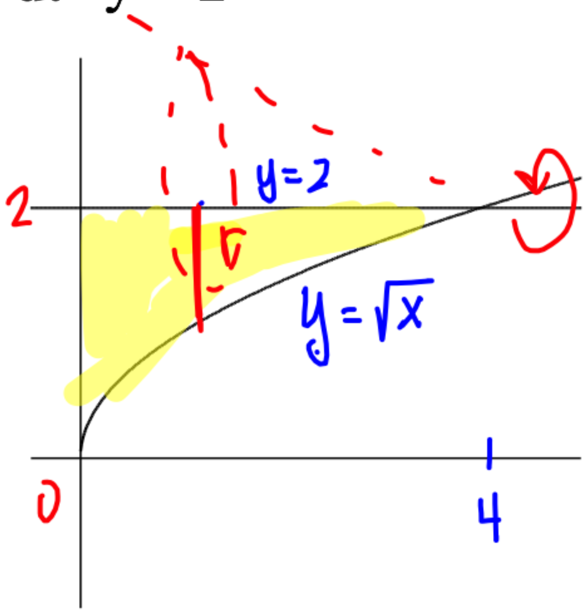
$$= \pi \left(\frac{64}{3} - \frac{32}{5} \right) = \frac{224\pi}{15}$$

$x = 32$

$$\frac{2x}{3} - \frac{x}{5}$$

$$\frac{10x - 3x}{15} = \frac{7(32)}{15}$$

d. $y = 2$



$$r = (2-\sqrt{x})$$

$$V = \pi \int_0^4 (2-\sqrt{x})^2 \, dx$$

$$= \pi \int_0^4 (4 - 4x^{1/2} + x) \, dx$$

$$= \pi \left(4x - \frac{8}{3}x^{3/2} + \frac{x^2}{2}\right)\Big|_0^4$$

$$= \pi \left(16 - \frac{64}{3} + 8\right)$$

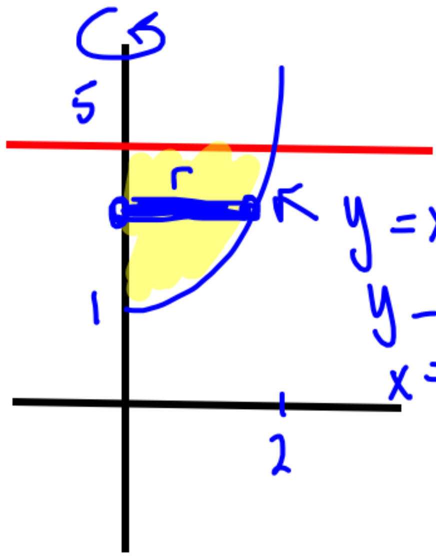
$$\pi \left(24 - \frac{64}{3}\right) = \boxed{\frac{8}{3}\pi}$$

$$\frac{72}{3} - \frac{64}{3}$$

Popper 02

1. The region R in the first quadrant is enclosed by the lines $x = 0$ and $y = 5$ and the curve $y = x^2 + 1$. The volume of the solid generated when R is revolved about the y-axis is

A) 6π B) 8π ~~C)~~ $\frac{34}{3}\pi$ D) 16π ~~E)~~ $\frac{544}{15}\pi$



$y = x^2 + 1$

$y - 1 = x^2$

$x = \sqrt{y-1}$

$r = \sqrt{y-1}$

$$V = \int_1^5 \pi \left(\sqrt{y-1}\right)^2 dy$$

$$= \pi \int_1^5 (y-1)\, dy$$

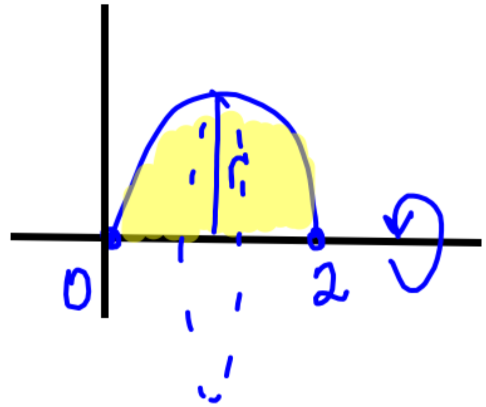
2. Let R be the region in the first quadrant bounded by the x-axis and the curve $y = 2x - x^2$. The volume produced when R is revolved about the x-axis is

A) $\frac{16\pi}{15}$ B) $\frac{8\pi}{3}$ C) $\frac{4\pi}{3}$ D) 16π E) 8π

$2x - x^2 = 0$

$x(2-x) = 0$

$x = 0, 2$



$r = 2x - x^2$

$$V = \int_0^2 \pi (2x - x^2)^2 \, dx$$

$$= \pi \int_0^2 (4x^2 - 4x^3 + x^4) \, dx$$

$$= \pi \left[\frac{4x^3}{3} - x^4 + \frac{x^5}{5} \right]_0^2$$

$$= \pi \left(\frac{32}{3} - 16 + \frac{32}{5} \right)$$

3. Given the region in the first quadrant bounded by the function $y = 4 - x^2$, set up the integral equation that finds the volume of the region when rotated about $y = 0$.

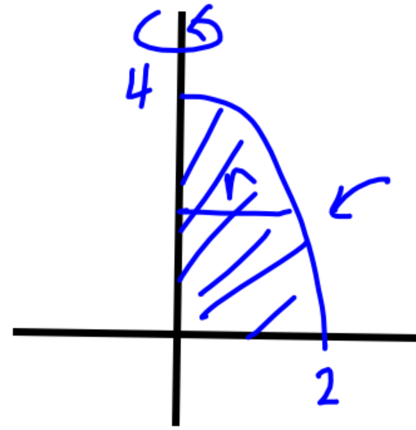
a. $V = \int_0^2 \left(4 - x^2\right)^2 \, dx$

b. $V = \pi \int_0^2 \left(4 - x^2\right)^2 \, dx$

c. $V = \pi \int_0^2 -\left(4 - x^2\right)^2 \, dx$

d. $V = \pi \int_0^2 \left(4 - x^2\right) dx$

4. Given the region in the first quadrant bounded by the function $y = 4 - x^2$, set up the integral equation that finds the volume of the region when rotated about $x = 0$.



$x = \sqrt{4-y}$

a. $V = \pi \int_0^2 (4-y)\,dy$

b. $V = \pi \int_0^4 (4-y)\,dy$

c. $V = \pi \int_0^4 \sqrt{4-y}\,dy$

d. $V = \pi \int_0^4 (0-(4-y))\,dy$

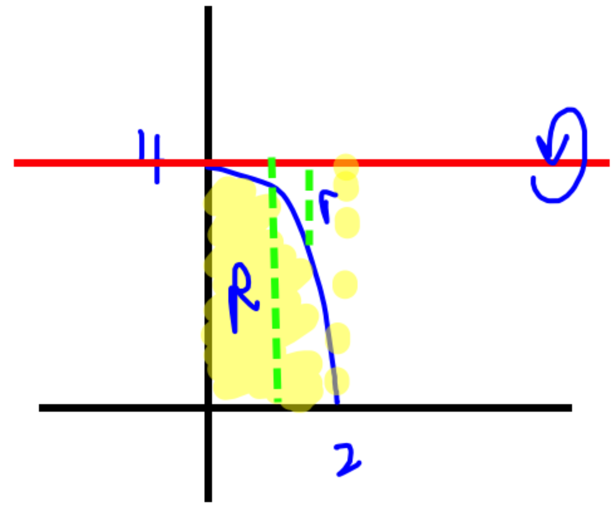
5. Given the region in the first quadrant bounded by the function $y = 4 - x^2$, set up the integral equation that finds the volume of the region when rotated about $y = 4$.

a. $V = \pi \int_0^2 \left(x^2 - 4^2\right) dx$

b. $V = \pi \int_0^2 \left(\left(x^2\right)^2 - 4^2\right) dx$

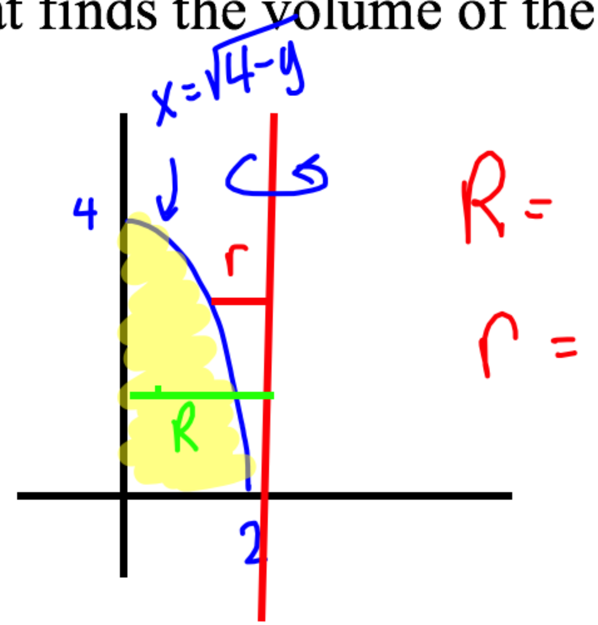
c. $V = \pi \int_0^2 \left(\left(4 - x^2\right)^2 - 4^2\right) dx$

d. $V = \pi \int_0^2 \left(4^2 - \left(x^2\right)^2\right) dx$



$\pi \int_a^b \left(R^2 - r^2\right) dx$

6. Given the region in the first quadrant bounded by the function $y = 4 - x^2$, set up the integral equation that finds the volume of the region when rotated about $x = 2$.



a. $V = \pi\int_0^4 \left(\left(4-y\right)-4^2\right)dy$

b. $V = \pi\int_0^4 \left(4^2-\left(y-2\right)\right)dy$

c. $V = \pi\int_0^4 \left(4-\left(2-\sqrt{4-y}\right)^2\right)dy$

d. $V = \pi\int_0^4 \left(4-y^2\right)dy$

+ Washer

In the Disc Method, the $\underbrace{\text{rectangle}}_{r}$ of revolution is perpendicular to the axis of revolution.

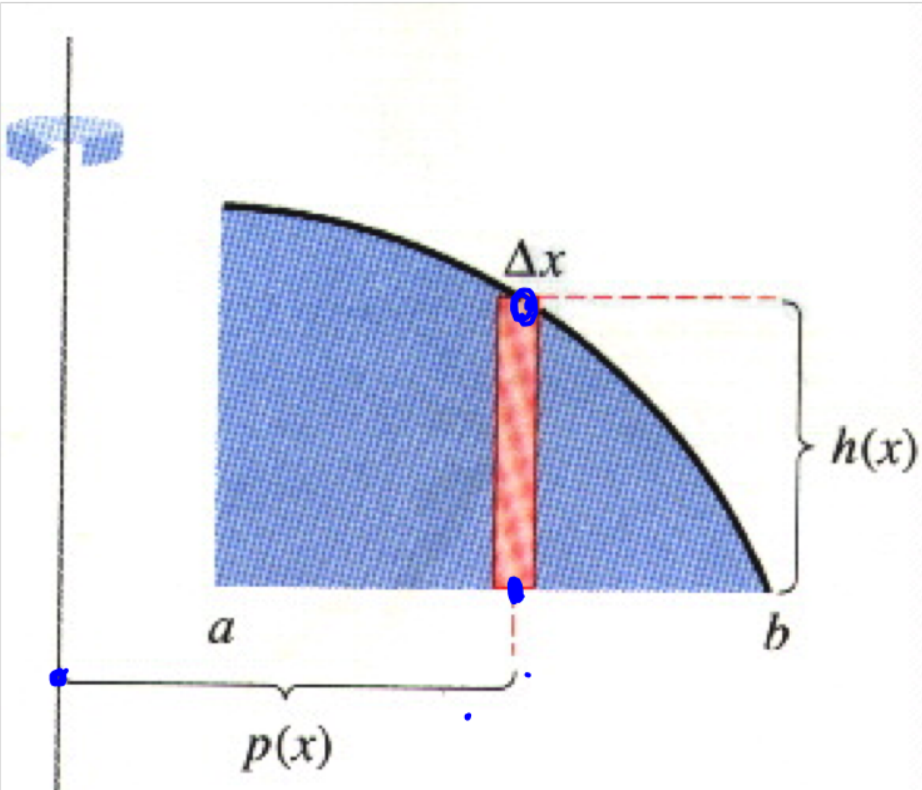
Now for a different method to find volume of revolution:

In the Shell Method, the rectangle of revolution is parallel to the axis of revolution.

r

Revolving about the y-axis or a vertical axis:

$$V = \int_a^b 2\pi\, p(x) h(x)\, dx$$



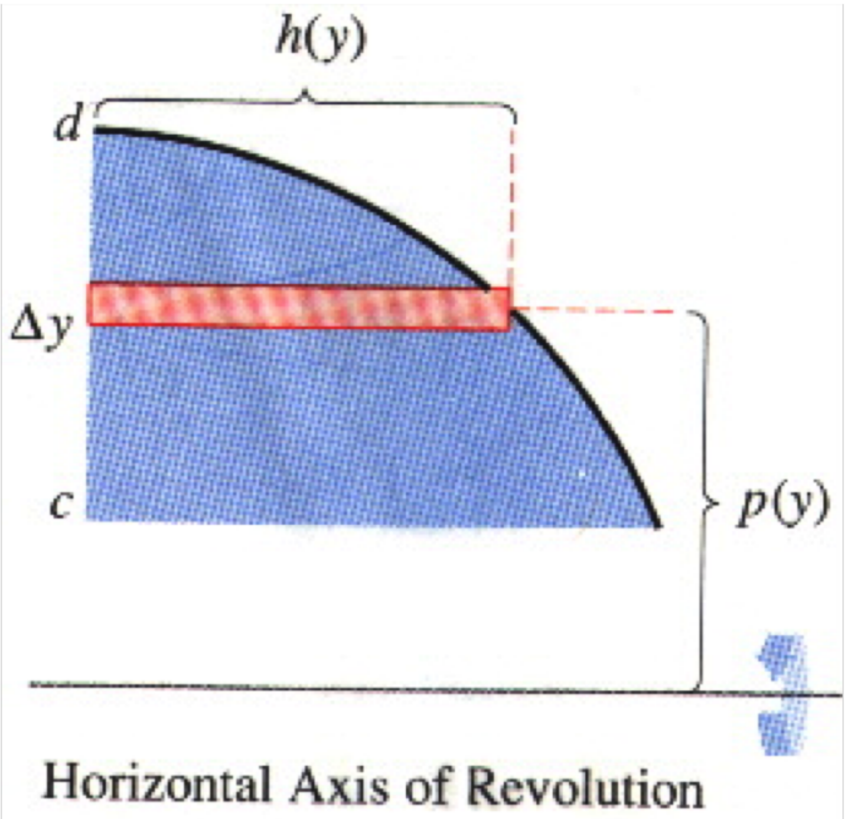
Vertical Axis of Revolution

if y-axis $p(x) = x$

$h(x) =$ top - bottom

Revolving about the x-axis or a horizontal axis:

$$V = \int_c^d 2\pi\, p(y) h(y)\, dy$$



x-axis

$p(y) = y$

$h(y) = \text{Right} - \text{left}$

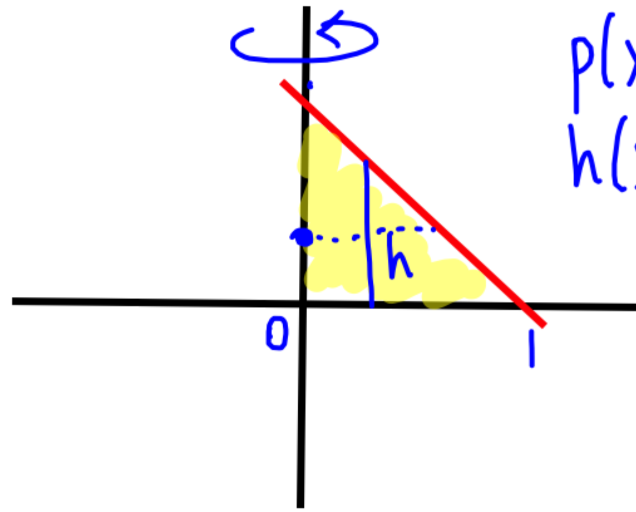
$p(x); p(y)$: Distance from the axis of revolution to center of revolution; radius

$h(x); h(y)$: Height of rectangle (big – little), (top – bottom), (right – left)

$dx; dy$: Width of rectangle

Find the volume of the solid formed by rotating about the y – axis using the shell method.

y = 1 – x, x = 0, y = 0



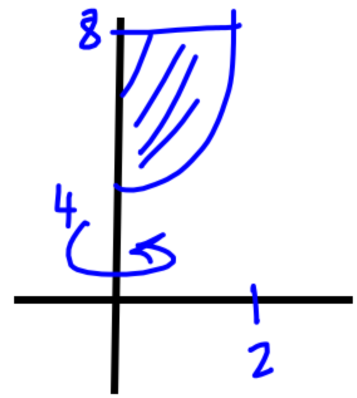
$p(x) = x$

$h(x) = 1 - x - 0$

$$V = \int_0^1 2\pi\, x\,(1-x)\,dx$$

Find the volume of the solid formed by rotating the region in the first quadrant about the y – axis using the shell method.

$y = x^2 + 4$, $x = 0$, $y = 8$



$p(x) = x$

$h(x) = 8 - (x^2 + 4)$

$= 4 - x^2$

$$V = \int_0^2 2\pi x \, (4 - x^2) \, dx$$

Find the volume of the solid formed by rotating about the x – axis using the shell method.

$y = 2 - x,\ x = 4,\ y = 0$

Give the formula for the volume of the solid formed by rotating about the y – axis using the shell method then by the disc method.

$y = x^2 + 1$, $x = 0$, $x = 1$, $y = 0$