Math 1432

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Office Hours:

Mondays 1-2pm,
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(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

Quiz 4 - 7.4 (Volume)
due Sat 2/6

Rev

Disc (no holes)

x-axis (horizontal axis)

$$V = \int_a^b \pi r^2 \, dx$$

y-axis (vertical axis)

$$V = \int_c^d \pi R^2 \, dy$$

$r = f(x)$ if x-axis

or $r = f(x) -$ axis

or $r =$ axis $- f(x)$

$R = g(y)$ if y-axis

$R = g(y) -$ axis

or

$R =$ axis $- g(y)$

Rev

Washer method (holes)

x-axis (horizontal)

$$V = \int_a^b \pi \left[R^2 - r^2\right] dx$$

y-axis (vertical)

$$V = \int_c^d \pi \left[R^2 - r^2\right] dy$$

R = dist from axis to outermost graph

r = dist from axis to innermost graph

Rev

Shell method

x-axis (horizontal)

$$V = \int_c^d 2\pi\, p(y)\, h(y)\, dy$$

p = dist from axis (x or y)

y-axis (vertical)

$$V = \int_a^b 2\pi\, p(x)\, h(x)\, dx$$

h = shaded region (top-bottom or Rt-left)

Cross Sections

cross sections are

$$V = \int_a^b A(x)\,dx$$ $\perp$ to x axis

$$V = \int_c^d A(y)\,dy$$ $\perp$ to y-axis

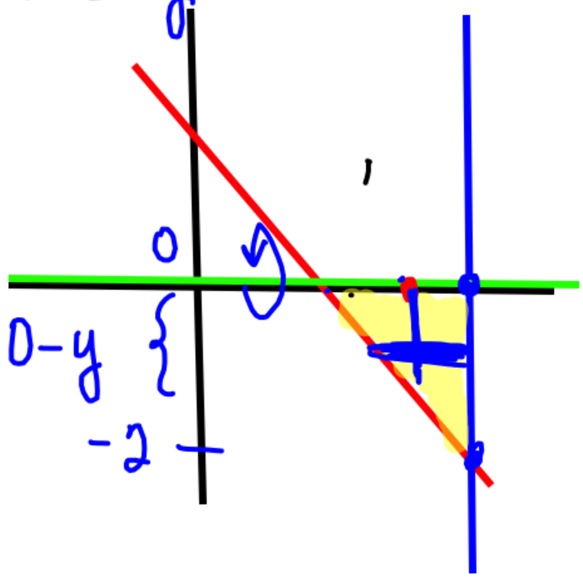
A = area of each cross section

Find the volume of the solid formed by rotating about the x – axis using the shell method.

$$V = \int_c^d 2\pi p(y) h(y)\, dy$$

y = 2 – x, x = 4, y = 0

$x = 2 - y$



$2 - x \Rightarrow$ plug in 4

$2 - 4 = -2$

$$\int_{-2}^{0} 2\pi(-y)(2+y)\,dy = \int_{-2}^{0} -2\pi(2y+y^2)\,dy$$

$h(y) = 4 - (2-y)$

$$= -2\pi \int_{-2}^{0} 2y + y^2\, dy$$

$$= -2\pi\left(y^2 + \frac{y^3}{3}\right)\Big|_{-2}^{0}$$

$$= -2\pi\left[0 - \left(4 - \frac{8}{3}\right)\right]$$

$$-2\pi\left(-\frac{4}{3}\right) = \boxed{\frac{8\pi}{3}}$$

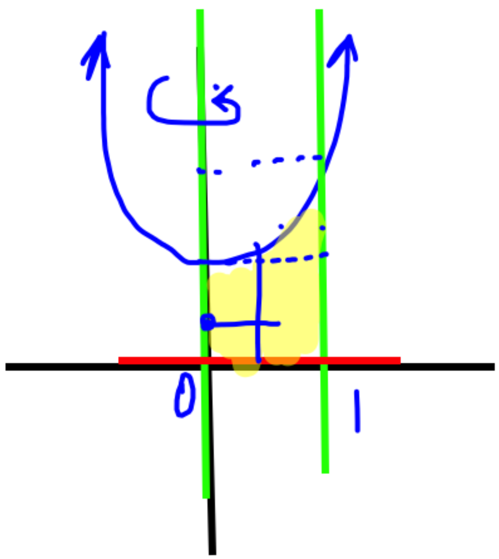
p dist from axis to y

0 - y

Disc : $V = \int_2^4 \pi(-2+x)^2\, dx$

Give the formula for the volume of the solid formed by rotating about the y – axis using the shell method then by the disc method.

$y = x^2 + 1$, $x = 0$, $x = 1$, $y = 0$



$$V = \int_0^1 2\pi (x)(x^2+1)\,dx$$

$$= 2\pi \int_0^1 (x^3 + x)\,dx$$

$$= 2\pi \left(\frac{x^4}{4} + \frac{x^2}{2} \right)\Big|_0^1$$

$$= 2\pi \left(\frac{1}{4} + \frac{1}{2} \right) = \frac{3\pi}{2}$$

Popper 03

1. Which of the following would give the volume of the region bounded by $y = 3x - x^2$ and y = 0 rotated about the x-axis?

a. $\pi \int_0^3 \left(3x - x^2\right)^2 dx$

b. $\pi \int_0^3 \left(9x^2 + x^4\right) dx$

c. $2\pi \int_0^3 y\sqrt{9 - 4y}\, dy$

d. $\pi \int_0^{\sqrt{3}} \left(3x - x^2\right) dx$

e. $\pi \int_0^{9/4} y^2\, dy$

2. Which of the following would represent the length of the inner radius for the volume of the region bounded by $y = 3x - x^2$ and y = x rotated about the x-axis?

a. x

b. $2x - x^2$

c. $3x - x^2$

d. $x^2 - 2x$

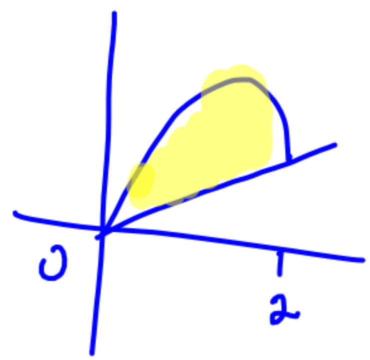
e. none of these

$3x - x^2 = x$

$2x - x^2 = 0$

$x(2 - x) = 0$

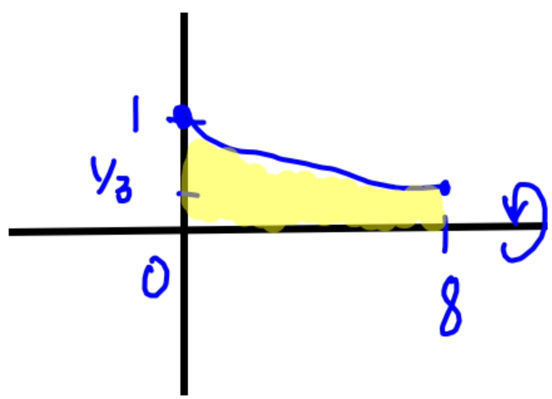
$x = 0, 2$



$V = \int_0^2 \pi (R^2 - r^2)\, dx$

Mixed examples:

The region bounded by the graph of $f(x)=\dfrac{1}{\sqrt{1+x}}$ and the $x-$axis for $0\le x\le 8$ is revolved about the $x-$axis. Find the volume of the solid that is generated.



$$V=\int_0^8 \pi\left(\frac{1}{\sqrt{1+x}}\right)^2 dx$$

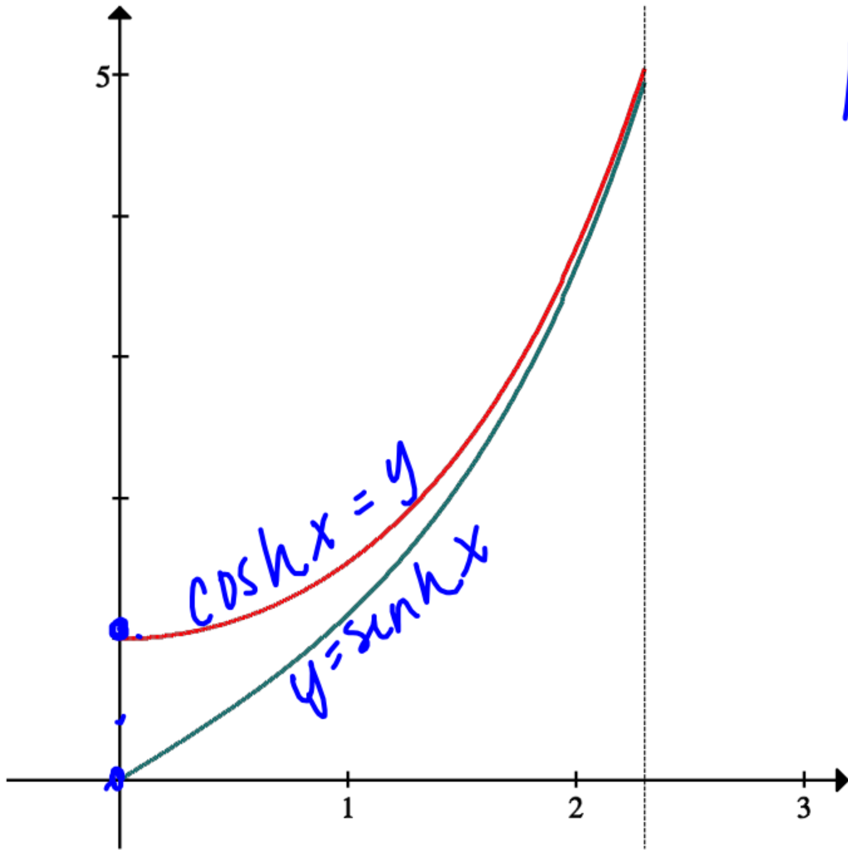
$$=\pi\int_0^8 \frac{1}{1+x}\,dx$$

$$=\pi \ln|1+x|\Big|_0^8$$

$$=\pi\left[\ln(9)-\ln(1)\right]$$

$$=\boxed{\pi\ln(9)}$$

Find the area bounded by the graphs of $f(x) = \sinh(x)$ and $f(x) = \cosh(x)$ for $0 \le x \le \ln 10$



$$A = \int_0^{\ln 10} (\cosh x - \sinh x)\, dx$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x - \sinh x = \frac{2e^{-x}}{2} = e^{-x}$$

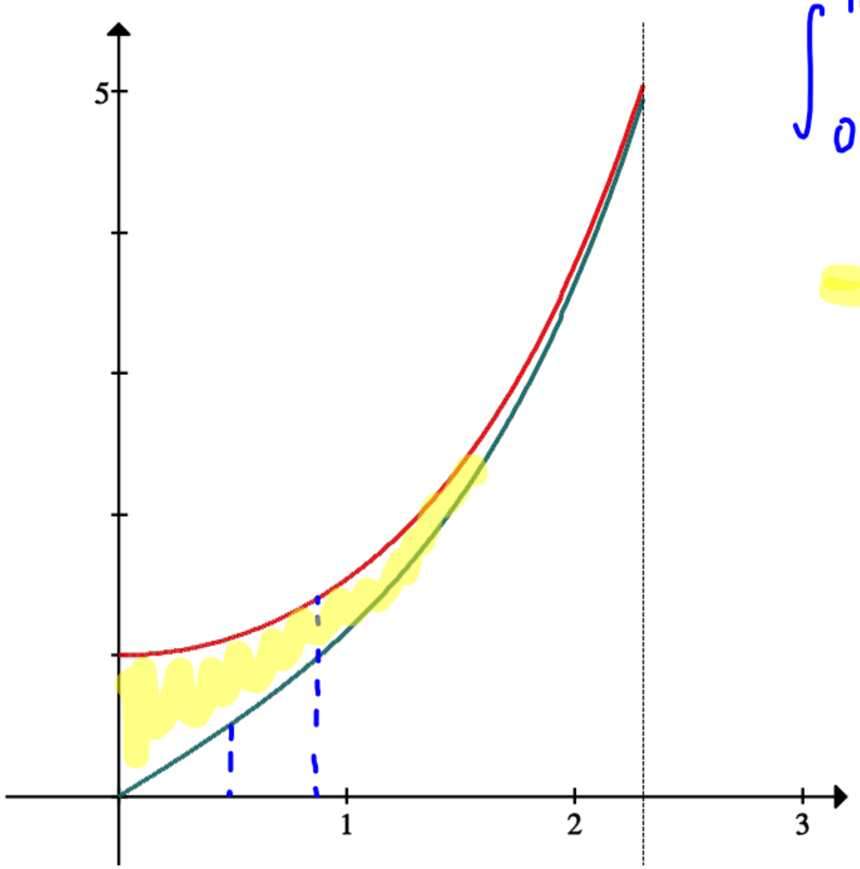
$$A = \int_0^{\ln 10} e^{-x}\, dx$$

$$= -e^{-x} \Big|_0^{\ln 10}$$

$$= -e^{-\ln 10} - -e^0 = -\frac{1}{10} + 1 = \frac{9}{10}$$

$$e^{-\ln 10} = e^{\ln 10^{-1}} = e^{\ln(1/10)}$$

Find the volume when the region bounded by the graphs of $f(x)=\sinh(x)$ and $f(x)=\cosh(x)$ is revolved around the $x-$axis for $0 \le x \le \ln 10$

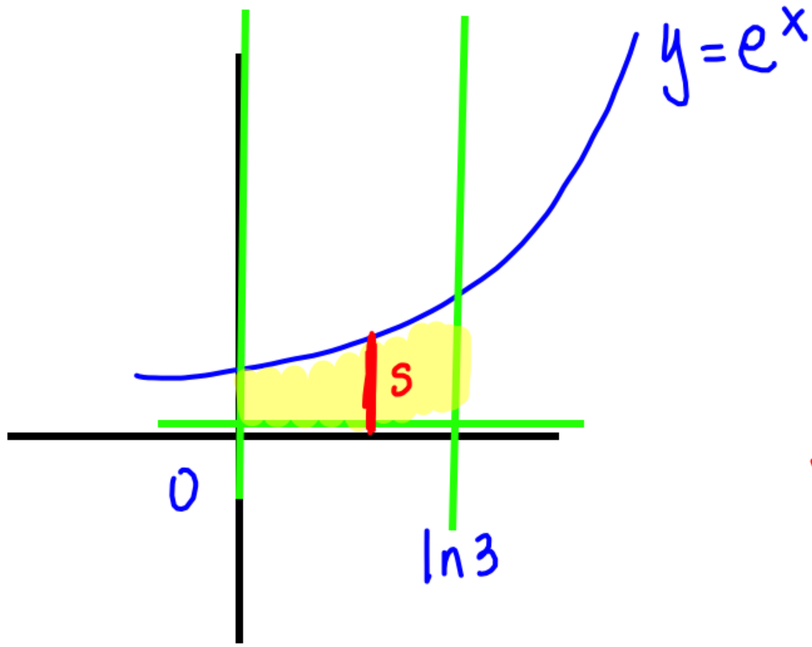


$$\int_0^{\ln 10} \pi\left(\cosh^2 x - \sinh^2 x\right) dx$$

$$= \pi \int_0^{\ln 10} dx = \pi x \Big|_0^{\ln 10}$$

$$= (\ln 10)\pi$$

The base of a solid is the region enclosed by $y = e^x$, the x-axis, the y-axis and the line $x = \ln 3$. Cross sections perpendicular to the x-axis are squares. Write an integral that represents the volume of the solid.



$s = e^x - 0 = e^x$

$A = s^2 = (e^x)^2 = e^{2x}$

$V = \int_0^{\ln 3} e^{2x}\,dx$

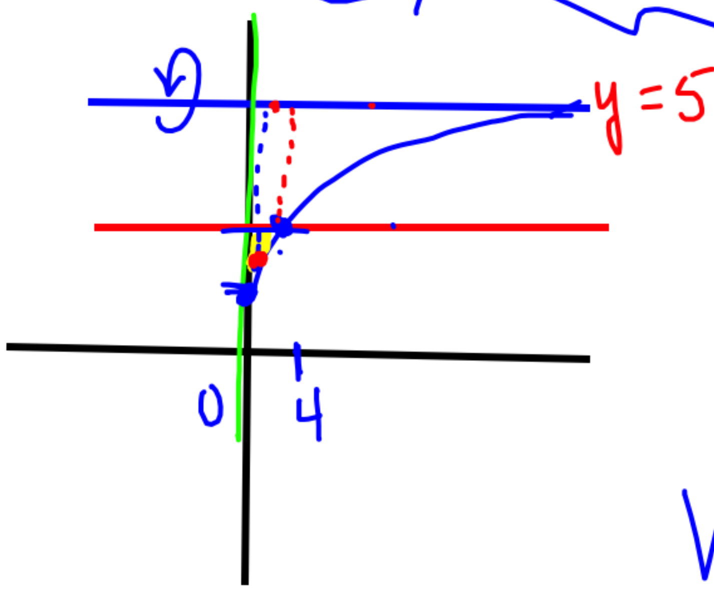
$\frac{1}{2} e^{2x} \Big|_0^{\ln 3}$

$2\ln 3 = \ln 3^2$

$= \frac{1}{2} e^{2\ln 3} - \frac{1}{2} e^0$

$\frac{1}{2} e^{\ln 9} - \frac{1}{2} = \frac{9}{2} - \frac{1}{2} = \boxed{4}$

Find the volume of the solid formed by rotating the region bounded by the graph of $y = \sqrt{x} + 1$, the y-axis, and the line y = 3 about the line y = 5.



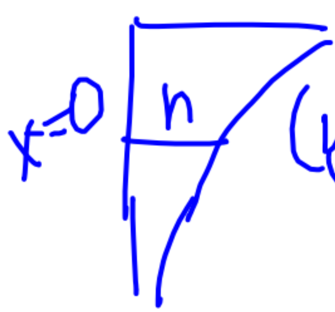
$$\sqrt{x} + 1 = 3$$
$$\sqrt{x} = 2$$
$$x = 4$$

$$V = \int_0^4 \pi \left[(4 - \sqrt{x})^2 - 2^2 \right] dx$$ (washer)

$R = 5 - (\sqrt{x} + 1)$

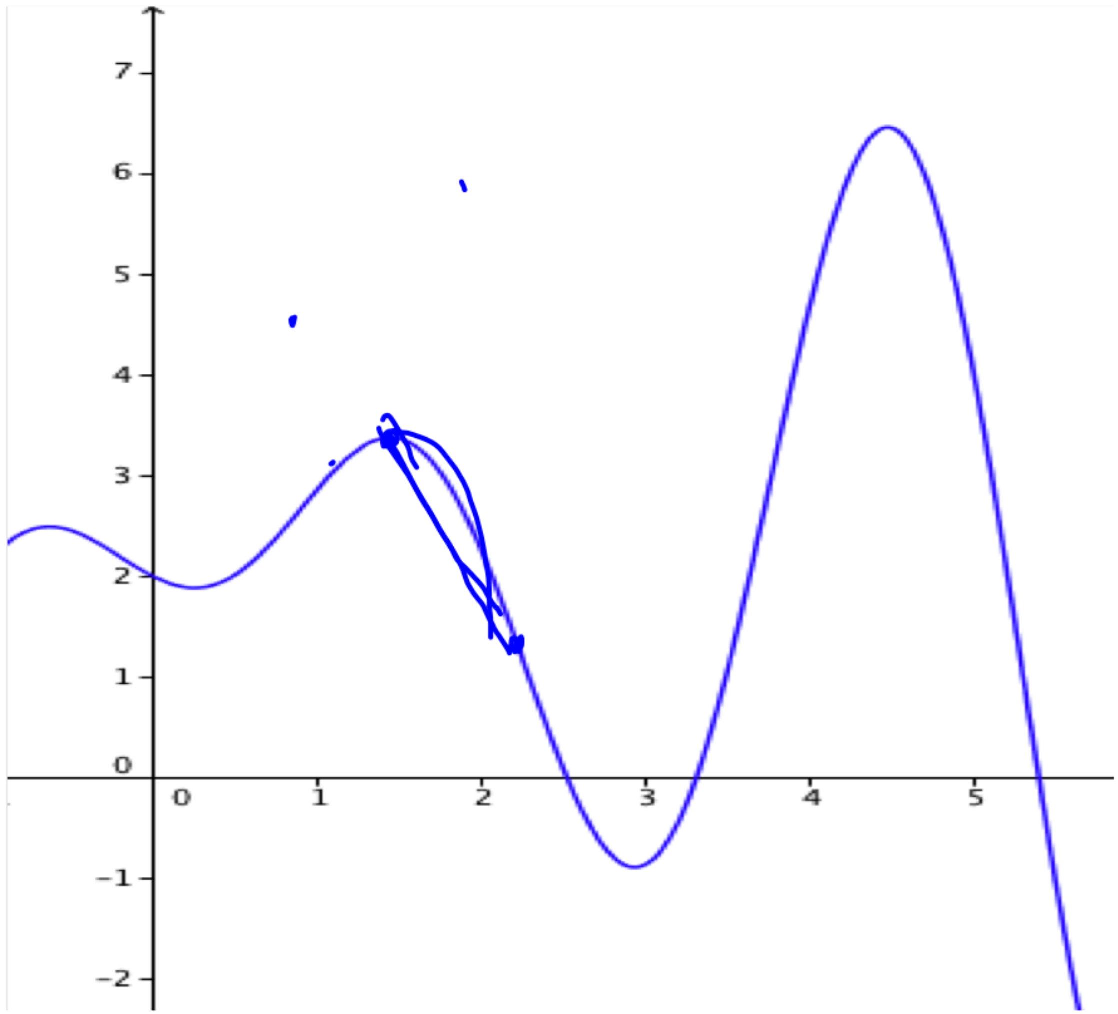
$r = 2$

$$V = \int_1^3 2\pi (5-y)(y-1)^2 \, dy$$



$(y-1)^2 = x$

Arc Length



Formula:

$$L = \int_C ds$$

where $\quad ds = \sqrt{1+(f'(x))^2}\, dx \quad$ for the curve C traced by $y = f(x), a \le x \le b$

or $\quad ds = \sqrt{1+(g'(y))^2}\, dx \quad$ for the curve C traced by $x = g(x), c \le y \le d$

Examples:

Give a formula for the length of the "curve" given by the graph of

$f(x) = 2x + 1$ for $1 \le x \le 3$.

$$L = \int_1^3 \sqrt{1 + 2^2}\, dx = \int_1^3 \sqrt{5}\, dx = \sqrt{5}\,x \Big|_1^3$$

$$f'(x) = 2$$

$$= \boxed{2\sqrt{5}}$$

Give a formula for the length of the curve given by the graph of $f(x) = x^2$ for $-1 \le x \le 1$.

$f'(x) = 2x$

$L = \int_{-1}^{1} \sqrt{1 + (2x)^2}\, dx$

Find the length of the curve $x = \frac{2}{3}(y-1)^{3/2}$ for $1 \le y \le 4$

$$x' = (y-1)^{1/2}$$

$$\int_1^4 \sqrt{1 + [(y-1)^{1/2}]^2}\, dy$$

$$= \int_1^4 \sqrt{1+y-1}\, dy = \int_1^4 \sqrt{y}\, dy$$

$$\frac{2}{3} y^{3/2} \Big|_1^4 = \frac{16}{3} - \frac{2}{3} = \boxed{14/3}$$

Free popper Friday!!
Poppers 3-6 all B.