Math 1432

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Office Hours:

Mondays 1-2pm,
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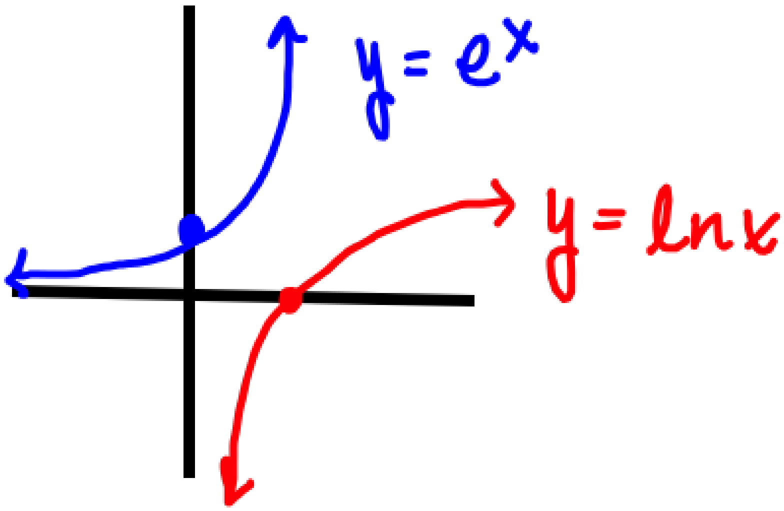
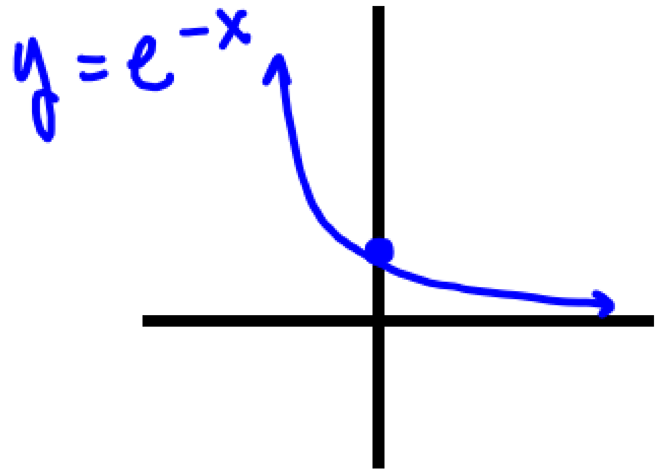
(also available by appointment)

Class webpage:
http://www.math.uh.edu/~bekki/Math1432.html





$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} e^{u(x)} = e^{u(x)} \cdot u'(x)$$

$$y = a^x$$

$$\ln y = \ln a^x$$

$$\ln y = x \cdot \ln a$$

$$\frac{y'}{y} = \ln a$$

$$y' = y \cdot \ln a$$

$$y' = a^x \ln a.$$

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1. Find $\frac{dy}{dx}$ *if* $y = 2^x$.

A. $2^x \ln 2$

B. $x2^{x-1}$

C. 2^x

D. $\ln 2^x$

E. None of these

2. Evaluate $\int_e^{4e} \frac{1}{x}\, dx = \ln|x| \Big|_e^{4e}$ ✱ $\ln xy = \ln x + \ln y$

A. $\ln 3e$

B. $\dfrac{15}{16e^2}$

C. $-\dfrac{3}{4e}$

D. $\ln 4$

E. None of these

3. Find y' if $y = e^{\sin x}$.

A. $e^{\sin x}$

B. $(\cos x)e^{\sin x}$

C. $(\sin x)e^{\sin x - 1}$

D. $e^{\cos x}$

E. none of these

4. Give the domain of the function $g(x) = \ln(1 - 3x)$

A. $x > 0$

$1 - 3x > 0$

B. $x > 1/3$

C. $x < 1/3$

D. $x < -1/3$

E. none of the above

Find the solution to this differential equation subject to the given initial condition.

$$y' = y - 2 \quad y(\overset{x}{0}) = \overset{y}{6}$$

$$\frac{dy}{dx} = y - 2$$

$$dy = (y-2)\,dx$$

$$\int \frac{1}{y-2}\,dy = \int dx$$

$$\ln|y-2| = x + C$$ ← general solution

$$\ln 4 = 0 + C$$

$$\ln 4 = C$$

Specific Soln:

$$\ln|y-2| = x + \ln 4$$

$$\rightarrow \ln|y-2| = x + \ln 4$$

$$e^{\ln|y-2|} = e^{(x+\ln 4)}$$

$$x^2 \cdot x^3 = x^5$$

$$y-2 = e^x \cdot e^{\ln 4}$$

$$y-2 = 4e^x$$

$$\boxed{y = 4e^x + 2}$$

Find a function that satisfies y’ = –2y and y(0) = 3

$$\frac{dy}{dx} = -2y$$

$$\int \frac{1}{y}\,dy = \int -2\,dx \qquad \text{OR} \qquad \frac{-1}{2y}\,dy = dx$$

$$\ln|y| = -2x + C$$

$$\ln 3 = 0 + C$$

$$\boxed{\ln|y| = -2x + \ln 3}$$

$$y = 3e^{-2x}$$

Find a function that satisfies $y' = e^{x+y}$ $\quad y(\underset{x}{0}) = \underset{y}{0}$

$$\frac{dy}{dx} = e^x \cdot e^y$$

$$\frac{1}{e^y}\,dy = e^x\,dx$$

$$\int e^{-y}\,dy = \int e^x\,dx$$

$$-e^{-y} = e^x + C$$

$$-1 = 1 + C$$

$$-2 = C$$

$$\boxed{-e^{-y} = e^x - 2}$$

or

$$e^{-y} = -e^x + 2$$

Write and solve a differential equation for the rate of change of y with respect to time is proportional to y, given that y > 0.

$$\frac{dy}{dt} = Ky$$

$$\int \frac{1}{y}\,dy = \int K\,dt$$

$$\ln y = Kt + C$$

$$y = e^{(Kt+C)} = e^{Kt} \cdot e^{C}$$

These type of differential equations are used in exponential growth and decay models.

Exponential Growth and Decay models are used in:

Population Growth/Decay
Radioactive Decay
Investments
Mixing problems
Newton's Law of Cooling

$$y = A(0)e^{Kt}$$

Formula:

$$A(t) = A_0 e^{kt}$$

A_0= A(0) = initial amount
k = growth/decay rate
+ −

Note that we have a quantity that changes at a rate proportional to itself!

At what rate r of continuous compounding does a sum of money double in 10 years?

$$A(t) = A_0 e^{rt}$$

$$2x = x e^{r(10)}$$

$$\ln(2) = \ln e^{r(10)}$$

$$\ln(2) = 10r$$

$$\frac{\ln(2)}{10} = r$$

$$A(t) = A_0 e^{Kt}$$

In a bacteria growing experiment, a biologist observes that the number of bacteria in a certain culture triples every 4 hours. After 12 hours, it is estimated that there are 1 million bacteria in the culture.

a. How many bacteria were present initially?

$$3x = x e^{K(4)}$$

$$3 = e^{4K}$$

$$\ln 3 = 4K$$

$$\left(\frac{\ln 3}{4}\right) = K$$

$$A(t) = A_0 e^{\left(\frac{\ln 3}{4}\right)t}$$

$$1{,}000{,}000 = A_0 e^{\left(\frac{\ln 3}{\cancel{4}}\right)\left(\frac{3}{\cancel{12}}\right)}$$

$3\ln 3 = \ln 27$

$$1{,}000{,}000 = A_0 \cancel{e^{\ln 27}} (27)$$

$$A_0 = \frac{1{,}000{,}000}{27}$$

b. What is the doubling time for the bacteria population?

$$A(t) = \frac{1000000}{27} e^{\left(\frac{\ln 3}{4}\right)t}$$

← formula for any time t

$$2\cancel{x} = \cancel{x} e^{\left(\frac{\ln 3}{4}\right)t}$$

$$\ln 2 = \frac{\ln 3}{4} t$$

$$\boxed{t = \frac{4\ln 2}{\ln 3}}$$

doubling time

A 100-liter tank initially full of water develops a leak at the bottom. Given that 10% of the water leaks out in the first 5 minutes, find the amount of water left in the tank 15 minutes after the leak develops if the water drains off at a rate that is proportional to the amount of water present.

$A(0) = 100\text{ L}$

$A(5) = 90\text{ L}$

$A(15) = ?$

$A(t) = 100e^{\left(\frac{\ln .9}{5}\right)t}$

$A(15) = 100e^{\left(\frac{\ln .9}{5}\right)(15)}$

$= 100e^{3\ln(.9)}$

$= 100e^{\ln(.9)^3} = \boxed{100(.9)^3}\text{ L}$

$A(t) = 100e^{Kt}$

$90 = 100e^{K(5)}$

$.9 = e^{5K}$

$\ln(.9) = \ln e^{5K} = 5K$

$\frac{\ln(.9)}{5} = K$

A certain species of virulent bacteria is being grown in a culture. It is observed that the rate of growth of the bacterial population is proportional to the number present. If there were 5000 bacteria in the initial population and the number doubled after the first 60 minutes, how many bacteria will be present after 4 hours?

↳ 1 hr

$$A(t) = 5000e^{kt}$$

$$10{,}000 = 5000e^{k(1)}$$

$$2 = e^{k}$$

$$\ln 2 = k$$

$$A(t) = 5000e^{(\ln 2)t}$$

$$A(4) = 5000e^{4\ln 2} = 5000e^{\ln 16}$$

$$= \underline{5000(16)} = 80{,}000$$

* double time

$$k = \frac{\ln 2}{t}$$

half life

$$k = \frac{\ln(1/2)}{t} = \frac{-\ln 2}{t}$$

After 3 days a sample of radon-222 decayed to 58% of its original amount. What is the half-life of radon-222? How long would it take the original sample to decay to 10% of its original amount?

$$A(3) = .58(A_0)$$

$$.58(A_0) = A_0 e^{K(3)}$$

$$.58 = e^{3K}$$

$$\ln(.58) = 3K$$

$$\frac{\ln(.58)}{3} = K$$

$$A(t) = A_0 e^{\frac{\ln(.58)}{3} t}$$

find t such that

$$\frac{1}{2} A_0 = A_0 e^{\left(\frac{\ln .58}{3}\right) t}$$

$$\frac{1}{2} = e^{\left(\frac{\ln .58}{3}\right) t}$$

$$\ln(1/2) = \left(\frac{\ln .58}{3}\right) t$$

$$\frac{\ln(1/2)}{\left(\frac{\ln(.58)}{3}\right)} = t$$

$$\frac{3 \ln(1/2)}{\ln(.58)} = t$$

$$.1 = e^{\left(\frac{\ln .58}{3}\right)t}$$

$$\ln(.1) = \left(\frac{\ln .58}{3}\right)t$$

$$\frac{3\ln(.1)}{\ln(.58)} = t$$

A deposit of \$1000 is made into a fund with an annual interest rate of 10 percent. Find the time (in years) necessary for the investment to double if the interest is compounded continuously. $A(t) = A_0 e^{r \cdot t}$

$$2000 = 1000 e^{(.10)t}$$

$$2 = e^{.1t}$$

$$\ln 2 = .1 \cdot t$$

$$10 \ln 2 = t$$

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5. If 225 grams of a substance doubles in 8 days, what is the growth factor (k)?

A. 225 / 8

B. (ln 2) / 8

C. ($-ln$ 2) / 8

D. ($-ln$ 2) / 225

E. none of these