Math 1432

Bekki George
bekki@math.uh.edu
639 PGH

Office Hours:

Mondays 1-2pm,
Fridays noon-1pm
(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

Popper07

Which of the following are improper integrals?

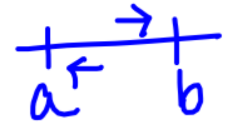
1. $\int_{-3}^{3} \frac{1}{x^{2/3}}\,dx$ **a) yes**   **b) no**

2. $\int_{1}^{9} \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ **a) yes**   **b) no**

3. $\int_{2}^{5} \frac{1}{(x-4)^2}\,dx$ **a) yes**   **b) no**

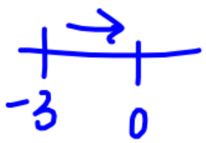
4. $\int_{0}^{\infty} \frac{1}{1+x^2}\,dx$ **a) yes**   **b) no**

7.7 – a few more notes



Proper Limit Notation:

$$\int_{-3}^{3} x^{-2/3}\,dx = \int_{-3}^{0} x^{-2/3}\,dx + \int_{0}^{3} x^{-2/3}\,dx$$



$$\lim_{b\to 0-}\int_{-3}^{b} x^{-2/3}\,dx + \lim_{a\to 0+}\int_{a}^{3} x^{-2/3}\,dx$$

$$\lim_{b\to 0-}\left[3x^{1/3}\right]_{-3}^{b} + \lim_{a\to 0+}\left[3x^{1/3}\right]_{a}^{3}$$

$$\lim_{b\to 0-}\left[3b^{1/3} - 3(-3)^{1/3}\right] + \lim_{a\to 0+}\left[3(3)^{1/3} - 3a^{1/3}\right]$$

$$3(3)^{1/3} + 3(3)^{1/3} = \boxed{6\sqrt[3]{3}}$$

$\star \; -p+1 = -(p-1)$

Important examples:

$$\int_1^{\infty} \frac{dx}{x^p} \qquad p = 1$$

$$\lim_{b\to\infty} \int_1^b \frac{dx}{x} = \lim_{b\to\infty} \left[\ln|x|\right]_1^b$$

$$= \lim_{b\to\infty} \left[\ln|b| - 0\right] = \underline{\text{diverges}}$$

$$\int_1^{\infty} \frac{dx}{x^p} \qquad p > 1$$

$$\lim_{b\to\infty} \int_1^b x^{-p}\,dx = \lim_{b\to\infty} \left[\frac{x^{-p+1}}{-p+1}\right]_1^b$$

$$= \lim_{b\to\infty} \left[\frac{1}{(1-p)\,x^{(p-1)}}\right]_1^b$$

$$= \lim_{b\to\infty} \left[\frac{1}{(1-p)\,b^{(p-1)}} - \frac{1}{(1-p)(1)^{p-1}}\right] = \frac{-1}{1-p}$$

$\to 0$

$$\int_1^{\infty} \frac{dx}{x^p} \qquad 0 < p < 1$$

divergesu

ex $\int_1^{\infty} \frac{1}{x^{2/3}}\,dx = \lim_{b \to \infty} \int_1^b x^{-2/3}\,dx$

$= \lim_{b \to \infty} \left[3x^{1/3}\right]_1^b = \lim_{b \to \infty} \left[3\underbrace{b^{1/3}}_{\to\infty} - 3\right]$

$=$ diverges

$$\int_1^{\infty} \frac{dx}{x^p}$$

Diverges for $p \le 1$
Converges for $p > 1$

Test 2 Review:

- Exam covers sections 7.1-7.7
- Review sheet is posted on class webpage ~~and is due for lab quiz grade this week (a completion grade)~~.
- Must sign up on CASA under proctored exams. You should get a confirmation email (if you didn't, you may not have registered correctly). Don't be late!!!
- No calculators.
- No formulas given and you cannot bring any notes/formulas to the exam.
- You should be working on review in recitation this week.
- Practice test is bonus (5%).

What have we covered in chapter 7?

7.1 – Integration Review

You need to know ALL integration formulas from calculus 1 (see chart from day 01).

You need to know how to use u-substitution correctly.

LOOK OVER 7.1 EXERCISES – BE ABLE TO WORK ALL OF THESE.

I will take questions from 7.1 in class.

8. $\int_{\pi/4}^{\pi/3} \frac{\sec^2 x}{\sqrt{\tan x}} dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$x = \pi/3 \rightarrow u = \tan \pi/3$

$x = \pi/4 \quad u = \tan \pi/4$

$\int_1^{\sqrt{3}} \frac{du}{\sqrt{u}} = \int_1^{\sqrt{3}} u^{-1/2} du$

$= 2u^{1/2} \Big|_1^{\sqrt{3}}$

$= 2(\sqrt{3})^{1/2} - 2(1)^{1/2}$

$2\sqrt[4]{3} - 2$

41. $\int \frac{x+e^{4\sqrt{x}}}{\sqrt{x}}\,dx$

$$= \int \frac{x^1}{x^{1/2}}\,dx + \frac{1}{2}\int \frac{2e^{4\sqrt{x}}}{\sqrt{x}}\,dx$$

$u = 4\sqrt{x}$

$du = \frac{2}{\sqrt{x}}\,dx$

$$= \int x^{1/2}\,dx + \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u$$

$$= \frac{2}{3}x^{3/2} + \frac{1}{2}e^{4\sqrt{x}} + C$$

7.2 – Average Value

Fact: Given a continuous function f defined on an interval $[a,b]$, the average value of the function on this interval is given by:

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)dx. \qquad (7.2.1)$$

Be able to work out problems like review and any problem from text.

c. $f(x)$ is positive with an area of $\frac{19}{3}$ for the region encloses between $f(x)$, the x - axis between $x=1$ and $x=5$.

$$\int_1^5 f(x)\,dx = \frac{19}{3}$$

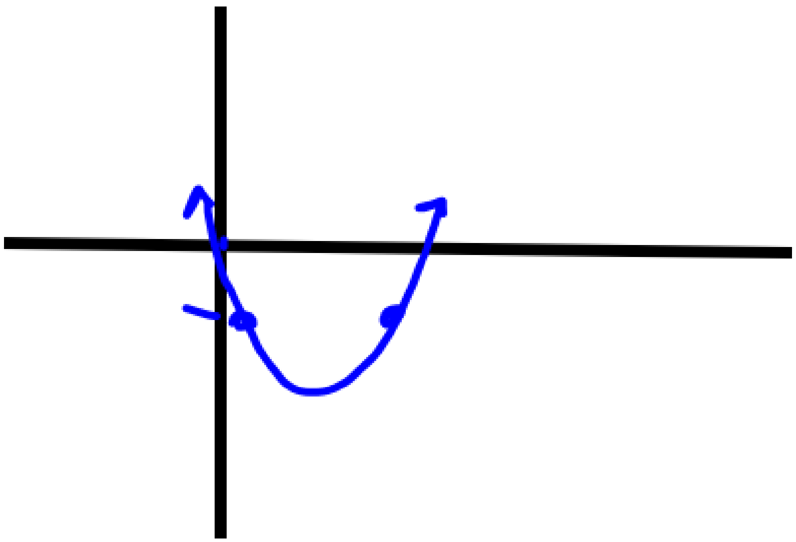
$$\frac{1}{5-1}\int_1^5 f(x)\,dx = \frac{1}{4}\left(\frac{19}{3}\right) = \frac{19}{12}$$

2. Given F(x) and the interval [a, b], graph F(x) over the interval, find the average value of F(x) on that interval and find the value of c that verifies the conclusion of the mean value theorem for integrals for the function F over the interval [a, b].

a. $F(x) = x^2 - x \quad [0,1]$

$$F(c) = \frac{1}{1-0}\int_0^1 (x^2 - x)\,dx = \left.\frac{x^3}{3} - \frac{x^2}{2}\right|_0^1$$

$$= \frac{1}{3} - \frac{1}{2} = \boxed{-\frac{1}{6}}$$ Avg value



find "c"

$x^2 - x = -1/6$

$x^2 - x + 1/4 = 0$

$6x^2 - 6x + 1 = 0$

$$x = \frac{6 \pm \sqrt{36 - 24}}{2(6)}$$

$$\boxed{\frac{1}{2} \pm \frac{2\sqrt{3}}{2(6)}}$$

29. A car starting from rest accelerates at a rate of 2 m/sec^2. Find the average speed of this object in the first 10 seconds. $[0, 10]$

$$a(t) = v'(t)$$

$$2 = v'(t)$$

$$v(t) = \int 2\,dt = 2t$$

$$\text{Avg speed} = \frac{1}{10-0}\int_0^{10} 2t\,dt$$

$$= \frac{1}{10}\left[t^2\right]_0^{10} = \frac{1}{10}(100-0) = \boxed{10 \text{ m/sec}}$$

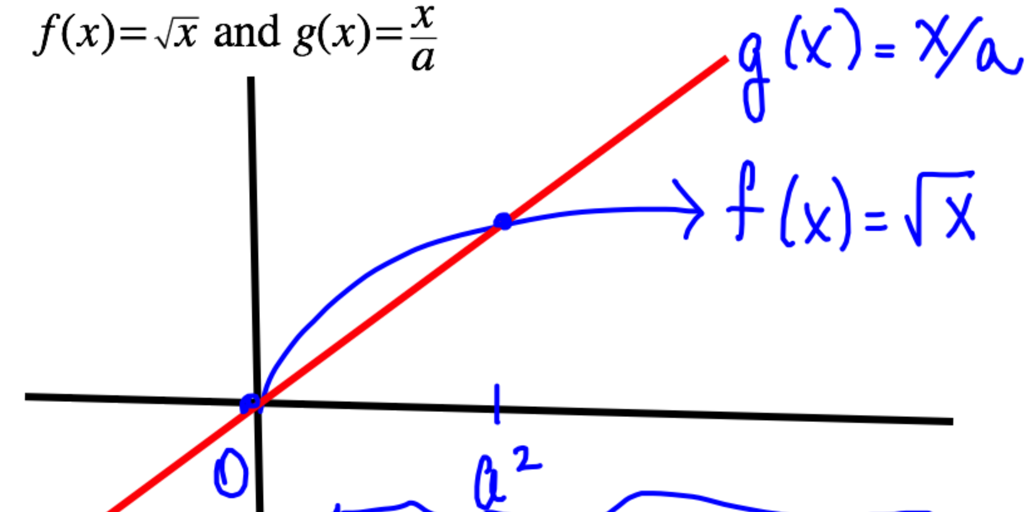
7.3 – Area

$$A = \int_a^b [f(x) - g(x)]\,dx$$

$$A = \int_c^d [h(y) - k(y)]\,dy$$

Be able to graph, set up and solve any area problems.

#3 from review sheet: Find the area of the region (c) between $f(x) = \sqrt{x}$ and $g(x) = \frac{x}{a}$



$$x/a = \sqrt{x}$$

$$x^2/a^2 = x$$

$$x^2/a^2 - x = 0$$

$$x\left(x/a^2 - 1\right) = 0$$

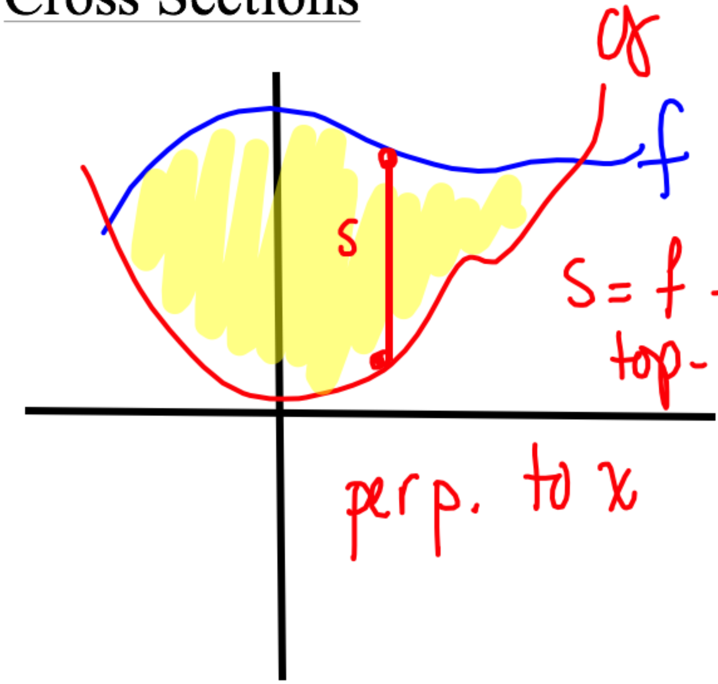
$$x = 0,\quad x/a^2 = 1 \Rightarrow x = a^2$$

$$\boxed{\int_0^{a^2} \left(\sqrt{x} - \frac{x}{a}\right) dx}$$

$$\frac{2}{3}x^{3/2} - \frac{x^2}{2a}\Big|_0^{a^2} = \frac{2}{3}(a^2)^{3/2} - \frac{(a^2)^2}{2a} = \frac{2}{3}a^3 - \frac{a^3}{2}$$

7.4 – Volume

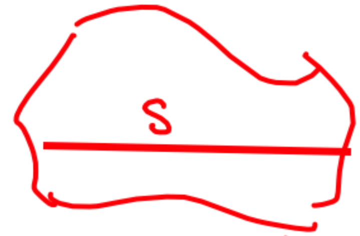
Cross Sections



$s = f - g$

top - bottom

perp. to x



$s = Rt - left$

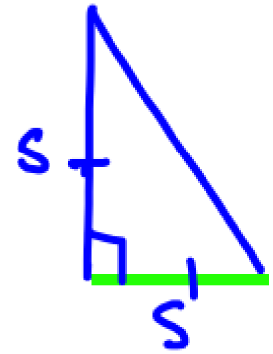
perp to y

What shape is cross section? · $A(s)$

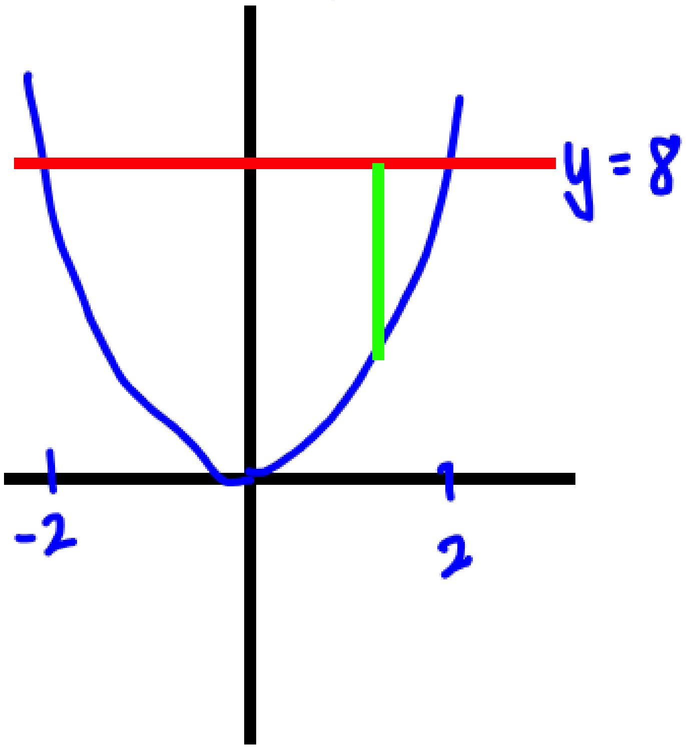
$$\int_a^b A \, d\_$$

12. The base of a solid is the region bounded by $y = 2x^2$ and $y = 8$. Find the volume of the solid given that the cross sections perpendicular to the $x-$axis are:

a. Squares
b. Semicircles
c. Right triangles with leg on the xy-plane. (isosc.



$A = \frac{1}{2}s^2$



$2x^2 = 8$

$x^2 = 4$

$x = \pm 2$

$s = (8 - 2x^2) \Rightarrow A = \frac{1}{2}(8 - 2x^2)^2$

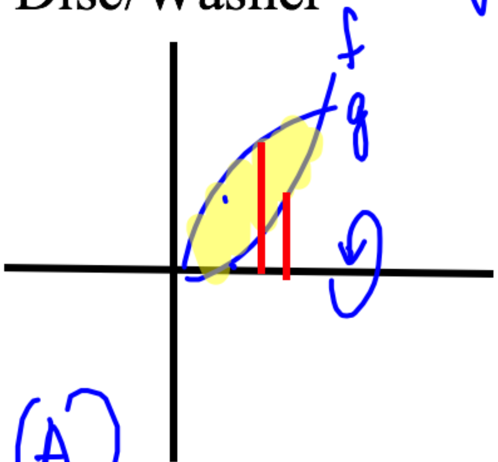
$$\int_{-2}^{2} \frac{1}{2}(64 - 32x^2 + 4x^4)\,dx$$

Of Revolution

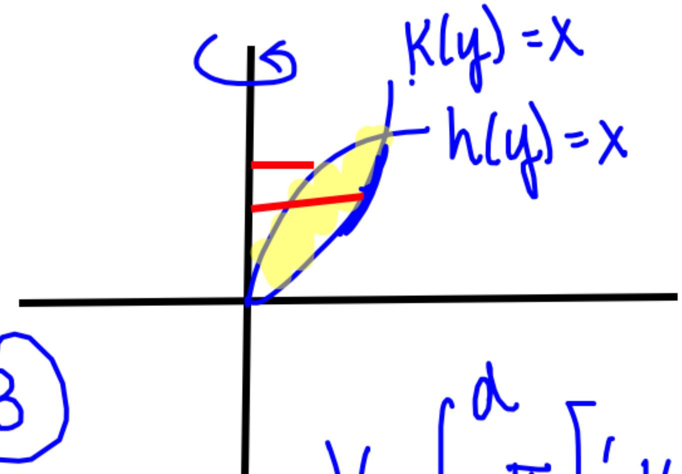
Disc/Washer

$$V = \int_a^b \pi \left[g(x)^2 - f(x)^2 \right] dx$$

$$R^2 \quad - r^2$$



(A)



(B)

$$V = \int_c^d \pi \left[(K(y))^2 - (h(y))^2 \right] dy$$

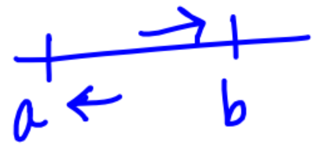
$$Rt^2 - left^2$$

$$R^2 - r^2$$

Shell

(A) $\int_c^d 2\pi y \left(K(y) - h(y) \right) dy$

(B) $\int_a^b 2\pi x \left(g(x) - f(x) \right) dx$



7.7 – a few more notes

Proper Limit Notation:

$$\int_2^5 \frac{1}{(x-4)^2}\,dx = \int_2^{4} \frac{1}{(x-4)^2}\,dx + \int_{4}^5 \frac{1}{(x-4)^2}\,dx$$

$$= \lim_{b\to 4^-} \int_2^b (x-4)^{-2}\,dx + \lim_{a\to 4^+} \int_a^5 (x-4)^2\,dx$$

$$= \lim_{b\to 4^-} \Big[-(x-4)^{-1}\Big]_2^b + \lim_{a\to 4^+} \left[-\frac{1}{x-4}\right]_a^5$$

$$= \lim_{b\to 4^-}\left(\underset{\infty}{\frac{-1}{b-4}} - \frac{-1}{2-4}\right) + \lim_{a\to 4^+}\left[\frac{-1}{5-4} - \underset{\infty}{\frac{-1}{a-4}}\right]$$

DIVERGES!

$* \quad -p+1 = -(p-1)$

Important examples:

$$\int_1^\infty \frac{dx}{x^p} \qquad p=1$$

$$\int_1^\infty \frac{1}{x}\,dx = \lim_{b\to\infty} \int_1^b \frac{1}{x}\,dx$$

$$= \lim_{b\to\infty} \Big[\ln|x|\Big]_1^b = \lim_{b\to\infty} [\ln b - 0]$$

$$= \text{diverges}$$

$$\int_1^\infty \frac{dx}{x^p} \qquad p>1$$

$$\int_1^\infty x^{-p}\,dx$$

$$= \lim_{b\to\infty} \int_1^b x^{-p}\,dx = \lim_{b\to\infty} \left[\frac{x^{-p+1}}{-p+1}\right]_1^b$$

$$= \lim_{b\to\infty} \left[\frac{1}{(-p+1)\,x^{p-1}}\right]_1^b = \lim_{b\to\infty} \left[\underbrace{\frac{1}{(1-p)\,b^{p-1}}}_{\to 0} - \frac{1}{(1-p)\,1^{p-1}}\right] = \frac{-1}{1-p}$$

$$\int_1^\infty \frac{dx}{x^p} \qquad 0 < p < 1 \qquad \ldots$$

$$\underline{\underline{\lim_{b\to\infty}}} \left[\frac{1}{(p-1)\, b^{p-1}} - \frac{1}{(p-1)\, 1^{p-1}} \right]$$

if $p < 1 \Rightarrow b$'s power is neg

which puts b in numerator

$$\infty - \frac{1}{p-1}$$

diverge

$$\int_1^\infty \frac{dx}{x^p}$$

Diverges for $p \le 1$

Converges for $p > 1$

What have we covered in chapter 7?

7.1 – Integration Review

You need to know ALL integration formulas from calculus 1 (see chart from day 01).

You need to know how to use u-substitution correctly.

LOOK OVER 7.1 EXERCISES – BE ABLE TO WORK ALL OF THESE.

I will take questions from 7.1 in class.

29. $\int \frac{x}{x^4+9}\,dx$

$u = x^2$

$du = 2x\,dx$

$a^2 = 9$

$a = 3$

$\cancel{u = x^4 + 9}$

$\cancel{du = 4x^3\,dx}$

$$\frac{1}{2}\int \frac{(2x)}{(x^2)^2+9}\,dx = \boxed{\frac{1}{2}\cdot\frac{1}{3}\arctan\left(\frac{x^2}{3}\right)+C}$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a}+C$$

37. $\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\,dx$

$~~u = 1 - \tan^2 x~~$

$du = \cancel{-2\tan x} \cdot \sec^2 x\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$$\int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C$$

$$= \arcsin(\tan x) + C$$

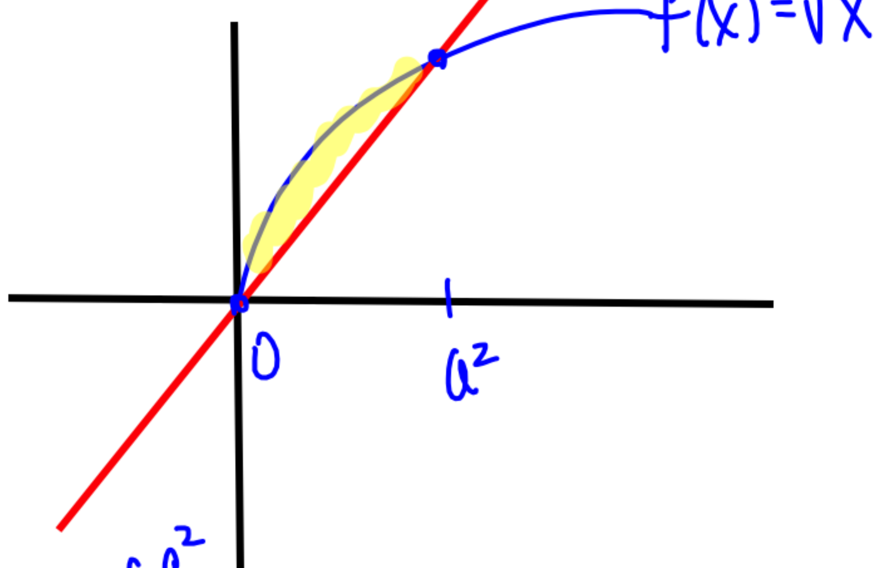
7.3 – Area

$$A=\int_a^b [f(x)-g(x)]\,dx$$ top - bottom

$$A=\int_c^d [h(y)-k(y)]\,dy$$ Rt - left

Be able to graph, set up and solve any area problems.

#3 from review sheet: Find the area of the region (c) between $f(x)=\sqrt{x}$ and $g(x)=\frac{x}{a}$



$$\frac{x}{a}=\sqrt{x}$$

$$\frac{x^2}{a^2}=x$$

$$\frac{x^2}{a^2}-x=0$$

$$x\left(\frac{x}{a^2}-1\right)=0$$

$$x=0,\quad \frac{x}{a^2}=1 \Rightarrow x=a^2$$

$$A=\int_0^{a^2}\left(\sqrt{x}-\frac{x}{a}\right)dx$$

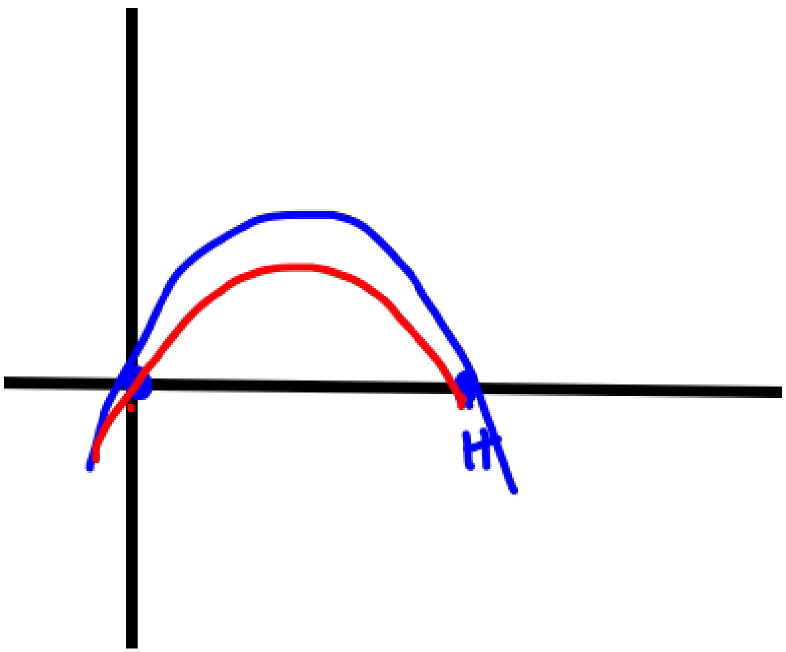
$$\frac{2}{3}x^{3/2}-\frac{x^2}{2a}\Big|_0^{a^2}=\frac{2}{3}(a^2)^{3/2}-\frac{a^4}{2a}=\frac{2}{3}a^3-\frac{1}{2}a^3=\frac{1}{6}a^3$$

26. $y = \sin x$, $y = \pi x - x^2$

$\sin x = \pi x - x^2$

$\sin 0 = 0$

$y(\pi/2) = \frac{\pi^2}{2} - \frac{\pi^2}{4} = \frac{\pi^2}{4} > 1$



$$\int_0^{\pi} \pi x - x^2 - \sin x \, dx$$

$$\frac{\pi x^2}{2} - \frac{x^3}{3} + \cos x \Big|_0^{\pi}$$

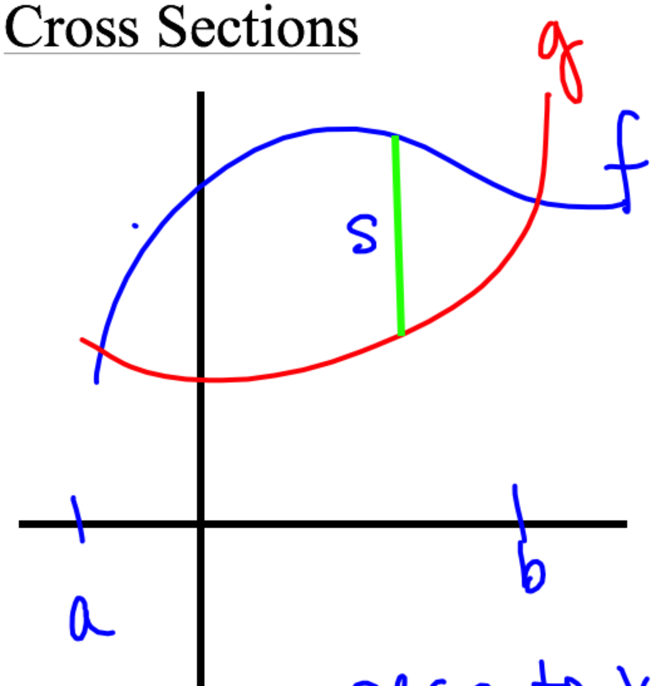
$$\left(\frac{\pi^3}{2} - \frac{\pi^3}{3} - 1\right) - (0 - 0 + 1)$$

$$\frac{\pi^3}{6} - 2$$

7.4 – Volume

Cross Sections

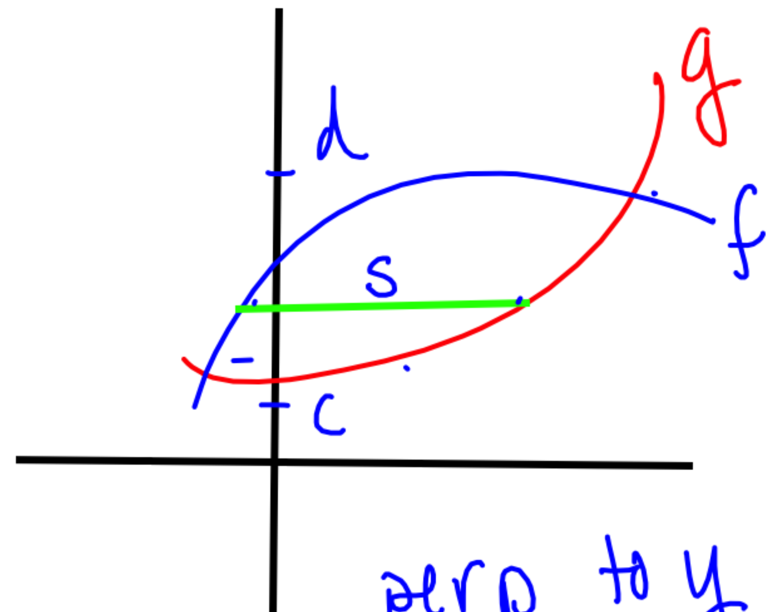
s = top - bottom



perp to x

$$\int_a^b A(x)\,dx$$

s = Rt - left



perp to y

$$\int_c^d A(y)\,dy$$

Of Revolution

Disc/Washer

no hole   hole

around x-axis : $V = \int_a^b \pi \left[R(x)^2 - r(x)^2 \right] dx$

around y-axis : $V = \int_c^d \pi \left[R(y)^2 - r(y)^2 \right] dy$

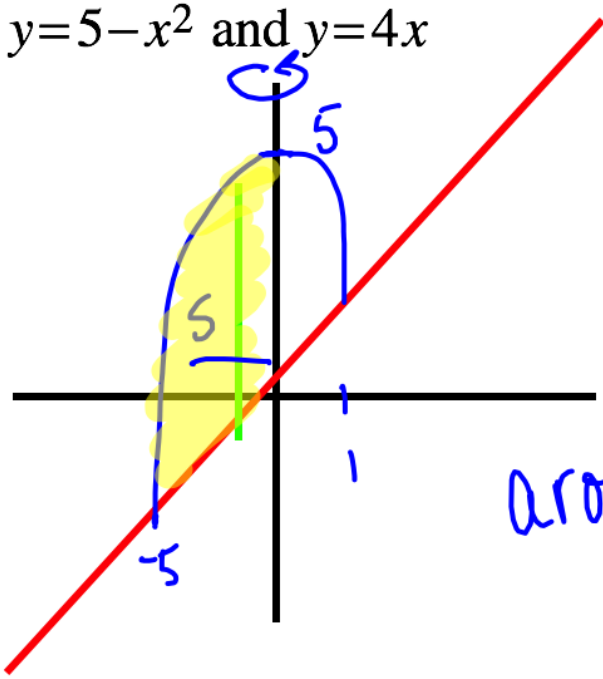
Shell

Around x-axis: $V = \int_c^d 2\pi y \,(\text{Rt} - \text{left})\, dy$

Around y-axis $V = \int_a^b 2\pi x \,(\text{top} - \text{bottom})\, dx$

R is the region bounded by the given graphs and the given axis. Sketch each graph then find the area of R, the volume when R is revolved about the x-axis and the volume when R is revolved about the y-axis

$y=5-x^2$ and $y=4x$



$5-x^2=4x$

$0=x^2+4x-5$

$0=(x+5)(x-1)$

$x=-5, 1$

around y (shell) $\int_{-5}^{0} 2\pi(-x)(5-x^2-4x)\,dx$

Give formula for volume of solid when cross sections perpendicular to the x-axis are semicircles.

$S=5-x^2-4x=$ diam

radius of semicircle is $\dfrac{5-x^2-4x}{2}$

$A=\frac{1}{2}\pi\left(\dfrac{5-x^2-4x}{2}\right)^2$

$\int_{-5}^{1}\frac{1}{2}\pi\left(\dfrac{5-x^2-4x}{2}\right)^2 dx$

7.5 – Arc Length, Centroids and Surface Area

(*you don't need to memorize surface area formula*)

Arc Length – know this formula

10. a. Give the formula for the arc length: $f(x)=\frac{2}{3}(x-1)^{3/2}, \quad x\in[1,2]$

Centroids – know formula and Pappus Thm

$y=8, y=4x, x=1$

7.6 – Differential Equations and Exponential Growth/Decay

Be able to solve a separable differential equation.

11. Find the general solution for:

(e) $y' = e^{2x}(1 + y^2)$

Exp Growth/Decay:

The half-life of radium-226 is 1620 years. How long will it take for the original amount to be reduced by 70%?

7.7 – Improper Integrals

Be able to identify an improper integral and write in proper limit notation.

Email questions for Wednesday's notes before Tuesday at 4pm

Wednesday we will review a bit more then start section 8.1!!

Popper07

#5-6 Both C.