Math 1432

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Office Hours:

Mondays 1-2pm,
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(also available by appointment)

Class webpage:

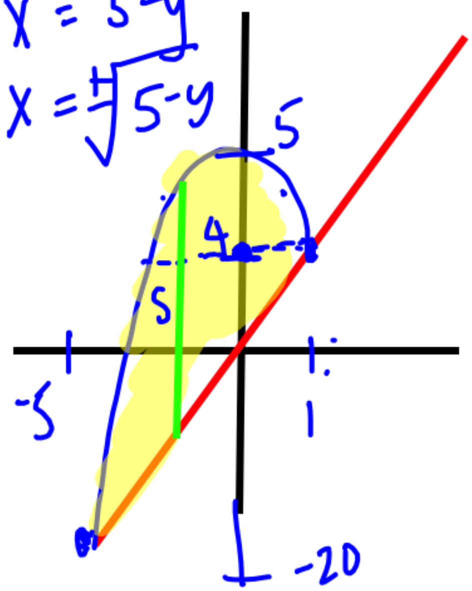
http://www.math.uh.edu/~bekki/Math1432.html

R is the region bounded by the given graphs and the given axis. Sketch each graph then find the area of R, ~~the volume when R is revolved about the x-axis and the volume when R is revolved about the y-axis~~

$y=5-x^2$ and $y=4x$

$x^2=5-y$

$x=\pm\sqrt{5-y}$



$5-x^2=4x$

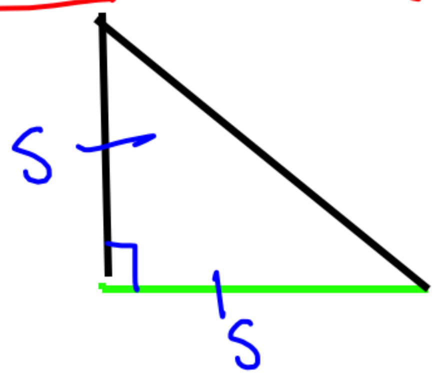
$0=x^2+4x-5$

$0=(x+5)(x-1)$

$$A=\int_{-5}^{1} 5-x^2-4x \, dx$$

$$A_y=\int_{-20}^{4} \frac{y}{4}-{}^{-}\sqrt{5-y}\, dy+\int_{4}^{5}\sqrt{5-y}-{}^{-}\sqrt{5-y}\, dy$$

Give formula for volume of solid when cross sections perpendicular to the x-axis are ~~semicircles~~. Isos. Rt Δ leg on xy plane



$S=5-x^2-4x$

$A=\frac{1}{2}\cdot S\cdot S=\frac{1}{2}S^2 \qquad A(x)=\frac{1}{2}(5-x^2-4x)^2$

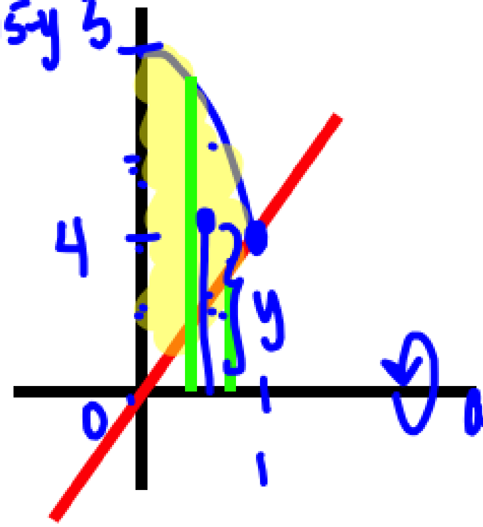
$$V=\int_{-5}^{1}\frac{1}{2}(5-x^2-4x)^2\,dx$$

1st Qud of R

R is the region bounded by the given graphs and the given axis. Sketch each graph then find the area of R, the volume when R is revolved about the x-axis and the volume when R is revolved about the y-axis

$y=5-x^2$ and $y=4x \rightarrow x = y/4$

$x = \pm\sqrt{5-y}$

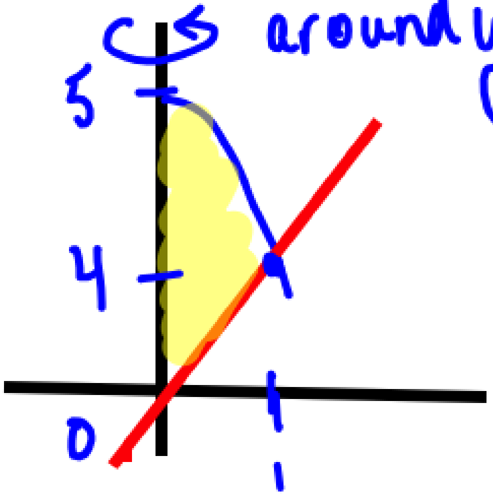


around x

washer $V = \int_0^1 \pi\left[(5-x^2)^2 - (4x)^2\right] dx$

shell $V = \int_0^4 2\pi y \left(\frac{y}{4}\right) dy + \int_4^5 2\pi y \left(\sqrt{5-y}\right) dy$

around y axis



disc $V = \int_0^4 \pi \left(\frac{y}{4}\right)^2 dy + \int_4^5 \pi \left(\sqrt{5-y}\right)^2 dy$

shell $V = \int_0^1 2\pi x (5 - x^2 - 4x)\, dx$

7.5 – Arc Length, Centroids and Surface Area

(*you don't need to memorize surface area formula*)

Arc Length – know this formula

$$\text{Arc length} = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$$

10. a. Give the formula for the arc length: $f(x)=\frac{2}{3}(x-1)^{3/2},\quad x\in[1,2]$

$$f'(x) = (x-1)^{1/2}$$

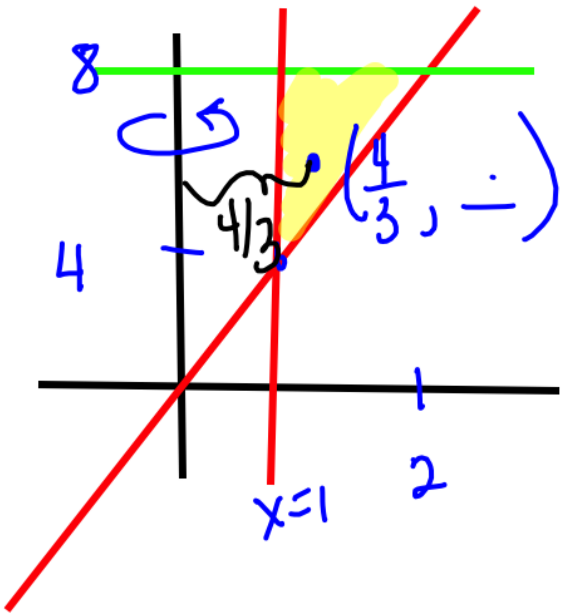
$$L = \int_1^2 \sqrt{1+(\sqrt{x-1})^2}\,dx = \int_1^2 \sqrt{x}\,dx$$

$$= \frac{2}{3}x^{3/2}\Big|_1^2 = \frac{2}{3}(2)^{3/2} - \frac{2}{3}$$

Centroids – know formula and Pappus Thm

$$V = 2\pi \bar{R} A$$

$y = 8, y = 4x, x = 1$



$$A = \int_1^2 (8 - 4x)\,dx = 8x - 2x^2 \Big|_1^2$$

$$= (16 - 8) - (8 - 2) = 2$$

$$\bar{x} = \int_1^2 x(8 - 4x)\,dx \,/\, 2$$

$$= \int_1^2 (8x - 4x^2)\,dx \,/\, 2$$

$$= \frac{\left[4x^2 - \frac{4}{3}x^3\right]_1^2}{2} = \frac{8/3}{2} = \frac{4}{3}$$

$$\bar{y} = \frac{\int_1^2 \frac{1}{2}\left(8^2 - (4x)^2\right)dx}{2}$$

$$V_y = 2\pi\left(\frac{4}{3}\right)(2) = \frac{16\pi}{3}$$

7.6 – Differential Equations and Exponential Growth/Decay

Be able to solve a separable differential equation.

11. Find the general solution for:

(e) $y' = e^{2x}(1+y^2)$

$$\frac{dy}{dx} = e^{2x}(1+y^2)$$

$$dy = e^{2x}(1+y^2)\,dx$$

$$\int \frac{1}{1+y^2}\,dy = \int e^{2x}\,dx$$

$$\arctan y = \frac{1}{2}e^{2x} + C$$

$$A(t) = A_0 e^{Kt}$$

Exp Growth/Decay:

The half-life of radium-226 is 1620 years. How long will it take for the original amount to be reduced by 70%?

half life: $K = \frac{\ln(1/2)}{t}$

double time: $K = \frac{\ln(2)}{t}$

$$\frac{1}{2} A_0 = A_0 e^{K(1620)}$$

$$\frac{1}{2} = e^{1620K}$$

$$\ln(1/2) = 1620K$$

$$\frac{\ln(1/2)}{1620} = K$$

$$K = \frac{\ln(.5)}{1620}$$

$$A(t) = A_0 e^{\left(\frac{\ln(.5)}{1620}\right)t}$$

30% left

$$.3 \cancel{A_0} = \cancel{A_0} e^{\frac{\ln(.5)}{1620} t}$$

$$\ln(.3) = \frac{\ln(.5)}{1620} t$$

$$\frac{1620(\ln .3)}{\ln(.5)} \text{ yrs} = t$$

7.7 – Improper Integrals

Be able to identify an improper integral and write in proper limit notation.

$$\int_{-2}^{0} \frac{1}{(x+2)^2}\,dx = \lim_{a \to -2^+} \int_{a}^{0} (x+2)^{-2}\,dx$$

$$= \lim_{a \to -2^+} -(x+2)^{-1} \Big|_a^0$$

$$= \lim_{a \to -2^+} \left(\frac{-1}{2} + \frac{+1}{a+2} \right) = \text{diverges}$$

0 in denom.

$u = x^2$

$du = 2x\,dx$

$(e^x)^2 = e^{2x}$

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1. $\frac{1}{2}\int 2x\,e^{x^2}\,dx =$

a. $e^{x^2} + C$

b. $2e^{x^2} + C$

c. $\frac{1}{2}e^{x^2} + C$

d. none of these

What if we had $\int x e^x \, dx$?

Section 8.1 - Integration by parts

("Undoing" the product rule)

$$\frac{d}{dx}(u \cdot v) = u'v + u \cdot v'$$

Lets start with the product rule:

$$\left[\frac{d}{dx}(uv) = u\left(\frac{d}{dx}v\right) + v\left(\frac{d}{dx}u\right)\right] dx$$

$$uv' + vu'$$

$$\int 1\, d(uv) = \int u\, dv + \int v\, du$$

$$uv = \int u\,dv + \int v\,du$$

$$uv - \int v\,du = \int u\,dv$$

So, the integration by parts formula is:

$$\boxed{\int u\,dv = uv - \int v\,du}$$

We use it to “undo” the product rule.

1. Reduction to integrate

   $x^n \sin(ax),\ x^n \cos(ax),\ x^n e^{ax},$

   $polynomial \cdot \sin(ax),\ polynomial \cdot \cos(ax),\ polynomial \cdot e^{ax}$

2. Cycling to integrate

   $\cos(ax)\sin(bx),\ \cos(ax)e^{bx},\ \sin(ax)e^{bx}$

3. Change of Form to integrate

   $\ln(x)f(x),\ \arctan(x)f(x),\ \arcsin(x)f(x)$

   (where $f(x)$ has a simple antiderivative)

How do you know what to pick for u and for dv?

"Shortcut"

ILATE

I –

L –

A –

T –

E –

1. $\int xe^x \, dx$

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2. $\int \frac{1}{9+x^2}\,dx =$

a. $\arcsin\left(\frac{x}{3}\right)+C$

b. $\arctan\left(\frac{x}{3}\right)+C$

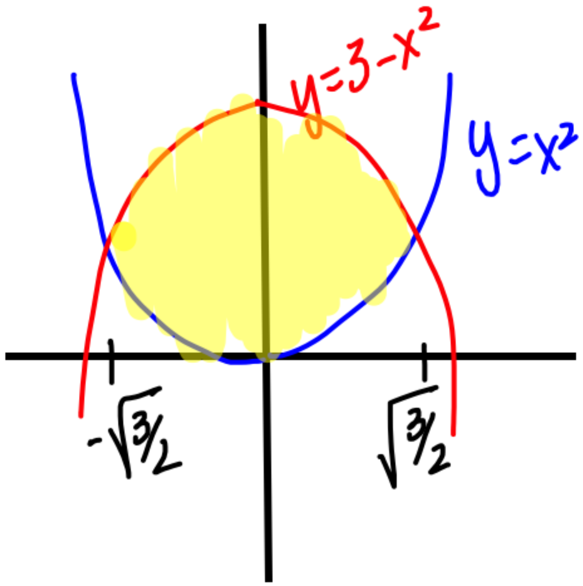
c. $\frac{1}{3}\arctan\left(\frac{x}{3}\right)+C$

d. none of these

$$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\arctan\frac{u}{a}+C$$

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \arcsin\frac{u}{a}+C$$

3. Give the formula for the area enclosed by $y=x^2$ and $y=3-x^2$



a) $A = \int_{-\sqrt{3/2}}^{\sqrt{3/2}} 2x^2-3 \, dx$

b) $A = \int_{-\sqrt{3/2}}^{\sqrt{3/2}} 3-2x^2 \, dx$

c) $A = \int_{-\sqrt{3/2}}^{\sqrt{3/2}} 3+2x^2 \, dx$

$$3-x^2=x^2$$

$$3=2x^2$$

$$\frac{3}{2}=x^2$$

$$\pm\sqrt{3/2}=x$$

4. Give the x value of the centroid for the region enclosed by $y=x^2$ and $y=3-x^2$

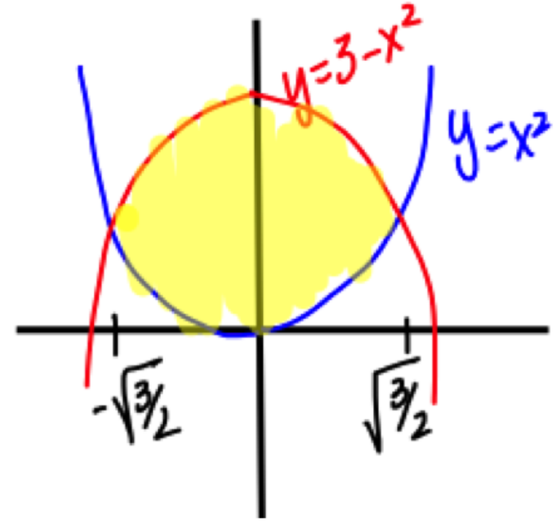
a. 0

b. ½

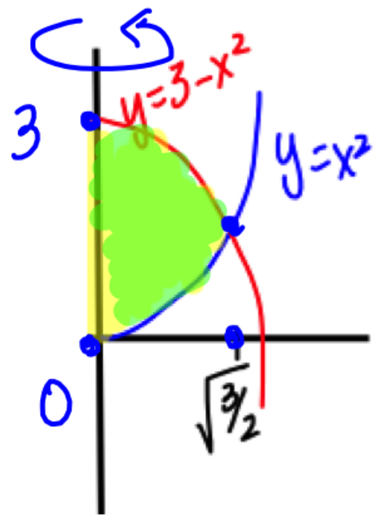
c. ¼

d. 1

e. none of these



5. Give the formula for the volume when the region enclosed by $y=x^2$ and $y=3-x^2$ *in the first quadrant* is revolved about the y – axis



shell method

A. $\int_0^{\sqrt{3/2}} 2\pi x\left(3-2x^2\right)dx$