Reminders

Review - online
Sunday @ 9am

Math 1432

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Office Hours:

Mondays 1-2pm,
Fridays noon-1pm
(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

Popper Wed - $\int x e^{x^2} dx \rightarrow$ usub.

What if we had $\int x e^x \, dx$?

Section 8.1 - Integration by parts

("Undoing" the product rule) $\frac{d}{dx}(u \cdot v) = u'v + uv'$

$$\frac{d}{dx}(u \cdot v) = u'v + u \cdot v'$$

Lets start with the product rule:

$$\left[\frac{d}{dx}(uv) = u\left(\frac{d}{dx}v\right) + v\left(\frac{d}{dx}u\right)\right]\cdot dx$$

$$uv' + vu'$$

$$d(uv) = u\,dv + v\,du$$

$$\int 1 \cdot d(uv) = \int u\,dv + \int v\,du$$

$$\int 1\,dx = x + C$$

$$\int 1\,dy = y + C$$

$$uv = \int u\,dv + \int v\,du$$

$$\boxed{uv - \int v\,du = \int u\,dv}$$

Integration by parts

So, the integration by parts formula is:

$$\boxed{\int u\,dv = uv - \int v\,du}$$

We use it to “undo” the product rule.

1. Reduction to integrate

   $x^n \sin(ax),\ x^n \cos(ax),\ x^n e^{ax},$

   $polynomial \cdot \sin(ax),\ polynomial \cdot \cos(ax),\ polynomial \cdot e^{ax}$

2. Cycling to integrate

   $\cos(ax)\sin(bx),\ \cos(ax)e^{bx},\ \sin(ax)e^{bx}$

3. Change of Form to integrate

   $\ln(x)f(x),\ \arctan(x)f(x),\ \arcsin(x)f(x)$

   (where $f(x)$ has a simple antiderivative)

$\int \arctan x\,dx$

$$\underbrace{\int u \cdot dv}_{\text{given}} = uv - \int v \cdot du$$

How do you know what to pick for *u* and for *dv*?

"Shortcut"

exp.

$$\int x e^x \, dx$$

algebraic

let $u = x$

ILATE

I – Inverses
L – logs
A – algebraic
T – trig
E – exponential

$uv - \int v\,du$

whoever is dv gets dx too

1. $\int \underset{A}{x}\,\underset{E}{e^x}\,dx$

$u = x$ $\quad dv = e^x\,dx$

$du = dx$

Integrate to find v

$\int dv = \int e^x dx$

$v = e^x$

$= \underset{u}{x} \cdot \underset{v}{e^x} - \int \underset{v}{e^x} \cdot \underset{du}{dx}$

$= \boxed{x \cdot e^x - e^x + C}$

$u = x$ $\quad dv = e^x\,dx$

$du = dx$ $\quad v = e^x$

1. $\int \underset{A}{x}\,\underset{T}{\sin}x\,dx$

$u = x \qquad dv = \sin x\,dx$

$du = dx \qquad v = -\cos x$

$$= -x\cos x + \int + \cos x\,dx$$

$$= \boxed{-x\cos x + \sin x + C}$$

	u	dv
$+$	x	$\sin x$
$-$	1	$-\cos x$
$+$	0	$-\sin x$

2. $\int \underset{A}{x^2} \underset{T}{\cos x}\, dx$

$u = x^2 \qquad dv = \cos x\, dx$

$du = 2x\, dx \qquad v = \sin x$

$$= x^2 \sin x - \underbrace{\int \underset{A}{2x}\, \underset{T}{\sin x}\, dx}$$

$u = 2x \qquad dv = \sin x\, dx$

$du = 2\, dx \qquad v = -\cos x$

$$= x^2 \sin x - \left[-2x\cos x + \underbrace{\int +2\cos x\, dx}_{2\sin x}\right]$$

$$\boxed{= x^2 \sin x + 2x\cos x - 2\sin x + C}$$

tabular method:

	u (deriv.)	dv (integrate)
$+$	x^2	$\cos x$
$-$	$2x$	$\sin x$
$+$	2	$-\cos x$
$-$	0	$-\sin x$

$x^2 \sin x + 2x\cos x$
$- 2\sin x + C$

3. $\int \overset{E}{e^x} \overset{T}{\cos x}\, dx$

$u = \cos x \qquad dv = e^x dx$

$du = -\sin x\, dx \qquad v = e^x$

$= e^x \cos x + \underbrace{\int e^x \sin x\, dx}$

$u = \sin x \qquad dv = e^x dx$

$du = \cos x\, dx \qquad v = e^x$

$\int e^x \cos x\, dx = e^x \cos x + e^x \sin x - \int e^x \cos x\, dx$

$+\int e^x \cos x\, dx \qquad\qquad + \int e^x \cos x\, dx$

$2\int e^x \cos x\, dx = e^x \cos x + e^x \sin x + C$

$\div 2 \qquad \int e^x \cos x\, dx = \frac{1}{2} e^x \cos x + \frac{1}{2} e^x \sin x + C$

$$x = 2 + 7 - x$$

$$+x \qquad\qquad\qquad +x$$

$$2x = 9$$

$u = \sec x \qquad dv = \sec^2 x\,dx$

$du = \sec x \tan x\,dx \qquad v = \tan x$

4. $\int \sec^3 x\,dx = \int \sec x \cdot \sec^2 x\,dx$

$= \sec x \tan x - \int \sec x \cdot \underbrace{\tan^2 x}_{(\sec^2 x - 1)}\,dx$

$= \sec x \tan x - \int (\sec^3 x - \sec x)\,dx$

$= \sec x \tan x - \int \sec^3 x\,dx + \int \sec x\,dx$

$\int \sec^3 x\,dx = \sec x \tan x - \int \sec^3 x\,dx + \ln|\sec x + \tan x| + C$

$+$

$2\int \sec^3 x\,dx = \sec x \tan x + \ln|\sec x + \tan x| + C$

$\int \sec^3 x\,dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$

5. $\int \underset{A}{x^2} \underset{L}{\ln x}\, dx$

$u = \ln x \qquad dv = x^2 dx$

$du = \frac{1}{x} dx \qquad V = \frac{1}{3}x^3$

$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \cdot \frac{1}{x}\, dx$

$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2\, dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$

6. $\int \ln x \, dx$

$u = \ln x$ $\qquad dv = dx$

$du = \frac{1}{x} dx$ $\qquad v = x$

$= x \ln x - \int x \cdot \frac{1}{x} dx$

$= x \ln x - \int dx$

$= x \ln x - x + C$

7. $\int \arctan x \, dx$

$u = \arctan x$ $\quad dv = dx$

$du = \frac{1}{1+x^2} dx$ $\quad V = x$

$= x \arctan x - \frac{1}{2} \int \frac{2x}{1+x^2} dx$

$w = 1+x^2$

$dw = 2x \, dx$

$= x \arctan x - \frac{1}{2} \ln(1+x^2) + C$

8. $\int x^3 e^{-x^2}\,dx = \int x^2 \cdot x e^{-x^2}\,dx$

$$= x^2\left(-\frac{1}{2}e^{-x^2}\right) - \int -\frac{1}{2}e^{-x^2}\,2x\,dx$$

$$= -\frac{1}{2}x^2 e^{-x^2} + \int x e^{-x^2}\,dx$$

$$= -\frac{1}{2}x^2 e^{-x^2} - \frac{1}{2}e^{-x^2} + C$$

	u	dv
+	$u = x^2$	$xe^{-x^2} = dv$
−	$du = 2x$	$-\frac{1}{2}e^{-x^2} = v$
+	2	?
−	0	

$$\int x e^{-x^2}\,dx$$
$$= -\frac{1}{2}e^{-x^2}$$

Integration by parts with definite integrals.

$$\int_a^b u\,dv = (uv)\Big|_a^b - \int_a^b v\,du$$

9. $\int_0^1 xe^x\,dx$

$u = x \qquad dv = e^x dx$

$du = dx \qquad v = e^x$

$= xe^x\Big|_0^1 - \int_0^1 e^x\,dx$

$= \left[xe^x - e^x\right]_0^1 = (e-e)-(0-1) = 1$

$\ln x^2 = 2\ln x$

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1. $\int x \ln x^2 \, dx =$

   a. $x^2 \ln x + C$

   b. $-\dfrac{x^2}{2} + C$

   c. $x \ln x + \dfrac{x^2}{2} + C$

   d. $x^2 \ln x - \dfrac{x^2}{2} + C$

   e. None of these

2-6 Choose B.