Math 1432

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Office Hours:

Mondays 1-2pm,
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(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

8.1

More IBP examples:

$$\boxed{\int u\,dv = uv - \int v\,du}$$

1. $\displaystyle\int_0^{\pi/2} \underset{A}{x^2}\, \underset{T}{\sin x}\, dx$

$u = x^2$ $\quad$ $dv = \sin x\,dx$

$du = 2x\,dx$ $\quad$ $v = -\cos x$

$$= -x^2\cos x \Big|_0^{\pi/2} - \int_0^{\pi/2} 2x(-\cos x)\,dx$$

$$= -x^2\cos x \Big|_0^{\pi/2} + \underbrace{\int_0^{\pi/2} 2x\cos x\,dx}_{2x\sin x \big|_0^{\pi/2} - \int_0^{\pi/2} 2\sin x\,dx}$$

$u = 2x$ $\quad$ $dv = \cos x\,dx$

$du = 2\,dx$ $\quad$ $v = \sin x$

$$= -x^2\cos x \Big|_0^{\pi/2} + 2x\sin x \Big|_0^{\pi/2} - \int_0^{\pi/2} 2\sin x\,dx$$

$$= -x^2\cos x + 2x\sin x + 2\cos x \Big|_0^{\pi/2} = -\left(\frac{\pi^2}{4}\right)(0) + \pi(1) + 2(0) - (0 + 0 + 2)$$

$$= \boxed{\pi - 2}$$

2. $\int (e^x + 2x)^2 \, dx = \int (e^{2x} + 4xe^x + 4x^2) \, dx$

$= \int e^{2x} \, dx + \int 4x^2 \, dx + \underbrace{\int 4xe^x \, dx}$

$u = 4x \quad dv = e^x dx$

$du = 4\,dx \quad v = e^x$

$= \frac{1}{2} e^{2x} + \frac{4}{3} x^3 + 4xe^x - \int 4e^x \, dx$

$= \frac{1}{2} e^{2x} + \frac{4}{3} x^3 + 4xe^x - 4e^x + C$

3. $\int \underset{A}{x^2} \underset{I}{\arctan x}\, dx$

$u = \arctan x \qquad dv = x^2\,dx$

$du = \frac{1}{1+x^2}\,dx \qquad v = \frac{x^3}{3}$

$$= \frac{x^3}{3}\arctan x - \frac{1}{3}\underbrace{\int \frac{x^3}{1+x^2}\,dx}$$

$$\begin{array}{r|l} & x \\ \hline x^2+1 & x^3 \\ & -(x^3+x) \\ \hline & -x \leftarrow \text{rem.} \end{array}$$

$$= \frac{x^3}{3}\arctan x - \frac{1}{3}\int x - \frac{x}{1+x^2}\,dx$$

$$= \frac{x^3}{3}\arctan x - \frac{1}{3}\int x\,dx + \frac{1}{3\cdot 2}\int \frac{2x}{1+x^2}\,dx$$

$u = 1+x^2$

$du = 2x\,dx$

$$= \frac{x^3}{3}\arctan x - \frac{x^2}{6} + \frac{1}{6}\ln(1+x^2) + C$$

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1. $\int xe^x \, dx =$

A E

a. $xe^x + C$

b. $xe^x - e^x + C$

c. $xe^x + e^x + C$

d. $e^x - xe^x + C$

e. none of these

2. Rewrite $\cos^2 x$ in terms of sine.

a. $\sin^2 x - 1$

b. $\sin^2 x + 1$

c. $1 - \sin^2 x$

d. $\sin^2 x$

e. $\sin x - 1$

$\cos^2 x + \sin^2 x = 1$

$\div \cos^2 x$

$1 + \tan^2 x = \sec^2 x$

$\div \sin^2 x$

$\cot^2 x + 1 = \csc^2 x$

8.2 Powers and Products of Trigonometric Functions

Recall the following identities:

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$\cos^2(x) = \frac{1 + \cos(2x)}{2} \qquad \sin^2(x) = \frac{1 - \cos(2x)}{2}$$

$$\sin(2x) = 2\sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

In this section, we will study techniques for evaluating integrals of the form

$$\int \sin^m x \cos^n x \, dx \qquad \text{and} \qquad \int \sec^m x \tan^n dx$$

where either m or n is a positive integer.

To find antiderivatives for these forms, try to break them into combinations of trigonometric integrals to which you can apply the Power Rule, which is

$$\int u^n \, du = \begin{cases} \dfrac{u^{n+1}}{n+1} + C & \text{if } \; n \neq 1 \\ \ln|u| + C & \text{if } \; n = -1 \end{cases}.$$

Integrals Involving Powers of Sine and Cosine

1. **If *m* or *n* odd:**

   a. *m* odd: rewrite $\sin^m x$ as $\sin^{m-1} x \sin x$ (*m*-1 is even so can use identity $\sin^2 x = 1 - \cos^2 x$)

   b. *n* odd: rewrite $\cos^m x$ as $\cos^{m-1} x \cos x$ (*n*-1 is even so can use identity $\cos^2 x = 1 - \sin^2 x$)

Examples:

$$\int \sin^3 x\,dx = \int \underline{\sin^2 x}\ \sin x\,dx$$

$$= \int (1-\cos^2 x)\sin x\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$$= \int \sin x\,dx - \int \cos^2 x \sin x\,dx \qquad -\int u^2\,du$$

$$= -\cos x + \int u^2\,du = -\cos x + \frac{u^3}{3} + C$$

$$= -\cos x + \tfrac{1}{3}\cos^3 x + C$$

$$\int \sin^3 x \cos^2 x \, dx = \int \underbrace{\sin^2 x}_{(1-\cos^2 x)} \cos^2 x \, \overbrace{\sin x \, dx}^{du}$$

$$= -\int (\cos^2 x - \cos^4 x) \, -\sin x \, dx$$

$$u = \cos x$$
$$du = -\sin x \, dx$$

$$= -\int (u^2 - u^4) \, du$$

$$= -\frac{1}{3} u^3 + \frac{1}{5} u^5 + C$$

$$= -\frac{1}{3} \cos^3 x + \frac{1}{5} \cos^5 x + C$$

$$\int \cos^5 x\,dx = \int \underbrace{\cos^4 x}_{(1-\sin^2 x)^2} \cos x\,dx$$

$\rightarrow u = \sin x$

$du = \cos x\,dx$

$$= \int (1 - 2\sin^2 x + \sin^4 x)\cos x\,dx$$

$$= \int (1 - 2u^2 + u^4)\,du$$

$$= u - \frac{2}{3}u^3 + \frac{1}{5}u^5 + C$$

$$= \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C$$

2. **If *m* and *n* even use these identities:**

$$\sin^2 x = \frac{1-\cos 2x}{2} \quad \text{and} \quad \cos^2 x = \frac{1+\cos 2x}{2}.$$

$$\int \cos^2 x\,dx = \int \frac{1}{2} + \frac{1}{2}\cos(2x)\;dx$$

$$= \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$$

$$\frac{1}{2}x + \frac{1}{4}(2\sin x\cos x) + C$$

$$\int \sin^2 x\,dx = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$$

$$\sin(2x) = 2\sin x\cos x$$

Note:

$$\int \sin^2 x\,dx = \frac{1}{2}x - \frac{1}{2}\sin x \cos x + C$$

$$\int \cos^2 x\,dx = \frac{1}{2}x + \frac{1}{2}\sin x \cos x + C$$

Integrals involving Secants and Tangents

$$\tan^2 x + 1 = \sec^2 x$$

For $\int \tan^m x \sec^n x dx$

a. n even: rewrite $\tan^m x \sec^n x$ as $\tan^m x \sec^{n-2} x \sec^2 x$ (then you can use identity $\sec^2 x = \tan^2 x + 1$)

b. m odd: rewrite as $\tan^{m-1} x \sec^{n-1} x \cdot \sec x \tan x$ (m-1 is even so can use identity $\tan^2 x = \sec^2 x - 1$)

c. m even and n odd: rewrite $\tan^m x$ using $\tan^2 x = \sec^2 x - 1$

Examples:

$$\int \tan^3(x)\,dx = \int \tan^2 x \cdot \tan x \, dx$$

$$= \int (\sec^2 x - 1) \tan x \, dx$$

$$= \underbrace{\int \tan x \sec^2 x \, dx} - \int \tan x \, dx$$

$u = \tan x$

$du = \sec^2 x \, dx$

$\int u \, du = u^2/2$

$$= \frac{1}{2} \tan^2 x - (-\ln|\cos x|) + C$$

or

$$= \frac{1}{2} \tan^2 x - \ln|\sec x| + C$$

☆ $$\int \tan x \, dx = -\ln|\cos x| + C$$

OR

$$\ln|\sec x| + C$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \sec^4 x\,dx = \int \sec^2 x \cdot \underline{\sec^2 x\,dx}$$

$$\int (1+\tan^2 x)\sec^2 x\,dx$$

$u = \tan x$

$du = \sec^2 x\,dx$

$$\int (1+u^2)\,du$$

$$= u + \frac{1}{3}u^3 + C$$

$$= \tan x + \frac{1}{3}\tan^3 x + C$$

$$\int \sec^4 x \tan^2 x \, dx = \int \underline{\underline{\sec^2 x}} \tan^2 x \quad \sec^2 x \, dx$$

$$= \int (1 + \tan^2 x) \tan^2 x \quad \sec^2 x \, dx$$

$$= \int (\tan^2 x + \tan^4 x) \sec^2 x \, dx$$

$$u = \tan x \qquad du = \sec^2 x \, dx$$

$$= \int (u^2 + u^4) \, du$$

$$= \frac{1}{3} u^3 + \frac{1}{5} u^5 + C$$

$$= \frac{1}{3} \tan^3 x + \frac{1}{5} \tan^5 x + C$$

Note:

$$\int \tan^n x\,dx = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x dx \quad n \geq 2$$

$$\int \sec^n x\,dx = \frac{1}{n-1}\sec^{n-2} x \tan x + \frac{n-2}{n-1}\int \sec^{n-2} x dx \quad n \geq 2$$

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3. $\int \cos x \sin^3 x \, dx$

a. $\frac{1}{2}\cos^2 x + C$

b. $\frac{1}{4}\cos^4 x + C$

c. $\frac{1}{4}\sin^4 x + C$

d. none of these

$\int \sec(2x) \cdot \tan(2x) \cdot \underbrace{\tan^2(2x)} \, dx$

4. Compute $\int \sec(2x)\tan^3(2x)\,dx$ $\frac{1}{2}\int \left(\sec^2(2x)-1\right) 2\sec(2x)\tan(2x)\,dx$

a. $\frac{1}{6}\sec^3(2x)+\frac{1}{2}\sec(2x)+C$

$u = \sec(2x)$

$du = 2\sec(2x)\tan(2x)\,dx$

b. $\frac{1}{6}\sec^3(2x)-\frac{1}{2}\sec(2x)+C$

$\frac{1}{2}\int (u^2-1)\,du$

c. $\frac{1}{4}\sec^2(2x)-\sec(2x)+C$

d. $\frac{1}{4}\sec^2(2x)+\sec(2x)+C$

e. None of the above

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