Math 1432

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Office Hours:

Mondays 1-2pm,
Fridays noon-1pm
(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

More examples with trig powers:

$$\int \frac{\sec x}{\tan^2 x}\,dx$$

$$\sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\int \frac{1}{\sin x}\cdot\frac{\cos x}{\sin x}\,dx$$

$$\int \csc x \cot x\,dx$$

$$\int \frac{1}{\cos x}\cdot\frac{\cos^2 x}{\sin^2 x}\,dx = \int \frac{\cos x}{\sin^2 x}\,dx$$

let $u = \sin x \quad du = \cos x\,dx$

$$\int \frac{1}{u^2}\,du = \frac{u^{-1}}{-1}$$

$$= \frac{-1}{\sin x} + C$$

$$= -\csc x + C$$

$$\int \sin^4 x \cos^5 x \, dx = \int \sin^4 x \cos^4 x \cos x \, dx$$

$$\int \sin^4 x (1-\sin^2 x)^2 \cos x \, dx$$

$$\int \sin^4 x (1 - 2\sin^2 x + \sin^4 x) \cos x \, dx$$

$$\int (\sin^4 x - 2\sin^6 x + \sin^8 x) \cos x \, dx$$

$u = \sin x \quad du = \cos x \, dx$

$$\int (u^4 - 2u^6 + u^8) \, du = \frac{u^5}{5} - \frac{2}{7}u^7 + \frac{u^9}{9} + C$$

$$= \frac{1}{5}\sin^5 x - \frac{2}{7}\sin^7 x + \frac{1}{9}\sin^9 x + C$$

$$\int \tan^3 x \, dx = \int \tan^2 x \tan x \, dx$$

$$= \int (\sec^2 x - 1) \tan x \, dx$$

$$= \int (\tan x \cdot \sec^2 x - \tan x) \, dx$$

$$= \int \tan x \underbrace{\sec^2 x \, dx} - \int \tan x \, dx$$

$u = \tan x \quad du = \sec^2 x \, dx$ ← no usub

$\int u \, du = \frac{1}{2} u^2$

$$= \frac{1}{2} \tan^2 x - \ln|\sec x| + C$$

$$\int \sec^5 x \tan x\, dx = \int \sec^4 x \cdot \underbrace{\sec x \tan x\, dx}$$

$u = \sec x$

$du = \sec x \tan x\, dx$

$$\int u^4\, du = \frac{1}{5} u^5 + C$$

$$= \frac{1}{5} \sec^5 x + C$$

Integrals involving Sine-Cosine Products with Different Angles

If you are given one of these where $m \neq n$

$$\int \sin(\underline{mx})\cos(\underline{nx})dx$$

$$\int \sin(mx)\sin(nx)dx$$

$$\int \cos(mx)\cos(nx)dx$$

$$\int \sin(2x)\cos(3x)\,dx$$

$$u = \sin(2x)$$

$$du = 2\cos(2x)\,dx$$

Use these formulas:

$$\sin A\cos B = \frac{1}{2}[\sin(A-B)+\sin(A+B)]$$

$$\sin A\sin B = \frac{1}{2}[\cos(A-B)-\cos(A+B)]$$

$$\cos A\cos B = \frac{1}{2}[\cos(A-B)+\cos(A+B)]$$

$$\int \cos(\overset{B}{2x})\sin(\overset{A}{3x})\,dx$$

$$\sin A\cos B = \frac{1}{2}[\sin(A-B)+\sin(A+B)]$$

$$\sin A\sin B = \frac{1}{2}[\cos(A-B)-\cos(A+B)]$$

$$\cos A\cos B = \frac{1}{2}[\cos(A-B)+\cos(A+B)]$$

$$= \int \frac{1}{2}[\sin(3x-2x)+\sin(3x+2x)]\,dx$$

$$= \frac{1}{2}\int (\sin x + \sin(5x))\,dx$$

$$= \frac{1}{2}\left(-\cos x - \frac{1}{5}\cos(5x)\right) + C$$

$$= -\frac{1}{2}\cos x - \frac{1}{10}\cos(5x) + C$$

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1. Rewrite sin (5x) sin (3x)

A. $\frac{1}{2}\left[\cos(2x)-\cos(8x)\right]$

B. $\frac{1}{2}\left[\cos(2x)+\cos(8x)\right]$

C. $\frac{1}{2}\left[\sin(2x)-\sin(8x)\right]$

D. $\frac{1}{2}\left[\sin(2x)+\sin(8x)\right]$

2. $\int \sin(5x)\sin(3x)\,dx =$

A. $\frac{1}{4}\sin(2x)-\frac{1}{16}\sin(8x)+C$

B. $-\frac{1}{4}\sin(2x)+\frac{1}{16}\sin(8x)+C$

C. $\frac{1}{2}\sin(2x)-\frac{1}{2}\sin(8x)+C$

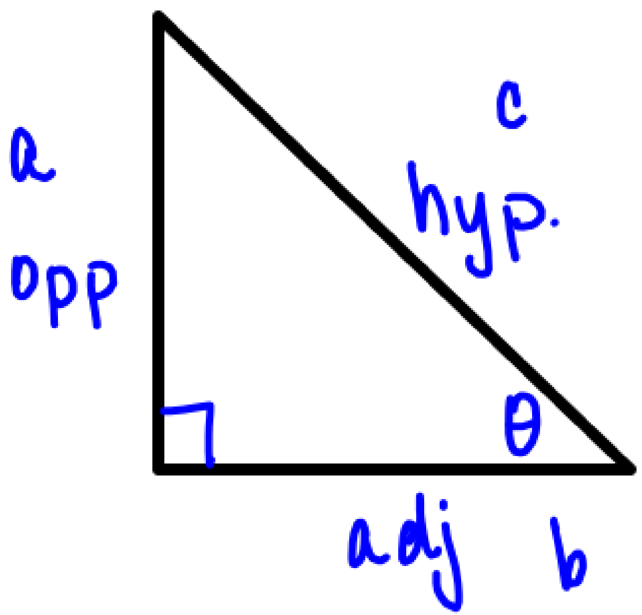
D. $-\frac{1}{2}\sin(2x)+\frac{1}{2}\sin(8x)+C$

E. none of these

$$\sin A\cos B=\frac{1}{2}[\sin(A-B)+\sin(A+B)]$$

$$\sin A\sin B=\frac{1}{2}[\cos(A-B)-\cos(A+B)]$$

$$\cos A\cos B=\frac{1}{2}[\cos(A-B)+\cos(A+B)]$$



$$a^2 + b^2 = c^2$$

$$\sin\theta = \frac{opp}{hyp}$$

$$\cos\theta = \frac{adj}{hyp}$$

$$\tan\theta = \frac{opp}{adj}$$

$$a = \sqrt{c^2 - b^2}$$

$$c = \sqrt{a^2 + b^2}$$

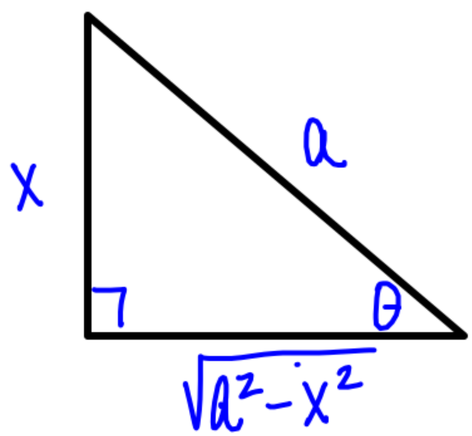
$$b = \sqrt{c^2 - a^2}$$

8.3 Trigonometric Substitution

$\sqrt{hyp^2 - leg^2}$

$\sqrt{leg^2 + leg^2}$

1) Given an integral involving $\sqrt{a^2 - x^2}$ use $x = a\sin\theta$.



$$\sin\theta = \frac{x}{a}$$

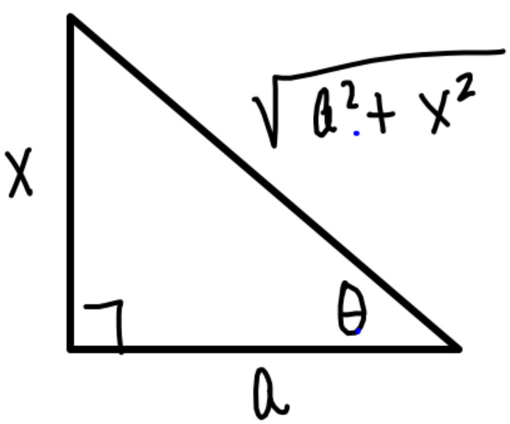
$$x = a\sin\theta \rightarrow dx = a\cos\theta \, d\theta$$

$$\boxed{\sqrt{a^2 - (a\sin\theta)^2}} = \sqrt{a^2 - a^2\sin^2\theta}$$

$$= \sqrt{a^2(1 - \sin^2\theta)} = \sqrt{a^2\cos^2\theta}$$

$$= \boxed{a\cos\theta}$$

2) Given an integral involving $\sqrt{a^2 + x^2}$ use $x = a\tan\theta$.



$$\frac{x}{a} = \tan\theta$$

$$x = a\tan\theta \rightarrow dx = a\sec^2\theta\, d\theta$$

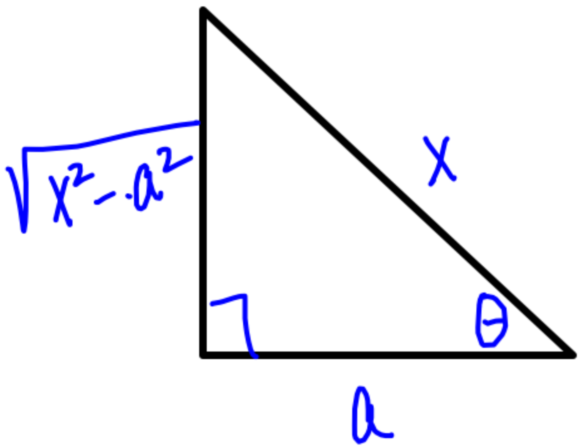
$$\sec\theta = \frac{\sqrt{a^2+x^2}}{a}$$

$$a\sec\theta = \sqrt{a^2+x^2}$$

$$\sqrt{a^2 + a^2\tan^2\theta} = \sqrt{a^2(1+\tan^2\theta)}$$

$$= \sqrt{a^2\sec^2\theta} = a\sec\theta$$

3) Given an integral involving $\sqrt{x^2 - a^2}$ use $x = a\sec\theta$.



$$\frac{x}{a} = \sec\theta$$

$$x = a\sec\theta$$

$$dx = a\sec\theta\tan\theta\,d\theta$$

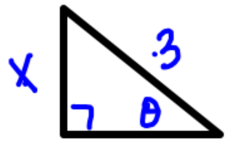
$$\sqrt{a^2\sec^2\theta - a^2} = \sqrt{a^2(\sec^2\theta - 1)}$$

$$= \sqrt{a^2\tan^2\theta} = a\tan\theta$$

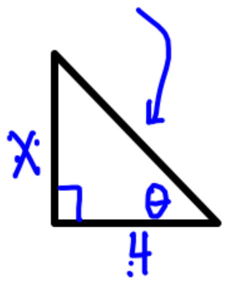
For the following, what substitution should we use?

$\sqrt{9-x^2}$

$\frac{x}{3} = \sin\theta \qquad x = 3\sin\theta$

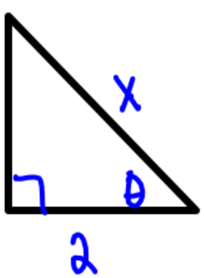


$\sqrt{16+x^2}$



$x = 4\tan\theta$

$\sqrt{x^2-4}$



$\frac{x}{2} = \sec\theta \qquad x = 2\sec\theta$

$\sqrt{a^2 - x^2}$ $\qquad$ $x = a\sin\theta$

$\sqrt{x^2 + a^2}$ $\qquad$ $x = a\tan\theta$

$\sqrt{x^2 - a^2}$ $\qquad$ $x = a\sec\theta$

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3. For $\int \sqrt{x^2 - 1}\, dx \quad$ x =?

A. sin θ

B. tan θ

C. sec θ

D. csc θ

E. none of these

4. What substitution should be used to compute $\int \frac{1}{\sqrt{x^2 - 9}} dx$?

a. $x = 3\sin(\theta)$

b. $x = 3\tan(\theta)$

c. $x = 3\sec(\theta)$

d. $x = 3\cot(\theta)$

e. None of the above

5. What substitution should be used to compute $\int \frac{1}{\sqrt{x^2 + 16}} dx$?

a. x = 4 sin (θ)

b. x = 4 tan (θ)

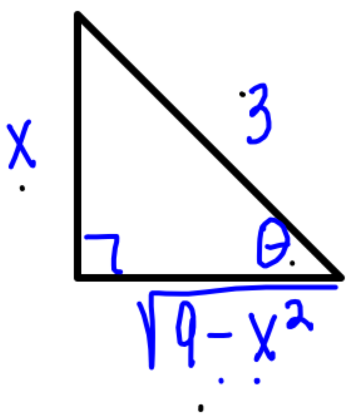
c. x = 4 sec (θ)

d. x = 4 cos (θ)

e. None of the above

Examples:

1. $\int \sqrt{9-x^2}\,dx$



$x = 3\sin\theta$

$\boxed{\dfrac{x}{3} = \sin\theta} \rightarrow \theta = \sin^{-1}\left(\dfrac{x}{3}\right)$

$x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$

$\dfrac{\sqrt{9-x^2}}{3} = \cos\theta$

$\sqrt{9-x^2} = 3\cos\theta$

$$= \int 3\cos\theta \cdot 3\cos\theta\, d\theta$$

$$= \int 9\cos^2\theta\, d\theta$$

$$= 9\int \cos^2\theta\, d\theta$$

$$= 9\left(\frac{1}{2}\theta + \frac{1}{2}\sin\theta\cos\theta\right) + C$$

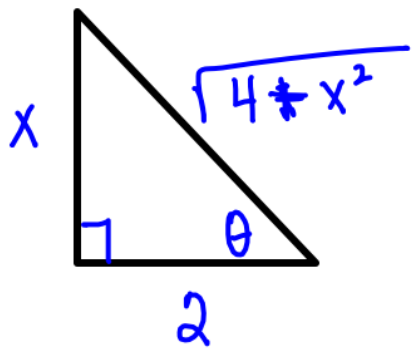
$$= \frac{9}{2}\theta + \frac{9}{2}\sin\theta\cos\theta + C$$

$$= \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + \frac{9}{2}\left(\frac{x}{3}\right)\left(\frac{\sqrt{9-x^2}}{3}\right) + C$$

$$= \frac{9}{2}\arcsin\left(\frac{x}{3}\right) + \frac{1}{2}x\sqrt{9-x^2} + C$$

2. $\displaystyle\int \frac{x^2}{\sqrt{x^2-1}}\,dx$

3. $\displaystyle\int \frac{x^2}{\sqrt{4+x^2}}\,dx = \int \frac{(2\tan\theta)^2}{2\sec\theta} \cdot 2\sec^2\theta \, d\theta$



$\frac{x}{2} = \tan\theta$

$\boxed{x = 2\tan\theta}$

$\boxed{dx = 2\sec^2\theta \, d\theta}$

$\frac{\sqrt{4+x^2}}{2} = \sec\theta$

$\boxed{\sqrt{4+x^2} = 2\sec\theta}$

$= \int 4\tan^2\theta \sec\theta \, d\theta$

$= \int 4(\sec^2\theta - 1)\sec\theta \, d\theta$

$= 4\int (\sec^3\theta - \sec\theta)\, d\theta$

#6 = D