Math 1432

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Office Hours:

Mondays 1-2pm,
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(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

	Test 2	Test 2 FR
max	42	58

$$PT2 \cdot .05 = \text{bonus}$$

4. $\boxed{\int \sec^3 x\,dx} = \int \sec x \cdot \sec^2 x\,dx$

$u = \sec x \qquad dv = \sec^2 x\,dx$

$du = \sec x \tan x\,dx \qquad v = \tan x$

$$= \sec x \tan x - \int \sec x \cdot \underset{(\sec^2 x - 1)}{\tan^2 x}\,dx$$

$$= \sec x \tan x - \int (\sec^3 x - \sec x)\,dx$$

$$= \sec x \tan x - \int \sec^3 x\,dx + \int \sec x\,dx$$

$$\int \sec^3 x\,dx = \sec x \tan x - \int \sec^3 x\,dx + \ln|\sec x + \tan x| + C$$

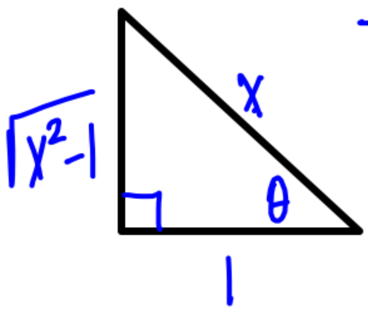
$$+$$

$$2\int \sec^3 x\,dx = \sec x \tan x + \ln|\sec x + \tan x| + C$$

$$\int \sec^3 x\,dx = \boxed{\frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C}$$

2. $\displaystyle\int \frac{x^2}{\sqrt{x^2-1}}\,dx = \int \frac{\sec^2\theta}{\tan\theta}\sec\theta\tan\theta\,d\theta$

$$= \int \sec^3\theta\,d\theta$$



$\rightarrow$ $x = \sec\theta$

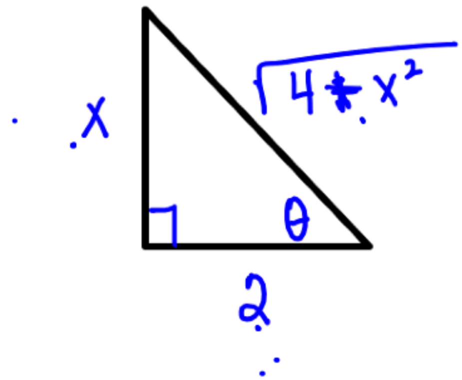
$dx = \sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-1} = \tan\theta$

$$= \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln\left|\sec\theta+\tan\theta\right| + C$$

$$= \frac{1}{2}(x)\left(\sqrt{x^2-1}\right) + \frac{1}{2}\ln\left|x+\sqrt{x^2-1}\right| + C$$

3. $\int \frac{x^2}{\sqrt{4+x^2}}\,dx = \int \frac{(2\tan\theta)^2}{2\sec\theta}\cdot 2\sec^2\theta\, d\theta$



$\frac{x}{2} = \tan\theta$

$\boxed{x = 2\tan\theta}$

$\boxed{dx = 2\sec^2\theta\, d\theta}$

$\frac{\sqrt{4+x^2}}{2} = \sec\theta$

$\boxed{\sqrt{4+x^2} = 2\sec\theta}$

$= \int 4\tan^2\theta \sec\theta\, d\theta$

$= \int 4(\sec^2\theta - 1)\sec\theta\, d\theta$

$= 4\int(\sec^3\theta - \sec\theta)\, d\theta$

$= 4\int \sec^3\theta\, d\theta - 4\int \sec\theta\, d\theta$

$= 4\left[\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta|\right] - 4\left(\ln|\sec\theta + \tan\theta|\right) + C$

$= 2\sec\theta\tan\theta - 2\ln|\sec\theta + \tan\theta| + C$

$= \cancel{2}\left(\frac{\sqrt{4+x^2}}{\cancel{2}}\right)\left(\frac{x}{2}\right) - 2\ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C$

$= \frac{x\sqrt{4+x^2}}{2} - 2\ln\left|\frac{\sqrt{4+x^2}+x}{2}\right| + C$

$$= \frac{x\sqrt{4+x^2}}{2} - 2\underbrace{\ln\left|\frac{\sqrt{4+x^2}+x}{2}\right|}_{\left(\ln\left|\sqrt{4+x^2}+x\right| - \ln 2\right)} + C$$

$\ln 2$ ← constant

$$= \frac{x\sqrt{4+x^2}}{2} - 2\ln\left|\sqrt{4+x^2}+x\right| + C$$

To summarize trig sub:

Given:	**Use:**
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$

$$\int \cos^2\theta \, d\theta = \frac{1}{2}\theta + \frac{1}{2}\sin\theta\cos\theta + C$$

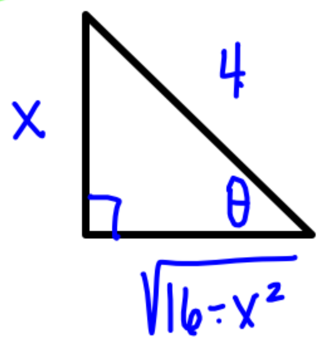
Examples:

$$\int \sqrt{16 - x^2}\, dx = \int 16\cos^2\theta \, d\theta = 16\left(\frac{1}{2}\theta + \frac{1}{2}\sin\theta\cos\theta\right) + C$$

$x = 4\sin\theta \Leftarrow \sin\theta = \frac{x}{4}$

$dx = 4\cos\theta \, d\theta$ $\quad \theta = \sin^{-1}\left(\frac{x}{4}\right)$

$$= 8\theta + 8\sin\theta\cos\theta + C \Leftarrow$$



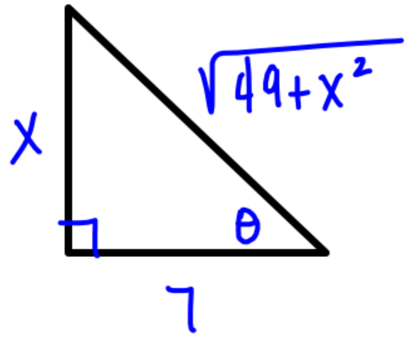
$$= 8\sin^{-1}\left(\frac{x}{4}\right) + 8\left(\frac{x}{4}\right)\left(\frac{\sqrt{16-x^2}}{4}\right) + C$$

$$= 8\sin^{-1}\left(\frac{x}{4}\right) + \frac{1}{2}x\sqrt{16-x^2} + C \Leftarrow$$

$$\frac{\sqrt{16-x^2}}{4} = \cos\theta$$

$$\sqrt{16-x^2} = 4\cos\theta$$

$$\int \frac{1}{x^2\sqrt{49+x^2}}\,dx$$



$$\boxed{x = 7\tan\theta}$$

$$dx = 7\sec^2\theta\, d\theta$$

$$\boxed{\sqrt{49+x^2} = 7\sec\theta}$$

$$\int \frac{1}{49\tan^2\theta \cdot \cancel{7\sec\theta}}\,\cancel{7}\sec^{\cancel{2}}\theta\, d\theta$$

$$\frac{1}{49}\int \frac{\cos^{\cancel{2}}\theta}{\sin^2\theta}\cdot\frac{1}{\cancel{\cos\theta}}\,d\theta$$

usub w/ $u = \sin\theta$

$du = \cos\theta\, d\theta$

$$\frac{1}{49}\int \frac{1}{u^2}\,du = -\frac{1}{u} + C$$

$$\left(\frac{1}{49}\right)-\frac{1}{\sin\theta} + C = -\frac{1}{49}\csc\theta + C$$

$$\frac{1}{49}\int \csc\theta\cot\theta\, d\theta$$

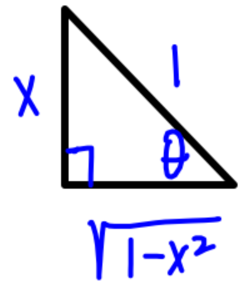
$$= -\frac{1}{49}\csc\theta + C$$

$$-\frac{1}{49}\left(\frac{\sqrt{49+x^2}}{x}\right) + C$$

x

$$\int_0^{1/2} \frac{x^2}{\left(1-x^2\right)^{3/2}}\,dx$$

$$\left(\sqrt{1-x^2}\right)^3$$



$x = \sin\theta$

$dx = \cos\theta\, d\theta$

$\sqrt{1-x^2} = \cos\theta$

$x = 1/2 \Rightarrow 1/2 = \sin\theta$

$\theta = \pi/6$

$x = 0 \Rightarrow 0 = \sin\theta$

$\theta = 0$

$$= \int_0^{\pi/6} \frac{\sin^2\theta}{\cos^3\theta} \cos\theta \, d\theta$$

$$= \int_0^{\pi/6} \tan^2\theta \, d\theta$$

$$= \int_0^{\pi/6} \left(\sec^2\theta - 1\right) d\theta$$

$$= \tan\theta - \theta \Big|_0^{\pi/6}$$

$$= \left(\tan\pi/6 - \pi/6\right) - \left(\tan 0 - 0\right)$$

$$= \boxed{\frac{1}{\sqrt{3}} - \pi/6}$$

$$\int \sqrt{2-x^2+4x}\,dx = \int \sqrt{4+2-(x^2-4x+4)}\,dx$$

$$= \int \sqrt{6-(x-2)^2}\,dx$$

$\sqrt{a^2-x^2}$

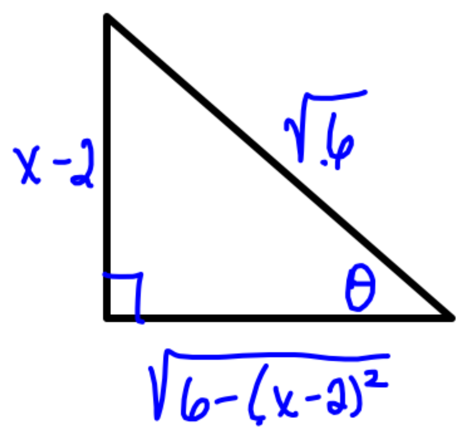
$x = a\sin\theta$

$x-2=\sqrt{6}\sin\theta$

$x=\sqrt{6}\sin\theta+2$

$dx=\sqrt{6}\cos\theta\,d\theta$

$\sqrt{6-(x-2)^2}=\sqrt{6}\cos\theta$



$$=\int 6\cos^2\theta\,d\theta$$

$$=6\left(\frac{1}{2}\theta+\frac{1}{2}\sin\theta\cos\theta\right)+C$$

$$=3\arcsin\left(\frac{x-2}{\sqrt{6}}\right)+3\left(\frac{x-2}{\sqrt{6}}\right)\left(\frac{\sqrt{6-(x-2)^2}}{\sqrt{6}}\right)+C$$

Popper12

1. $\frac{1}{3}\int \frac{3\left(x^2+1\right)}{x^3+3x-4}\,dx$

$u = x^3+3x-4$

$du = (3x^2+3)\,dx$

a. $3\ln\left|x^3+3x-4\right|$

b. $3\ln\left|x^3+3x-4\right|+C$

c. $\frac{1}{3}\ln\left|x^3+3x-4\right|$

d. $\frac{1}{3}\ln\left|x^3+3x-4\right|+C$

2. $\int \frac{x^3 - 5}{x} dx$ $= \int x^2 - \frac{5}{x} \, dx$

a. $\frac{x^3}{3} - \frac{5}{x^2} + C$

b. $\frac{x^3}{3} + \frac{5}{x^2} + C$

c. $\frac{x^3}{3} - 5 \ln x + C$

d. $\frac{x^3}{3} - 5 \ln|x| + C$

e. $\frac{x^3}{3} + 5 \ln|x| + C$

8.4 Rational Functions and Partial Fraction Decomposition

Rational functions are defined as functions in the form $R(x) = \frac{F(x)}{G(x)}$, where F(x) and G(x) are polynomials.

Rational functions are said to be *proper* if the degree of the numerator is less then the degree of the denominator (otherwise they are improper).

Theorem:

If F(x) and G(x) are polynomials and the degree of F(x) is larger than or equal to the degree of G(x), then there are polynomials *q*(x) (quotient) and r(x) (remainder) such that

$$\frac{F(x)}{G(x)} = \mathrm{q}(x) + \frac{r(x)}{G(x)}$$

where the degree of r(x) is smaller than the degree of G(x).

Example:

$$= x^2 + x + 3 + \frac{5x^2+6x+1}{x^3-x^2-2x}$$

Write $\dfrac{x^5+1}{x^3-x^2-2x}$ in terms of its quotient and remainder.

$$
\begin{array}{r l}
 & x^2 + x + 3 \\
x^3 - x^2 - 2x \enclose{longdiv}{} & x^5 \qquad\qquad\qquad + 1 \\
 & -\ x^5 + x^4 + 2x^3 \\
\hline
 & x^4 + 2x^3 + 1 \\
 & -\ x^4 + x^3 + 2x^2 \\
\hline
 & 3x^3 + 2x^2 + 1 \\
 & -\ 3x^3 + 3x^2 + 6x \\
\hline
 & 5x^2 + 6x + 1
\end{array}
$$

Write $\dfrac{x^2+x-1}{x^2+1}$ in terms of its quotient and remainder. $= 1 + \dfrac{x-2}{x^2+1}$

$$
\begin{array}{r|l}
 & \quad 1 \\
\hline
x^2+1 & x^2 + x - 1 \\
 & -x^2 \qquad - 1 \\
\hline
 & \qquad x - 2
\end{array}
$$

Compute: $\displaystyle\int \frac{x^2+x-1}{x^2+1}\,dx = \int 1 + \frac{x}{x^2+1} - \frac{2}{x^2+1}\,dx$

$$= x + \frac{1}{2}\ln(x^2+1) - 2\arctan x + C$$

$\star \displaystyle\int \frac{1}{1+x^2}\,dx = \arctan x + C$

Partial Fractions:

Example: $\dfrac{3}{x}+\dfrac{4}{x+1}=\dfrac{3(x+1)+4x}{x(x+1)}=\dfrac{7x+3}{x(x+1)}$

What if we have $\dfrac{7x+3}{x(x+1)}$ and want the original two fractions?

$$\frac{7x+3}{x(x+1)}=\underbrace{\frac{A}{x}+\frac{B}{x+1}}_{\text{form of PFD}}=\frac{3}{x}+\frac{4}{x+1}$$

How do we find A and B?

poppers 3-5 C