Quiz 10
due 2/24

Math 1432

Bekki George
bekki@math.uh.edu
639 PGH

Office Hours:

Mondays 1-2pm,
Fridays noon-1pm
(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

Pop Quiz - you have 10 minutes

Pop Quiz 1C

Name:

ID:

1. $\int \tan^2 x \, dx$

2. $\int x \sin x \, dx$

3. If you have substitution $x = 2\sin\theta$ and after integration get $\frac{1}{2}\theta - 4\cos\theta + C$ what is answer in terms of x?

Compute: $\int \frac{3x^3 - 2}{x^2 + 4} dx$

$$\begin{array}{r} 3x \\ x^2+4 \overline{\smash{)}\, 3x^3 - 2} \\ -\ (3x^3 + 12x) \\ \hline -12x - 2 \end{array}$$

$$= \int 3x + \frac{-12x - 2}{x^2+4}\, dx$$

$$= \int 3x - \frac{\overset{6\cdot(2x)}{\cancel{12x}}}{x^2+4} - \frac{2}{x^2+4}\, dx$$

$u = x^2+4$

$du = 2x\,dx \quad 6\int \frac{1}{u} du$

$$\int \frac{b}{a^2+x^2} dx$$

$$b \cdot \frac{1}{a} \arctan \frac{x}{a} + C$$

$$= \frac{3x^2}{2} - 6 \ln(x^2+4) - \cancel{2} \cdot \frac{1}{\cancel{2}} \arctan \frac{x}{2} + C$$

Partial Fractions:

Example: $\underbrace{\dfrac{3}{x}+\dfrac{4}{x+1}}=\dfrac{3(x+1)+4x}{x(x+1)}=\dfrac{7x+3}{x(x+1)} = \dfrac{7x+3}{x^2+x}$

What if we have $\dfrac{7x+3}{x(x+1)}$ and want the original two fractions?

$$\dfrac{7x+3}{x(x+1)}=\underbrace{\dfrac{A}{x}+\dfrac{B}{x+1}}_{\text{Form}}=\dfrac{3}{x}+\dfrac{4}{x+1}$$

How do we find A and B?

In general, each linear factor of the form $(x - \alpha)^k$ in the denominator gives rise to an expression of the form

$$\frac{A_1}{(x-a)} + \frac{A_2}{(x-a)^2} + \ldots + \frac{A_k}{(x-a)^k}$$

Give the form of the partial fraction decomposition for:

1a. $\dfrac{5x^2 - 6x + 1}{(x+1)(x+2)(x+3)} = \dfrac{A}{x+1} + \dfrac{B}{x+2} + \dfrac{C}{x+3}$

1b. $\dfrac{2x^2 - 3x + 1}{(x+1)(x+2)^2(x+3)} = \dfrac{A}{x+1} + \dfrac{B}{x+2} + \dfrac{C}{(x+2)^2} + \dfrac{D}{x+3}$

Rewrite using partial fractions:

$$\boxed{5x-10} \over (x-4)(x+1)$$ $$= \frac{2}{x-4} + \frac{3}{x+1} = \frac{\boxed{A(x+1)+B(x-4)}}{(x-4)(x+1)}$$

$$Ax + A + Bx - 4B = 5x - 10$$

$$Ax + Bx = 5x \rightarrow A + B = 5$$

$$A - 4B = -10 \rightarrow A = 4B - 10$$

$$4B - 10 + B = 5$$

$$5B = 15$$

$$B = 3$$

$$A = 2$$

Killer-x method:

$$A(x+1) + B(x-4) = 5x - 10$$

$x = -1$: $A(0) + B(-5) = -5 - 10$ $\quad -5B = -15$ $\quad B = 3$

$x = 4$: $5A + B(0) = 20 - 10$

$$5A = 10$$

$$A = 2$$

$$\frac{3x^2+20x+25}{(x-1)(x+2)(x+3)} = \boxed{\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{x+3}}$$ Form

$$A(x+2)(x+3)+B(x-1)(x+3)+C(x-1)(x+2)=3x^2+20x+25$$

$x=-2$: $A(0)+B(-3)(1)+C(0)=12-40+25$

$-3B=-3 \rightarrow \boxed{B=1}$

$x=-3$: $A(0)+B(0)+4C=27-60+25$

$4C=-8$ $\boxed{C=-2}$

$x=1$: $A(3)(4)+B(0)+C(0)=3+20+25$

$12A=48$ $\boxed{A=4}$

ans: $\frac{4}{x-1}+\frac{1}{x+2}-\frac{2}{x+3}$ ← find the PFD

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1. Give the form of the partial fraction decomposition for:

$$\frac{3x^2+20x+25}{(x-1)(x+2)(x+3)^2}$$

a. $\dfrac{A}{(x-1)}+\dfrac{B}{(x+2)}+\dfrac{C}{(x+3)}$

b. $\dfrac{A}{(x-1)}+\dfrac{B}{(x+2)}+\dfrac{C}{(x+3)^2}$

~~c.~~ $\dfrac{A}{(x-1)}+\dfrac{B}{(x+2)}+\dfrac{C}{(x+3)}+\dfrac{C}{(x+3)^2}$

d. $\dfrac{A}{(x-1)}+\dfrac{B}{(x+2)}+\dfrac{C}{(x+3)}+\dfrac{D}{(x+3)^2}$

Compute:

$$\int \frac{4x+41}{x^2+3x-10}\,dx = \int \frac{A}{x+5} + \frac{B}{x-2}\,dx$$

$(x+5)(x-2)$

$$A(x-2) + B(x+5) = 4x+41$$

$x=2$: $A(0) + 7B = 8+41 = 49 \rightarrow B = 7$

$x=-5$: $-7A + B(0) = -20+41 = 21 \rightarrow A = -3$

$$\int \frac{-3}{x+5} + \frac{7}{x-2}\,dx = -3\ln|x+5| + 7\ln|x-2| + C$$

$$\star \int \frac{b}{x+a}\,dx = b\ln|x+a| + C \qquad \int \frac{b}{(x+a)^2}\,dx = \frac{-b}{x+a} + C$$

$$\int \frac{b}{x^2+a}\,dx = \frac{b}{\sqrt{a}}\arctan\frac{x}{\sqrt{a}} + C$$

$$\int \frac{x^2+1}{x(x^2-1)}\,dx = \int \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}\,dx$$

$(x-1)(x+1)$

$$A(x-1)(x+1) + B(x)(x+1) + C(x)(x-1) = \boxed{x^2+1}$$

$x=1:\quad A(0) + 2B + C(0) = 2 \rightarrow \boxed{B=1}$

$x=-1:\quad A(0) + B(0) + 2C = 2 \rightarrow \boxed{C=1}$

$x=0:\quad -A + B(0) + C(0) = 1 \rightarrow \boxed{A=-1}$

$$\int \frac{-1}{x} + \frac{1}{x-1} + \frac{1}{x+1}\,dx = \boxed{-\ln|x| + \ln|x-1| + \ln|x+1| + C}$$

$$= \ln\left|\frac{x^2-1}{x}\right| + C$$

$$\int \frac{x^2+3}{x^2-3x+2}\,dx = \int 1 + \frac{3x+1}{(x-2)(x-1)}\,dx$$

$$\begin{array}{r|l} & \quad 1 \\ x^2-3x+2 & x^2 \qquad +3 \\ & \underline{x^2 - 3x + 2} \\ & \qquad 3x+1 \end{array}$$

$$= \int 1 + \frac{A}{x-2} + \frac{B}{x-1}\,dx$$

$$A(x-1) + B(x-2) = 3x+1$$

$x=1$: $-B = 4 \rightarrow B = -4$

$x=2$: $A = 7$

$$= \int 1 + \frac{7}{x-2} + \frac{-4}{x-1}\,dx$$

$$= x + 7\ln|x-2| - 4\ln|x-1| + C$$

$$\int \frac{dx}{x^2 + x - 2} = \int \frac{A}{x+2} + \frac{B}{x-1} \, dx$$

$$\int \frac{x+1}{x^3 - x^2} dx$$

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2. Give the form of the PFD for: $\dfrac{1}{x^2 - x - 6}$

a. $\dfrac{A}{x+3} + \dfrac{B}{x-2}$

b. $\dfrac{A}{x-3} + \dfrac{B}{x+2}$

c. $\dfrac{A}{x-6} + \dfrac{B}{x+1}$

d. $\dfrac{A}{x-1} + \dfrac{B}{x+6}$

$A(x+2) + B(x-3) = 1$

$x=-2$: $A(0) + B(-5) = 1$

$B = -1/5$

$x = 3$: $A(5) + B(0) = 1$

$A = 1/5$

3. Find the PFD for: $\dfrac{1}{x^2 - x - 6}$

a. $\dfrac{1}{x+3} + \dfrac{1}{x-2}$

b. $\dfrac{1/5}{x-3} + \dfrac{1/5}{x+2}$

c. $\boxed{\dfrac{1/5}{x-3} - \dfrac{1/5}{x+2}}$

d. $\dfrac{5}{x-3} - \dfrac{5}{x+2}$

4. Compute: $\displaystyle\int \frac{1}{x^2 - x - 6}\,dx$

a. $\dfrac{1}{5}\ln(x-3) - \dfrac{1}{5}\ln(x+2) + C$

b. $\ln(x-3)^{1/5} - \ln(x+2)^{1/5} + C$

c. $\ln\dfrac{(x-3)^{1/5}}{(x+2)^{1/5}} + C$

d. $\ln\sqrt[5]{\dfrac{x-3}{x+2}} + C$

e. All of these are correct!

Now, if the denominator has an **irreducible quadratic factor** of the form $x^2 + bx + c$ then we have a term in the form $\dfrac{Ax+B}{x^2+bx+c}$

Give the form of the PFD for the following:

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$\frac{1}{(x-2)^2(x^2+2)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{Cx+D}{x^2+2}$$

$$\frac{1}{(x^2-1)(x^2+4)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+4}$$

5. Give the form of the PFD for: $\dfrac{1}{(x-2)^2(x^2+2)}$

a. $\dfrac{A}{(x-2)^2}+\dfrac{B}{(x^2+2)}$

b. $\dfrac{A}{(x-2)}+\dfrac{B}{(x-2)^2}+\dfrac{C}{(x^2+2)}$

c. $\dfrac{A}{(x-2)}+\dfrac{B}{(x-2)^2}+\dfrac{Cx+D}{(x^2+2)}$

d. $\dfrac{A}{(x-2)^2}+\dfrac{Bx+C}{(x^2+2)}$

More examples. Give the PFD and then integrate:

$$\int \frac{1}{x(x^2+1)}dx$$

$$\int \frac{x^2+2x+7}{(x-1)^2(x^2+4)}dx$$

$$\int \frac{e^{x}}{e^{2x}+5e^{x}+6}\,dx$$

6. If the degree in the numerator of a rational expression is equal or higher than the degree in the denominator, you should do this first:

a. Simplify with division

b. Use partial fraction decomposition

c. Integrate

$$\int \tan^2 x\,dx = \int(\sec^2 x - 1)\,dx = \int \sec^2 x\,dx - \int dx$$

$$= \tan x - x + C$$

$$\int x \ln x\,dx$$

A L

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{x^2}{2}$

$$\frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx$$

$$\frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx$$

$$\frac{x^2}{2}\ln x - \frac{1}{4}x^2 + C$$

$$\int x e^x\,dx$$

A E

$u = x \qquad dv = e^x\,dx$

$du = dx \qquad v = e^x$

$$xe^x - \int e^x\,dx$$

$$xe^x - e^x + C$$

$$\int x \sin x\,dx$$

A T

$u = x \qquad dv = \sin x\,dx$

$du = dx \qquad v = -\cos x$

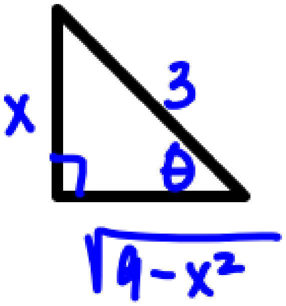
$$-x\cos x + \int +\cos x\,dx$$

$$-x\cos x + \sin x + C$$

Give substitution & answer to integral

$x = 3\sin\theta$

$\frac{1}{2}\theta + 9\sec\theta + C$



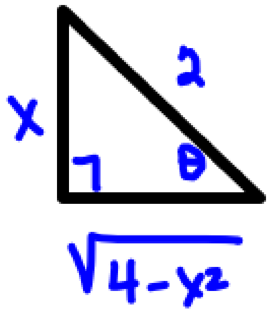
$\frac{x}{3} = \sin\theta$

$\theta = \sin^{-1}\frac{x}{3}$

$\frac{1}{2}\sin^{-1}\left(\frac{x}{3}\right) + 9\left(\frac{3}{\sqrt{9-x^2}}\right) + C$

$x = 2\sin\theta$

$\frac{1}{2}\theta - 4\cos\theta + C$



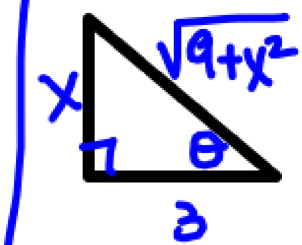
$\frac{x}{2} = \sin\theta$

$\theta = \sin^{-1}\frac{x}{2}$

$\frac{1}{2}\arcsin\frac{x}{2} - 4\left(\frac{\sqrt{4-x^2}}{2}\right) + C$

$x = 3\tan\theta$

$\frac{1}{2}\theta + 9\sin\theta + C$



$\frac{x}{3} = \tan\theta$

$\theta = \tan^{-1}\frac{x}{3}$

$\frac{1}{2}\tan^{-1}\frac{x}{3} + 9\left(\frac{x}{\sqrt{9+x^2}}\right) + C$