Math 1432

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Office Hours:

Mondays 1-2pm,
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(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

$$\int \frac{dx}{x^2+x-2} = \int \frac{\boxed{1}}{(x+2)(x-1)}\,dx = \int \underbrace{\frac{A}{x+2} + \frac{B}{x-1}}_{\text{FORM}}\,dx$$

$$A(x-1) + B(x+2) = \boxed{1}$$

$x=1:\quad A(0) + 3B = 1 \to B = 1/3$

$x=-2:\quad -3A + B(0) = 1 \to A = -1/3$

$$\int \frac{-1/3}{x+2} + \frac{1/3}{x-1}\,dx$$

$$= \frac{-1}{3}\ln|x+2| + \frac{1}{3}\ln|x-1| + C$$

or

$$\ln \sqrt[3]{\left|\frac{x-1}{x+2}\right|} + C$$

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$$\int \frac{b}{x+a}\,dx = b\ln|x+a| + C$$

$$\int \frac{b}{(x+a)^2}\,dx = \frac{-b}{x+a} + C$$

$$\int \frac{b}{x^2+a^2}\,dx = \frac{b}{a}\arctan\frac{x}{a} + C$$

$$\int \frac{x+1}{x^3-x^2}\,dx = \int \frac{x+1}{x^2(x-1)}\,dx = \int \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}\,dx$$

$$\int \frac{-2}{x} - \frac{1}{x^2} + \frac{2}{x-1}\,dx$$

$$A(x)(x-1) + B(x-1) + C(x^2) = x+1$$

$x=0$: $\quad -B = 1 \rightarrow B = -1$

$x=1$: $\quad C = 2$

$x=2$: $\quad 2A + B + 4C = 3$

$$2A - 1 + 8 = 3$$

$$2A + 7 = 3$$

$$2A = -4$$

$$A = -2$$

$$= -2\ln|x| + \frac{1}{x} + 2\ln|x-1| + C$$

Now, if the denominator has an $\boxed{\textbf{irreducible}}$ **quadratic factor** of the form x^2+bx+c then we have a term in the form $\dfrac{Ax+B}{x^2+bx+c}$

Give the form of the PFD for the following:

$$\frac{1}{x\underbrace{(x^2+1)}} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$\frac{1}{(x-2)^2(x^2+2)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{Cx+D}{x^2+2}$$

$$\frac{1}{\underbrace{(x^2-1)}_{(x-1)(x+1)}(x^2+4)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+4}$$

More examples. Give the PFD and then integrate:

$$\int \frac{1}{x(x^2+1)}dx = \int \frac{A}{x} + \frac{Bx+C}{x^2+1}\,dx$$

$$A(x^2+1) + (Bx+C)(x) = 1$$

$x=0$: $A = 1$

$x=1$: $2A + B + C = 1$

$2 + B + C = 1$

$B + C = -1$

$x=-1$: $2A + B - C = 1$

$B - C = -1$

$+ \Rightarrow 2B = -2$

$B = -1$

$-1 - C = -1 \rightarrow C = 0$

$$\int \frac{1}{x} + \frac{1}{2}\,\frac{-2x}{x^2+1}\,dx$$

$u = x^2+1$

$du = 2x\,dx$

$-\frac{1}{2}\int \frac{1}{u}\,du$

$$= \ln|x| - \frac{1}{2}\ln(x^2+1) + C$$

$$\int \frac{x^2+2x+7}{(x-1)^2(x^2+4)}dx = \int \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+4}\,dx$$

$$A(x-1)(x^2+4) + B(x^2+4) + (Cx+D)(x-1)^2 = x^2+2x+7$$

$x=1$: $5B = 1+2+7 = 10 \rightarrow \boxed{B=2}$

$x=0$: $-4A + 4(2) + D = 7$

$\boxed{-4A+D=-1}$

$-15A = 0 \rightarrow \boxed{A=0}$

$x=2$: $8A + 8(2) + 2C + D = 15$

$(8A+2C+D=-1)\cdot -6$: $-48A-12C-6D=6$

$x=3$ $26A + 13(2) + 12C + 4D = 9+6+7$

$26A+12C+4D=-4$

$+$ $-22A-2D=2$

$\boxed{-11A-D=1}$

$-D=1$

$\boxed{D=-1}$

$0+12C-4=-4$

$12C=0$ $\boxed{C=0}$

$$\int \frac{2}{(x-1)^2} + \frac{-1}{x^2+4}\, dx$$

$$= \frac{-2}{x-1} - \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$$

$$\int \frac{e^x}{e^{2x}+5e^x+6}\,dx = -\ln|e^x+3| + \ln|e^x+2| + C$$

$$= \ln\left|\frac{e^x+2}{e^x+3}\right| + C$$

$u = e^x$

$du = e^x\,dx$

$$\int \frac{1}{u^2+5u+6}\,du = \int \frac{1}{(u+3)(u+2)}\,du = \int \frac{A}{u+3} + \frac{B}{u+2}\,du$$

$A(u+2) + B(u+3) = 1$

$u = -2: \quad B = 1$

$u = -3: \quad -A = 1 \rightarrow A = -1$

$$\int \frac{-1}{u+3} + \frac{1}{u+2}\,du$$

$$= -\ln|u+3| + \ln|u+2| + C$$

Today we will work many different integral problems.
Before coming to class take time to identify the technique you think should be used on each.

Popper 14

For the following problems, answer A, B or C to identify the type of integration we should use to solve.

A. $\int u^p\,du = \frac{u^{p+1}}{p+1} + C,\ p \neq 1$ **B.** $\int \frac{du}{u} = \ln|u| + C$ **C.** $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$

1. $\int \frac{dx}{17+x^2}$

2. $\int \frac{\ln x}{x}\,dx$ A $u = \ln x$, $du = \frac{1}{x}dx$, $= \int u\,du$

3. $\int \frac{1}{11+6x^2}dx$

4. $\int \frac{(\ln x)^5}{x}dx$

5. $\int \frac{2}{27+x^2}\mathrm{dx}$

6. $\int \frac{3x}{13+x^4}dx$

$u = x^2$
$du = 2x\,dx$
$\frac{3}{2}\int \frac{du}{13+u^2}$

7. $\int \frac{3x}{13+x^2}dx$

8. $\int \frac{3+x^4}{15x+x^5}dx$ $\frac{1}{5}\int \frac{1}{u}du$

$u = 15x + x^5$
$du = (15+5x^4)dx$
$\frac{1}{5}du = (3+x^4)dx$

9. $\int \frac{3x}{(13+x^2)^2}dx$

Now let's work some out…

$$\int \frac{dx}{17+x^2} = \frac{1}{\sqrt{17}} \arctan\frac{x}{\sqrt{17}} + C$$

a^2

$a = \sqrt{17}$

$$\int \frac{\ln x}{x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\int \ln x \cdot \frac{1}{x}\,dx = \int u\,du = \frac{u^2}{2} + C$$

$$\frac{(\ln x)^2}{2} + C$$

$$\int \frac{1}{11+6x^2}dx$$

$a^2 = 11$

$a = \sqrt{11}$

$u^2 = 6x^2$

$u = \sqrt{6}\,x$

$du = \sqrt{6}\,dx$

$$\frac{1}{\sqrt{6}} \int \frac{1}{\sqrt{11}^2 + u^2}du$$

$$= \frac{1}{\sqrt{6}} \cdot \frac{1}{\sqrt{11}} \arctan \frac{u}{\sqrt{11}} + C \quad \rightarrow \frac{1}{\sqrt{66}} \arctan \frac{\sqrt{6}\,x}{\sqrt{11}} + C$$

$$\int \frac{(\ln x)^5}{x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\int u^5\,du = \frac{u^6}{6} + C$$

$$\frac{(\ln x)^6}{6} + C$$

$$\int \frac{2}{27+x^2}\,dx \quad = \quad 2\cdot\frac{1}{3\sqrt{3}}\arctan\frac{x}{3\sqrt{3}} + C$$

$$\int \frac{3x}{13+x^4}\,dx = \frac{3}{2}\int \frac{2x}{\sqrt{13}^2+(x^2)^2}\,dx$$

$u = x^2$

$du = 2x\,dx$

$$= \frac{3}{2}\cdot\frac{1}{\sqrt{13}}\arctan\frac{x^2}{\sqrt{13}} + C$$

$$\frac{3}{2}\int \frac{2\cancel{3}x}{13+x^2}\,dx$$

$$u = 13 + x^2$$

$$du = 2x\,dx$$

$$\frac{3}{2}\int \frac{1}{u}\,du = \frac{3}{2}\ln\left(13+x^2\right) + C$$

$$\int \frac{3+x^4}{15x+x^5}\,dx \qquad \frac{1}{5}\int \frac{1}{u}\,du = \frac{1}{5}\ln\left|15x+x^5\right| + C$$

$$u = 15x + x^5$$

$$du = 15 + 5x^4\,dx = 5(3+x^4)\,dx$$

$$\frac{3}{2}\int \frac{2\cancel{3}x}{\left(13+x^2\right)^2}\,dx$$

$$\frac{3}{2}\int \frac{du}{u^2} = -\frac{3}{2u} + C$$

$u = 13 + x^2$

$du = 2x\,dx$

$$\frac{-3}{2\left(13+x^2\right)} + C$$

Popper 14

For the following problems, answer A, B, C, D or E to identify the technique we should use to solve each integral.

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A. u-sub   B. trig powers   C. trig sub (triangle)   D. PFD   E. IBP

10. $\int \frac{x^2+2x+7}{(x-1)^2(x^2+4)}dx$ **11.** $\int x\ln x\,dx$ **12.** $\int \sec^4(2x)dx$ **13.** $\int \frac{1}{x^2\sqrt{4+x^2}}dx$

14. $\int \sec^3(2x)dx$ **15.** $\int \frac{1}{x\sqrt{4-x^2}}dx$ **16.** $\int \frac{11x-73}{x^2-11x+24}dx$

IBP

$$\int \underset{u}{\sec(2x)}\ \underset{dv}{\sec^2(2x)\,dx}$$

$\int x \ln x \, dx$

A L

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{x^2}{2}$

$$\frac{x^2}{2}\ln x - \int \frac{x^{\cancel{2}}}{2}\cdot\frac{1}{\cancel{x}}\,dx$$

$-$

$$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$$

$$\int \sec^4(2x)\,dx = \int \sec^2(2x)\cdot \underline{\sec^2(2x)\,dx}$$

$$= \frac{1}{2}\int \left(1+\tan^2(2x)\right)\cdot 2\sec^2(2x)\,dx$$

$$u = \tan(2x)$$

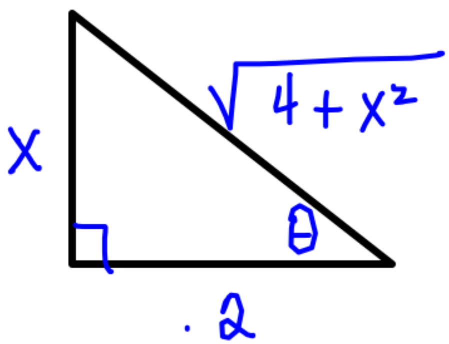
$$du = 2\sec^2(2x)\,dx$$

$$\frac{1}{2}\int 1+u^2\,du$$

$$\frac{1}{2}\left(u+u^3/3\right)+C$$

$$\underline{\frac{1}{2}\tan(2x) + 1/6\,\tan^3(2x) + C}$$

$$\int \frac{1}{x^2\sqrt{4+x^2}}\,dx \longrightarrow \int \frac{1}{4\tan^2\theta \cdot 2\sec\theta} \cdot 2\sec^2\theta\, d\theta$$



$$\frac{x}{2} = \tan\theta$$

$$x = 2\tan\theta$$

$$dx = 2\sec^2\theta\, d\theta$$

$$\sqrt{4+x^2} = 2\sec\theta$$

$$\frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\, d\theta$$

$$\frac{1}{4}\int \frac{1}{\cancel{\cos\theta}} \cdot \frac{\cos^{\cancel{2}}\theta}{\sin^2\theta}\, d\theta$$

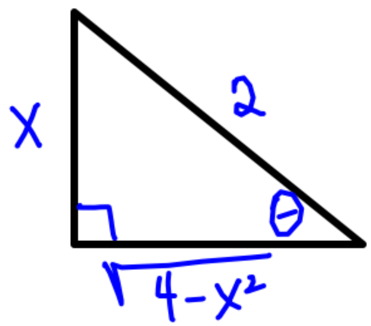
$$= \frac{1}{4}\int \cot\theta \csc\theta\, d\theta$$

+/0

$$= -\frac{1}{4}\csc\theta + C$$

$$= -\frac{1}{4}\left(\frac{\sqrt{4+x^2}}{x}\right) + C$$

$$\int \frac{1}{x\sqrt{4-x^2}}\,dx \rightarrow \int \frac{1}{2\sin\theta \cdot 2\cos\theta} \cdot 2\cos\theta \, d\theta$$



$$= \frac{1}{2}\int \csc\theta \, d\theta$$

$$\frac{H}{O} \qquad \frac{A}{O}$$

$$= -\frac{1}{2} \ln|\csc\theta + \cot\theta| + C$$

$$\frac{x}{2} = \sin\theta$$

$$x = 2\sin\theta$$

$$dx = 2\cos\theta \, d\theta$$

$$\sqrt{4-x^2} = 2\cos\theta$$

$$= -\frac{1}{2} \ln\left| \frac{2}{x} + \frac{\sqrt{4-x^2}}{x} \right| + C$$

$$= -\frac{1}{2} \ln\left| \frac{2+\sqrt{4-x^2}}{x} \right| + C$$