Math 1432

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Office Hours:

Mondays 1-2pm,
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(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

POPPER15

1. Give the quotient associated with $\dfrac{x^3}{(x+1)(x^2+1)}$ from the long division process.

a. -1

b. $x+2$

c. 1

d. $2x-1$

e. None of these

Section 8.5
Numerical Integration

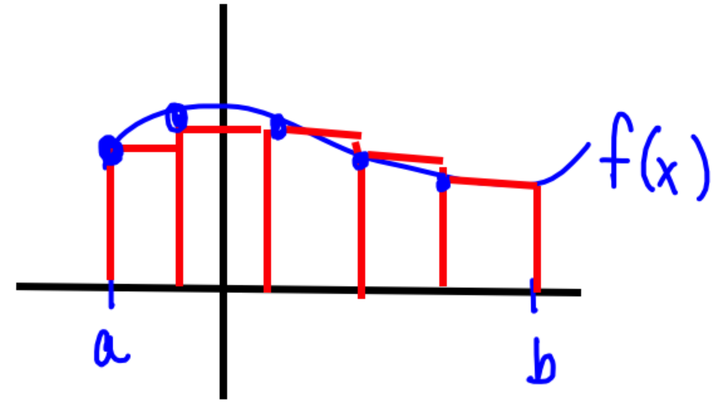
Recall:

1. The left hand endpoint method.
2. The right hand endpoint method.
3. The midpoint method.

New:

4. The trapezoid method.
5. Simpson's method

Left Hand Endpoint Method



Using n subdivisions, approximate $\int_a^b f(x)\,dx$.

The width of each rectangle is $\dfrac{b-a}{n}$.

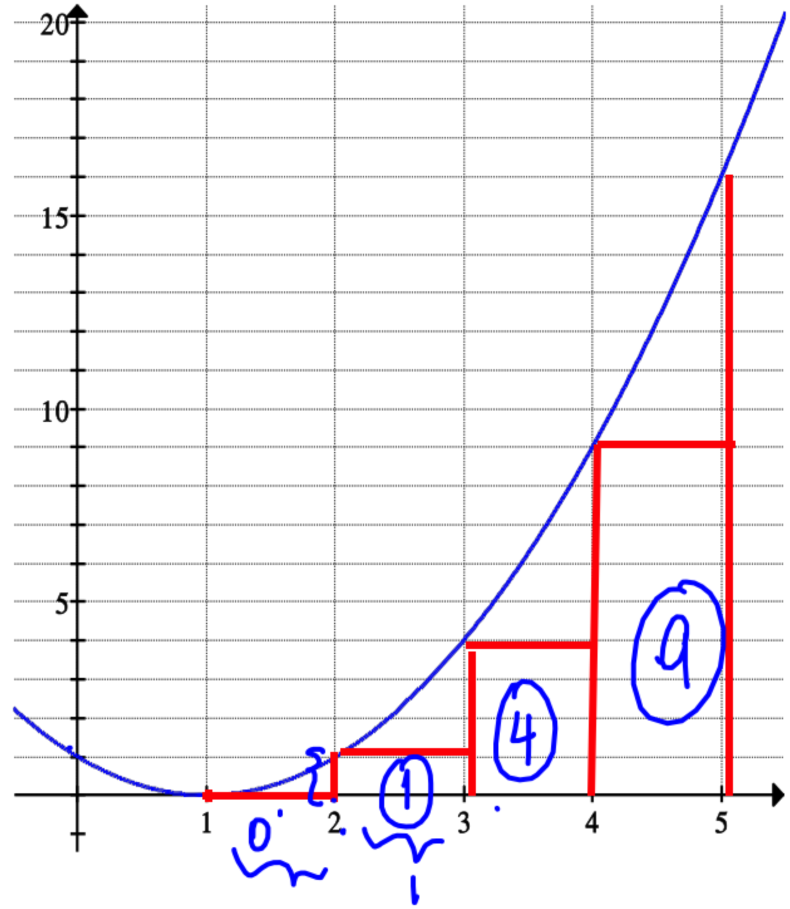
The height of each rectangle is the function value of the x value on the LEFT side of the rectangle.

If the function is positive, the top left corner of the rectangle is ON the curve.

Using $n = 4$, approximate

$$\int_1^5 (x-1)^2\, dx \quad \approx 14$$

using the Left Hand Endpoint Method.



Δx (width of each rect)

$$= \frac{b-a}{n} = \frac{5-1}{4} = 1$$

$f(\text{left endpt}) \cdot \Delta x = A$

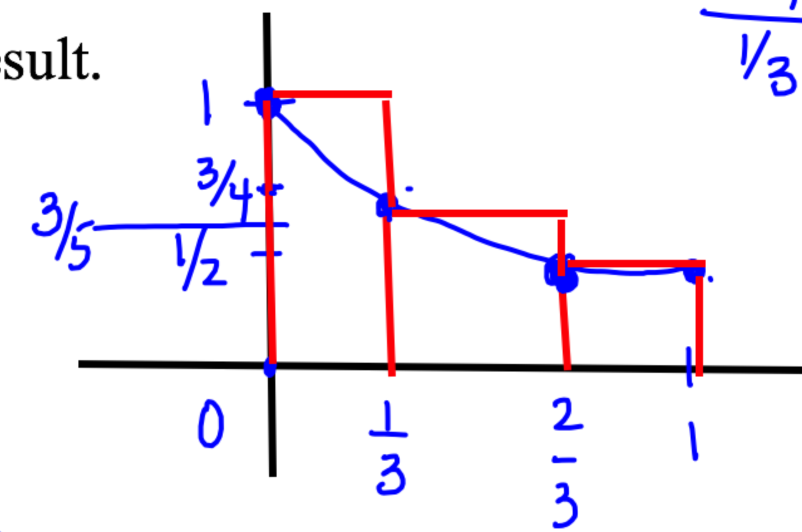
$f(x) = (x-1)^2$

$f(2) = 1$

$f(3) = 4$

2. Use the left hand endpoint method with $n = 3$ to approximate $\int_0^1 \frac{1}{x+1}\,dx$ and give the result.

$$\frac{1}{1/3+1} = 3/4$$



$$\frac{1}{2/3+1} = 3/5$$

a. 49/60

b. 47/60

c. 4/5

d. none of these

$$A\ \sum f(l)\cdot \Delta x = 1/3 \cdot 1 + 1/3 \cdot 3/4 + 1/3 \cdot 3/5$$

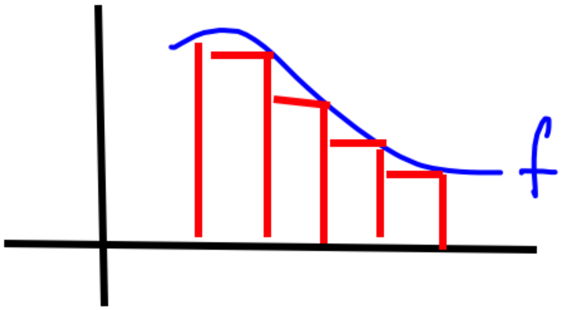
Sum

$$= 1/3 + 1/4 + 1/5$$

$$= \frac{\quad + \quad + \quad}{60}$$

Right Hand Endpoint Method

Using n subdivisions, approximate $\int_a^b f(x)\,dx$.



The width of each rectangle is $\dfrac{b-a}{n}$.

The height of each rectangle is the function value of the x value on the RIGHT side of the rectangle.

If the function is positive, the top right corner of the rectangle is ON the curve.

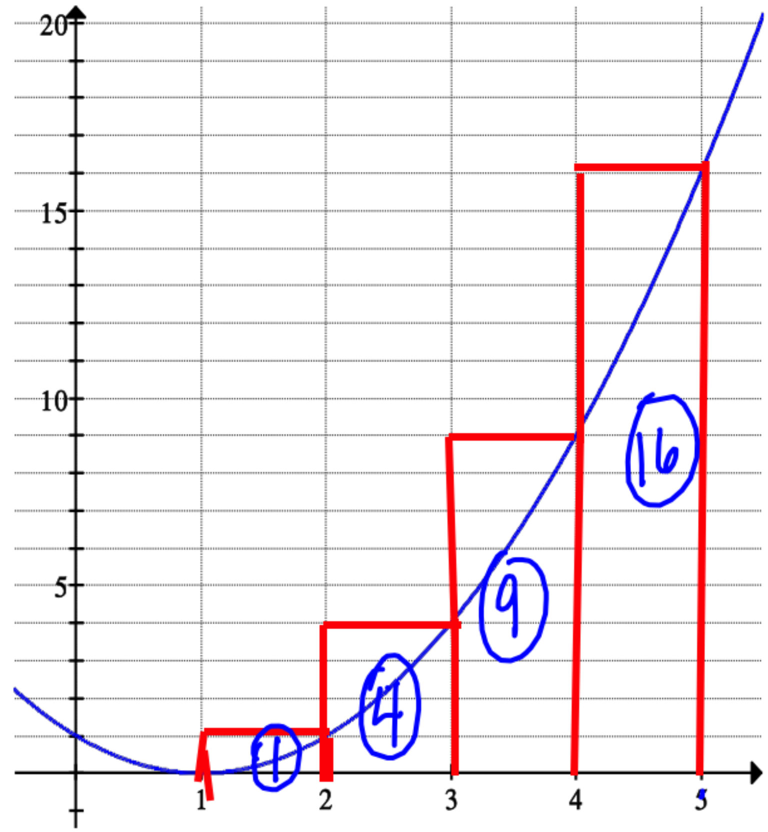
Using $n = 4$, approximate

$$\int_1^5 (x-1)^2 \, dx$$

using the Right Hand Endpoint Method.

$\Delta x = 1$

$1(1+4+9+16) = 30$



3. Use the right hand endpoint method with $n = 3$ to approximate

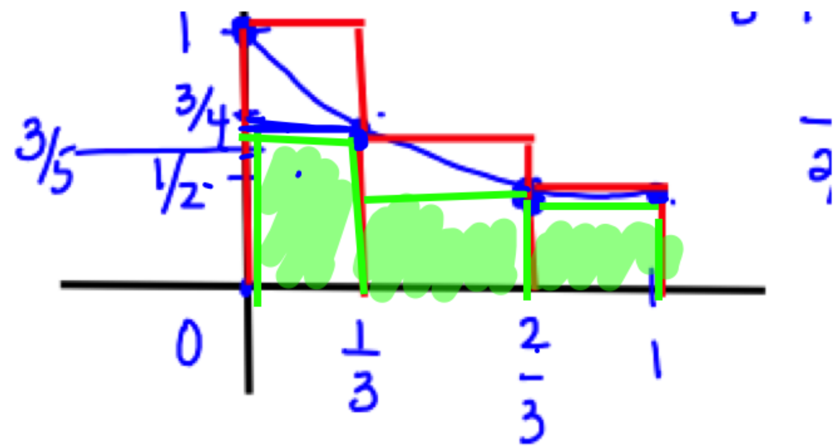
$$\int_0^1 \frac{1}{x+1}\,dx$$ and give the result.

a. 37/60

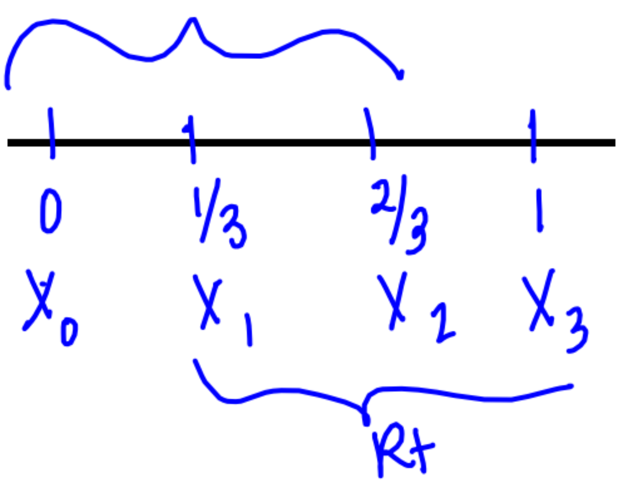
b. 39/60

c. 1/2

d. none of these

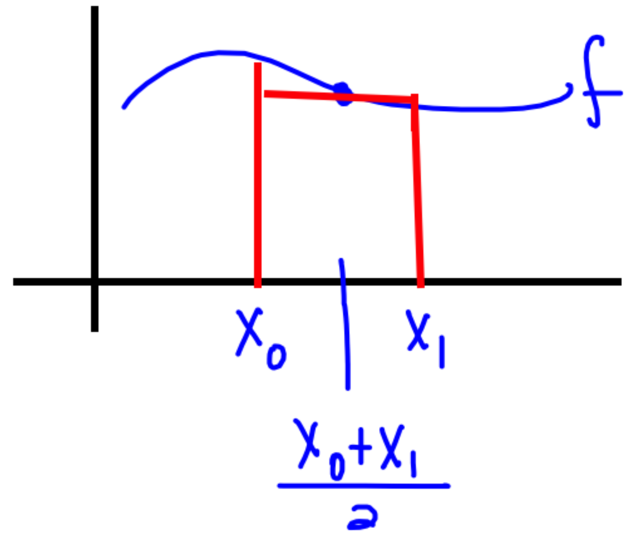


$$\frac{1}{3}\left(3/4 + 3/5 + 1/2\right)$$



Midpoint Method

Using n subdivisions, approximate $\int_a^b f(x)\,dx$.



The width of each rectangle is $\frac{b-a}{n}$.

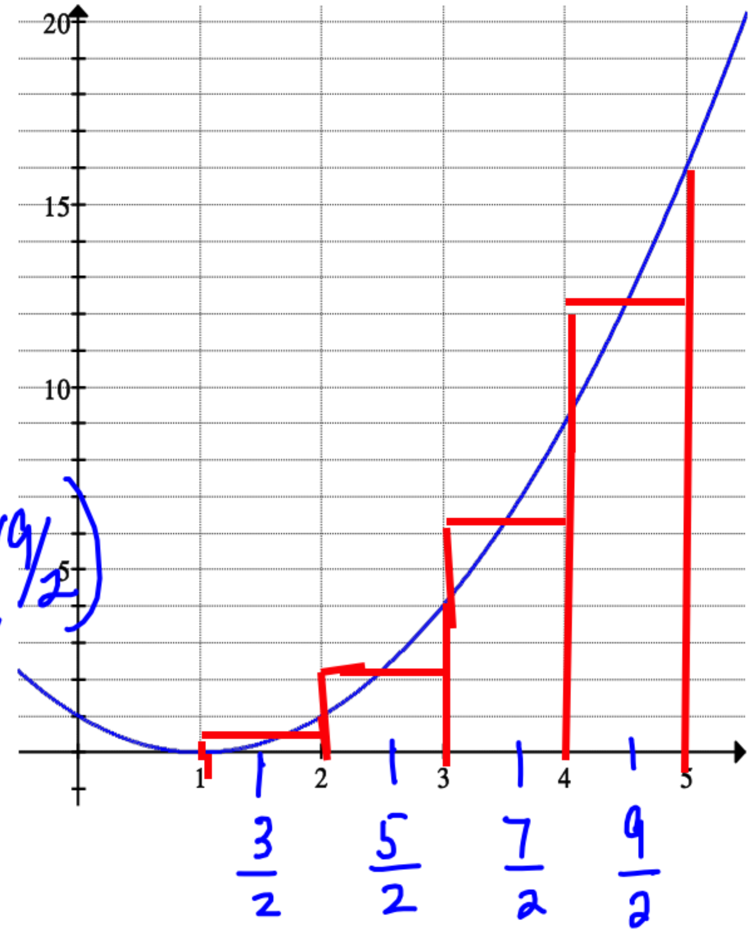
The height of each rectangle is the function value of the x value at the MIDPOINT of the width of the rectangle.

The midpoint of the side of the rectangle is ON the curve.

Using $n = 4$, approximate

$$\int_1^5 (x-1)^2 \, dx$$

using the Midpoint Method.



$\Delta x \cdot \left(f(3/2) + f(5/2) + f(7/2) + f(9/2) \right)$

$= 1 \left(1/4 + 9/4 + 25/4 + 49/4 \right)$

$= 1 \left(84/4 \right) = \boxed{21}$

Δx still 1

4. Use the midpoint method with $n = 2$ to approximate $\displaystyle\int_1^2 \frac{1}{x}\,dx$ and give the result.

a. 26/35

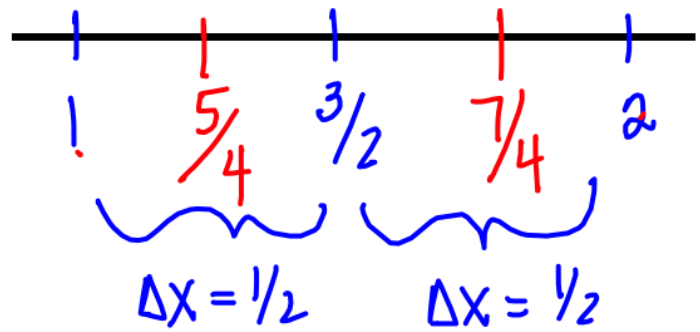
b. 24/35

c. 6/7

d. 5/7

e. none of these

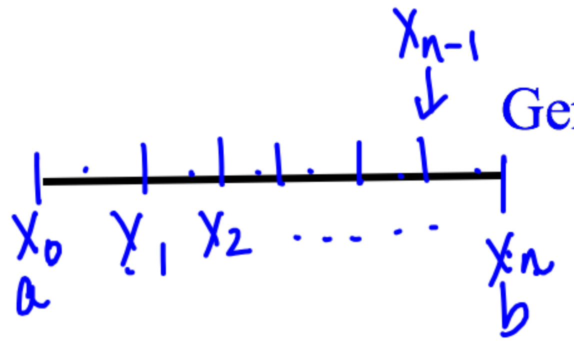
$f(x) = 1/x$



$\frac{1}{2}\left(f(5/4) + f(7/4)\right)$

$\frac{1}{2}\left(4/5 + 4/7\right)$

$\frac{1}{2}\left(\frac{}{35}\right)$



General Formulas to approximate

$$\int_a^b f(x)\,dx$$

Left Hand Endpoint Method:

Δx

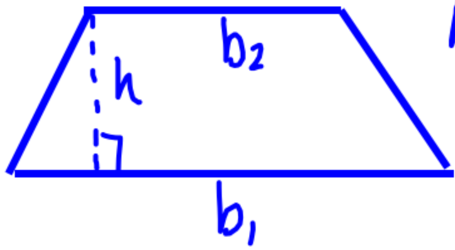
$$L_n = \frac{b-a}{n}\left[f(x_0)+f(x_1)+\cdots+f(x_{n-1})\right]$$

Right Hand Endpoint Method:

$$R_n = \frac{b-a}{n}\left[f(x_1)+f(x_2)+\cdots+f(x_n)\right]$$

Midpoint Method:

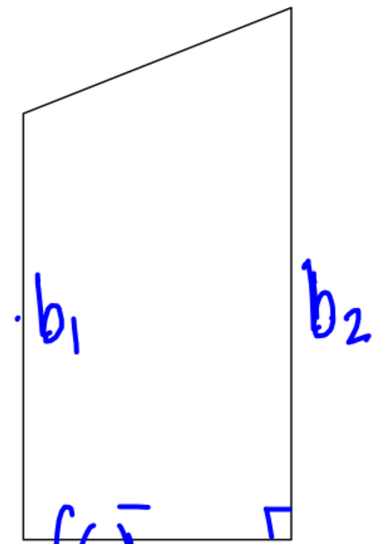
$$M_n = \frac{b-a}{n}\left[f\left(\frac{x_0+x_1}{2}\right)+\cdots+f\left(\frac{x_{n-1}+x_n}{2}\right)\right]$$



$A = \frac{1}{2}h(b_1 + b_2)$

Trapezoid Method

Area of a trapezoid $= \frac{1}{2}h\left(b_1 + b_2\right)$



$$\Delta x \left(\frac{b_1 + b_2}{2}\right)$$

$$\Delta x \left(\frac{f(x_0) + f(x_1)}{2}\right) + \Delta x \left(\frac{f(x_1) + f(x_2)}{2}\right) + \cdots$$

In this method, instead of using rectangles, we use trapezoids.

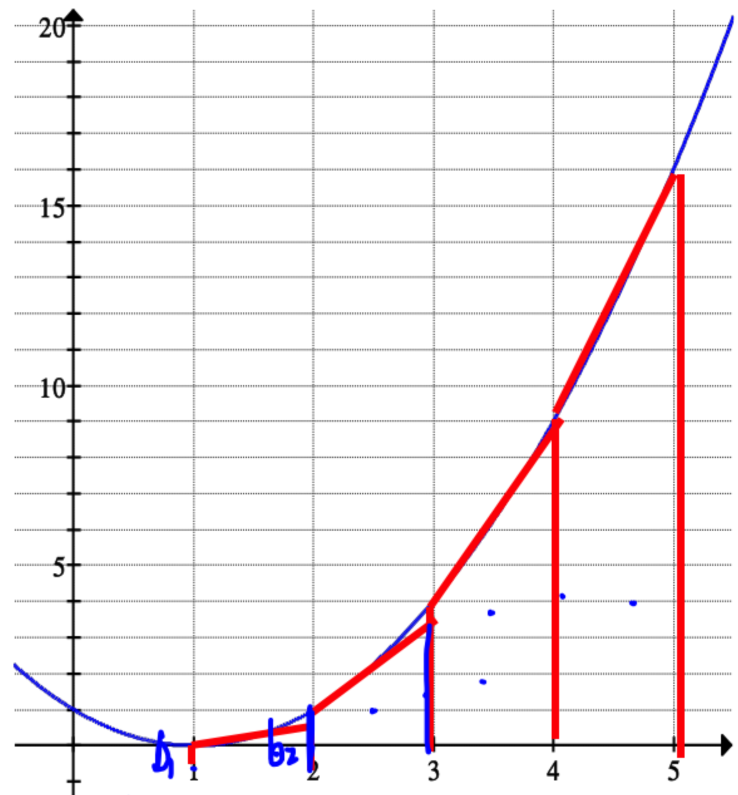
The bases are the parallel sides. Their lengths are the function values.

In our graphs, the height is going to be the length of the subdivision $\frac{b-a}{n}$.

Using n = 4, approximate

$$\int_1^5 (x-1)^2 \, dx$$

using the Trapezoid Method.



$$1\left[\frac{f(1)+f(2)}{2}+\frac{f(2)+f(3)}{2}+\frac{f(3)+f(4)}{2}+\frac{f(4)+f(5)}{2}\right]$$

$$\frac{1}{2}\left[f(1)+2f(2)+2f(3)+2f(4)+f(5)\right]$$

$$\frac{1}{2}\left(0+2\cdot 1+2\cdot 4+2\cdot 9+16\right)=22$$

5. Use the trapezoid method with $n = 2$ to approximate $\int_0^1 \underbrace{\frac{1}{x^2+1}}_{f(x)}\,dx$ and give the result.

a. 33/40

b. 7/8

c. 31/40

d. none of these

$$\begin{array}{ccc} 0 & \frac{1}{2} & 1 \\ x_0 & x_1 & x_2 \end{array}$$

$$\frac{b-a}{2n}\left(f(0) + 2f\left(\tfrac{1}{2}\right) + f(1)\right)$$

$$\frac{1}{4}\left(1 + 2\cdot\frac{4}{5} + \frac{1}{2}\right)$$

The trapezoid method (trapezoidal rule)
General Formula to approximate

$$\int_a^b f(x)\,dx$$

$$T_n = \frac{b-a}{n}\left[\frac{f(x_0)+f(x_1)}{2} + \frac{f(x_1)+f(x_2)}{2} + \cdots + \frac{f(x_{n-1})+f(x_n)}{2}\right]$$

$$= \frac{b-a}{2n}\,[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)].$$

Note: This is the average of the left hand estimate and the right hand estimate.

Trapezoid Rule Error Estimate

From your book:

As is shown in texts on numerical analysis, if *f* is continuous on [a, b] and twice differentiable on (a, b), then the theoretical error of the trapezoidal rule,

$$E_n^T = \int_a^b f(x)\,dx - T_n,$$

Can be written

$$E_n^T = -\frac{(b-a)^3}{12n^2} f''(c),$$

??

where c is some number between a and b. Usually we cannot pinpoint c any further. However, if f'' is bounded on [a, b], say $|f''(x)| \le M$ for $a \le x \le b$, then

max value $f''(x)$ can be for x in [a, b]

$$\left|E_n^T\right| \le \frac{(b-a)^3}{12n^2} M.$$

Example: Give a value of n that will guarantee the Trapezoid method approximates $\int_0^{\frac{\pi}{2}} \sin(2x)\,dx$ within $\underline{10^{-4}}$. $\frac{1}{10^4}$

$$\left|E_n^T\right| \le \frac{(b-a)^3}{12n^2} M \le \frac{1}{10^4}$$

Want $E_n^T \le \frac{1}{10^4}$

$f(x) = \sin(2x)$

$f'(x) = 2\cos(2x)$

$f''(x) = -4\sin(2x)$

$|f''(c)| \le 4$

$$\frac{(\pi/2 - 0)^3}{12 \cdot n^2}(4) \le \frac{1}{10^4}$$

$$\pi^3/8\,(4) \le \frac{12n^2}{10^4}$$

$$\frac{10^4}{12}\left(\frac{\pi^3}{2}\right) \le n^2$$

$$12919.28 \le n^2$$

$$113.66 \le n$$

$$\boxed{n = 114}$$

6. Give a smallest value of n that will guarantee the Trapezoid method approximates $\int_0^1 \frac{1}{x+1}\,dx$ within 10^{-2}.

$$\left|E_n^T\right| \le \frac{(b-a)^3}{12n^2} M$$

a. ~~10~~ 5
b. 20
c. 30
d. none of these

$$f(x) = \frac{1}{x+1}$$

$$f'(x) = \frac{-1}{(x+1)^2}$$

$$f''(x) = \left|\frac{2}{(x+1)^3}\right| \le 2$$

$$\frac{(1-0)^3}{12n^2}(2) \le \frac{1}{100}$$

$$\frac{100}{6} \le n^2$$

$$16\tfrac{2}{3} \le n^2$$

$$5 \le n$$