Reminder
HW Keys
links posted
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discussion board

Math 1432

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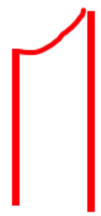
Office Hours:

Mondays 1-2pm,
Fridays noon-1pm
(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html



Simpson's Method
Fit a parabola to every section.

$$S_n = \frac{b-a}{3n}\left[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+\ldots+2f(x_{n-2})+4f(x_{n-1})+f(x_n)\right]$$

$\boxed{*n \text{ must be even}*}$

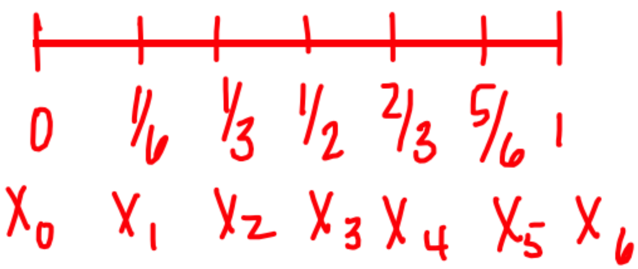
Use Simpson's method with $n = 6$ to approximate $\int_0^1 \frac{1}{x+1}\,dx$.

$\Delta x = \frac{1-0}{6} = \frac{1}{6}$

$= \ln|x+1| \Big|_0^1$

$= \ln 2 - \ln 1 = \ln 2$

$= .69315$



$S_6 = \frac{1-0}{3(6)}\Big[f(0) + 4f(1/6) + 2f(1/3) + 4f(1/2) + 2f(2/3)$
$+ 4f(5/6) + f(1)\Big] = \frac{14411}{20790} \approx .69$

Theoretical error - Simpson's Rule: The theoretical error of Simpson's Rule

$$S_n^T = \int_a^b f(x)\,dx - S_n$$

is given by

$$S_n^T = \frac{(b-a)^5}{180n^4} f^{(4)}(c)$$

for some $c \in (a,b)$. As above, we usually do not know c, but, if $f^{(4)}$ is bounded on $[a,b]$, say $\left|f^{(4)}(x)\right| \le M$ for all $x \in [a,b]$, then

max value of 4th deriv. on $[a,b]$

$$\boxed{|S_n^T| \le \frac{(b-a)^5}{180n^4} M.}$$

Give a value of n that will guarantee Simpson's method approximates $\int_0^{\frac{\pi}{2}} \sin(2x)\,dx$ within 10^{-4}. $\left|E_n^S\right| \le \frac{(b-a)^5}{180n^4} M$ where $\left|f^{(4)}(x)\right| \le M$ for $a \le x \le b$.

$f(x) = \sin 2x$
$f'(x) = 2\cos 2x$
$f''(x) = -4\sin 2x$
$f'''(x) = -8\cos 2x$
$\rightarrow f^{(4)}(x) = 16\sin 2x$

$M = 16$

$$\frac{(\pi/2 - 0)^5}{180n^4}(16) \le \frac{1}{10^4}$$

approx π w/ 3 $\rightarrow$

$$\frac{\frac{\pi^5}{\cancel{32}\,2}(\cancel{16})(10^4)}{180} \le n^4$$

$$\frac{2430000}{360} \le n^4$$

$$6750 \le n^4$$

$$9.06 \le n \Rightarrow n = 10$$

General Formulas to approximate

$$\int_a^b f(x)\,dx$$

Left Hand Endpoint Method:

$$L_n = \frac{b-a}{n}\left[f(x_0) + f(x_1) + \cdots + f(x_{n-1})\right]$$

Right Hand Endpoint Method:

$$R_n = \frac{b-a}{n}\left[f(x_1) + f(x_2) + \cdots + f(x_n)\right]$$

Midpoint Method:

$$M_n = \frac{b-a}{n}\left[f\left(\frac{x_0 + x_1}{2}\right) + \cdots + f\left(\frac{x_{n-1} + x_n}{2}\right)\right]$$

Trapezoid Method:

$$T_n = \frac{b-a}{2n}\left[f\left(x_0\right) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f\left(x_n\right) \right]$$

Simpson's Rule:

$$S_n = \frac{b-a}{3n}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n) \right]$$

Error Estimate

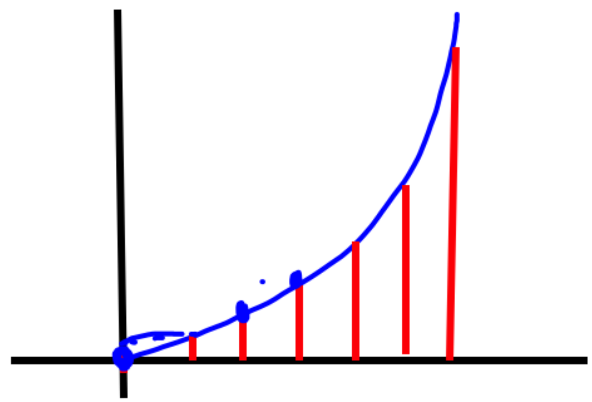
Trapezoid: $E_n^T = \frac{(b-a)^3}{12n^2} f''(c), \quad \left|E_n^T\right| \le \frac{(b-a)^3}{12n^2} M$

Simpson's: $E_n^S = \frac{(b-a)^5}{180n^4} f^{(4)}(c), \quad \left|E_n^S\right| \le \frac{(b-a)^5}{180n^4} M$

POPPER16

1. Which method will have the smallest error?
   a. Left endpoint
   b. Right endpoint
   c. Midpoint
   d. Trapezoid
   e. Simpsons

2. Which method will give the largest estimate for $\int_0^2 x^2\,dx$ with $n = 10$?
   a. Left endpoint
   b. Right endpoint
   c. Midpoint
   d. Trapezoid
   e. Simpsons



x_0 x_1 . . . .
0 0.2

More Examples:

Use the table below to approximate $\int_0^2 f(x)\,dx$ with $n = 10$.

$f(x_0)$

(a) using the trapezoid method

(b) using Simpson's rule.

x	$f(x)$
0	1.8
0.2	1.8
0.4	2.4
0.6	1.5
0.8	2.1
1	2.5
1.2	2.3
1.4	2.2
1.6	1.7
1.8	2.1
2	2.5

$f(x_{10})$

a) $\frac{2-0}{2(10)}\left[f(0)+2f(0.2)+2f(0.4)\cdots+2f(1.8)+f(2)\right]$

$\frac{1}{10}\left[1.8+2(1.8)+2(2.4)+2(1.5)+\cdots+2.5\right]$

$= 4.15$

b) $\frac{2-0}{3(10)}\left[f(0)+4f(0.2)+2f(0.4)+\cdots\cdots+4f(1.8)+f(2)\right]$

$\frac{2}{30}\left[1.8+4(1.8)+2(2.4)+\cdots+4(2.1)+2.5\right]$

$= 4.1133$

Estimate the error if T_8 is used to calculate $\int_0^5 \cos(3x)\,dx$

$n=8 \qquad a=0 \qquad b=5$

$f(x) = \cos(3x)$

$f'(x) = -3\sin(3x)$

$f''(x) = -9\cos(3x)$

$$E_8^T = \frac{(5-0)^3}{12(8)^2}\left|-9\cos(3c)\right|$$ for some c between 0 + 5

$$\leq \frac{125}{768}(9) = 1.465 \text{ (largest error)}$$

Estimate the error if S_8 is used to calculate $\int_0^5 \cos(3x)\,dx$

$f'''(x) = 27\sin(3x)$

$f^{4}(x) = 81\cos(3x)$

$$E_8^S = \frac{(5-0)^5}{180(8)^4}\left|81\cos(3c)\right|$$ for some $0 \leq c \leq 5$

$$\leq \frac{5^5}{180(8)^4}(81) = .343$$

$f(x) = \cos(2x)$ $f'(x) = -2\sin(2x)$ $f''(x) = -4\cos(2x)$

$f'''(x) = 8\sin(2x)$ $f^{4}(x) = 16\cos(2x)$

Find n so that T_n is guaranteed to approximate $\int_0^3 \cos(2x)\,dx$ to within 0.03

$$\frac{(3-0)^3}{12n^2}(\max|f''|) = \frac{27}{12n^2}(4) \le .03$$

$$\frac{9}{n^2} \le \frac{3}{100}$$

$$\cancel{900}/_{3} \le n^2$$

$$300 \le n^2$$

$$n = 18$$

Find n so that S_n is guaranteed to approximate $\int_0^3 \cos(2x)\,dx$ to within 0.03

$$\frac{(3-0)^5}{180n^4}(\max|f^4|) = \frac{3^5}{180n^4}(16) \le \frac{3}{100}$$

$$720 \le n^4$$

$$5.18 \le n$$

$$n = 6$$

POPPER16

3. What comes next in the *sequence* 3, 6, 11, 18, 27, 38,…?

1st 2nd 3rd

   a. 42
   b. 51
   c. 47
   d. 67
   e. none of these

4. What is the formula for a_n for the *sequence* 3, 6, 11, 18, 27, 38,…?

nth term

   a. $n + 2$
   b. $n^2 + 1$
   c. $n^2 + 2$
   d. $n^2 - 1$
   e. none of these

$\boxed{\text{Sequences}}$ are LISTS of objects. The objects could be numbers or something else. The list in poppers 1 and 2 are sequences of numbers. Each number "has a place".

Formally, a sequence of numbers is a function from the positive integers (or natural numbers) to the real numbers:

$$f(n) = a_n, \ n \in \mathbb{N} \ (n = 1, 2, 3, \ldots)$$

One of the most important aspects (from our perspective) will be something called the "limit of a sequence".

Some facts:

- Sequences of numbers do not have to have a pattern or nice behavior.
- Most sequences that we deal with will have a pattern and “reasonably” nice behavior.
- The pattern will come from a *generating formula*.
- The nice behavior will come in the from of a *limiting behavior*.
- There are a variety of ways to denote a sequence: a_n, $\{ a_n \}$ or $\{ a_n \}^{\infty}_{n=1}$
- We will be concerned with infinite sequences.

Example: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots$

What is the function associated with this sequence?

$f(n) = \frac{1}{n}$ $\qquad a_n = \frac{1}{n}$ $\qquad \left\{\frac{1}{n}\right\}_{n=1}^{\infty}$

Example: $-1, 1, -1, 1, -1, 1, -1, 1, \ldots$

What is the function associated with this sequence?

$(-1)^n$ $\qquad \cos(\pi n)$ $\qquad -\sin\left(\frac{\pi}{2}(2n+1)\right)$

Give the first 3 terms of each of the following sequences.

$$a_n = \frac{1}{n+2}$$

$a_1 = \frac{1}{3}$ $a_2 = \frac{1}{4}$ $a_3 = \frac{1}{5}$

$$a_n = \frac{n}{1-2n}$$

$a_1 = -1$ $a_2 = -\frac{2}{3}$ $a_3 = -\frac{3}{5}$

$$a_n = \frac{(-1)^n}{n}$$

$a_1 = -1$ $a_2 = \frac{1}{2}$ $a_3 = \frac{-1}{3}$

Terms:

Bounded sequence or set – The sequence or set fits inside an interval.

Upper bound – A number greater than or equal to all the elements of the sequence or set.

Least Upper Bound (LUB) – Smallest number greater than or equal to all the elements of the sequence or set.

Lower bound – A number less than or equal to all the elements of the sequence or set.

Greatest Lower Bound (GLB) – Largest number less than or equal to all the elements of the sequence or set.

Give several lower bounds for [–2, 3). $-10, \ -100, \ -52$

Give several upper bounds for [–2, 3). $5, \ 1024, \ 15$

Give the LUB and GLB of [–2, 3).

$GLB = -2$

$LUB = 3$

Give the LUB and GLB for $\{ x \mid x^2 < 4\}$. $= (-2, 2)$

$x^2 - 4 < 0$

$(x-2)(x+2) < 0$

$+ \quad - \quad +$

$-2 \quad 2$

$GLB = -2$

$LUB = 2$

5. Give the LUB for [–3, 1).

a. 3
b. –3
c. 1
d. –1
e. none of these