Math 1432

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Mondays 1-2pm,
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Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

The sequence $\{a_n\}$ is said to be

- *increasing* if $a_n < a_{n+1}$ for all n,
- *non-decreasing* if $a_n \leq a_{n+1}$ for all n,
- *decreasing* if $a_n > a_{n+1}$ for all n,
- *non-increasing* if $a_n \geq a_{n+1}$ for all n.

A sequence that satisfies any of these conditions is called monotonic.

Increasing Sequence: $a_1 < a_2 < a_3 < \ldots < a_n < a_{n+1} < \ldots$

Non-decreasing Sequence: $a_1 \leq a_2 \leq a_3 \leq \ldots \leq a_n \leq a_{n+1} \leq \ldots$

Decreasing Sequence: $a_1 > a_2 > a_3 > \ldots > a_n > a_{n+1} > \ldots$

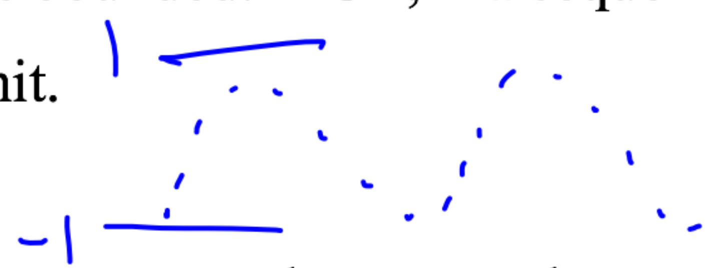
Non-increasing Sequence: $a_1 \geq a_2 \geq a_3 \geq \ldots \geq a_n \geq a_{n+1} \geq \ldots$

lower

In an increasing sequence, the GLB is the first term, and if there is a limit, then it is the LUB.

In a decreasing sequence, the LUB is the first term, and if there is a limit, then it is the GLB.

If a sequence has a limit, then the sequence is bounded. BUT, if a sequence is bounded, it does not necessarily have a limit.



The limit of a sequence is a number that the sequence values a_n tend towards as $n \to \infty$.

Find the GLB and LUB if they exist:

1. $S = \{1, 2, 3, 4\}$ GLB = 1 LUB = 4

2. $[-4, 2]$ GLB = -4 LUB = 2

3. $(-\infty, 8)$ GLB - none LUB = 8

4. $(5, \infty)$ GLB = 5 LUB - none

$$.\overline{9} = 1$$

$$
\begin{array}{rl}
 & \frac{1}{3} = .\overline{3} \\
+ & \frac{2}{3} = .\overline{6} \\
\hline
 & 1 = .\overline{9}
\end{array}
$$

5. $S = \{x : x \leq 16\}$ $= (-\infty, 16]$ GLB - none LUB $= 16$

6. $\left\{1 - \frac{2}{n}\right\}_{n=1}^{\infty}$ $= -1, \; 0, \; \frac{1}{3}, \; 1/2, \; \ldots, \; \frac{98}{100}$

$1-\frac{2}{1} \quad 1-\frac{2}{2} \quad 1-\frac{2}{3} \quad 1-\frac{2}{4} \quad 1-\frac{2}{100}$

GLB $= -1$ LUB $= 1$

If a sequence has a limit, then it is bounded.

WARNING!!! The converse is not necessarily true!!!

Determine the boundedness and monotonicity. → Incr, decr, not mono.

$$\left\{\frac{1}{n^2}\right\}_{n=1}^{\infty}$$

$1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16},$ decr.

LUB = 1

GLB = 0

$$\left\{(-1)^n\right\}$$

$-1, 1, -1, 1$ GLB = −1, LUB = 1

not monotonic

$$\left\{2^{n+1}\right\}$$

$4, 8, 16, \ldots\ldots$ Incr

GLB = 4 no LUB

$(-1)^{odd} = -1$

$\left\{(-1)^{2n+1}\sqrt{n}\right\}$ ← odd $= \left\{-\sqrt{n}\right\} = -1, -\sqrt{2}, -\sqrt{3}, -2, \ldots, -19\ldots$

decr.

LUB = −1 GLB DNE

$\left\{\frac{2n-1}{3n+2}\right\}_{n=1}^{\infty}$ $\frac{1}{5}, \frac{3}{8}, \frac{5}{11}, \frac{7}{14}, \ldots$ incr

GLB = 1/5

LUB = 2/3

$$\lim_{n\to\infty} \frac{2n-1}{3n+2} = \frac{2}{3}$$

$\left\{\frac{1}{2^n}\right\}_{n=1}^{\infty}$ $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$ decr

LUB = 1/2 GLB = 0

Using Geogebra (https://www.geogebra.org/)

Sequence[<Expression>, <Variable>, <Start Value>, <End Value>]

Sequence[(n, formula), n, 1, <End Value>]

$\{sin(n)\}_{n=1}^{\infty}$. not monotonic GLB = -1 LUB = 1

no limit

Give the limit of the sequence $\left\{ sin(n\pi) \right\}_{n=1}^{\infty}$. = 0, 0, 0, 0

$\underline{\underline{0}}$

Give the LUB and GLB for $\left\{ n\, sin(n) \right\}_{n=1}^{\infty}$.

none

Determine whether the sequence $\left\{\frac{2n+(-1)^n}{n}\right\}_{n=1}^{\infty}$ is

$\left\{\frac{2n}{n}+\frac{(-1)^n}{n}\right\}$

$\left\{2+\frac{(-1)^n}{n}\right\}$

a. bounded LUB $= 2\frac{1}{2}$

GLB $= 1$

$1, 2\frac{1}{2}, 1\frac{2}{3}, 2\frac{1}{4}$

b. monotone no

c. then give the limit if it exists. 2

Determine whether the sequence $\left\{\dfrac{\sqrt{n+1}}{\sqrt{n}}\right\}_{n=1}^{\infty}$ is

$$\sqrt{\frac{n}{n}+\frac{1}{n}}$$

$$\sqrt{1+\frac{1}{n}} \quad \frac{1}{n} \to 0$$

a. bounded

$LUB = \sqrt{2}$

$GLB = 1$

$\sqrt{2}, \sqrt{1.5}, \sqrt{4/3}$

b. monotone

decr.

c. then give the limit if it exists.

$\sqrt{1} = 1$

Give the limit (if it exists) of $\left\{\frac{\sin(n)}{n}\right\}_{n=1}^{\infty}$ $= 0$

Give the limit (if it exists) of $\left\{\left(\frac{2}{n}\right)^{n}\right\}_{n=1}^{\infty}$ $\frac{2^n}{n^n} \to 0$

Give the limit (if it exists) of $\left\{\left(\frac{2n^2-3n+6}{3n-16n^2+12}\right)\right\}_{n=1}^{\infty}$ $\Rightarrow \frac{2}{-16} = \frac{-1}{8}$

Give the limit (if it exists) of $\left\{n^n\right\}_{n=1}^{\infty}$. DNE

Find a formula for the general term a_n of $\left\{\frac{3}{2}, -\frac{9}{4}, \frac{27}{8}, -\frac{81}{16}, \ldots\right\}$ assuming the pattern of the first few terms continues.

Popper 17

1. Give the limit (if it exists) of $\left\{ \dfrac{2n^2 + 6n}{\sqrt{2}\,n - n^2} \right\}_{n=1}^{\infty}$

a. –2

b. 0

c. 1

d. 1/2

2. Give the limit (if it exists) of $\left\{n^{2n}\right\}_{n=1}^{\infty}$

a. –2

b. DNE

c. 1

d. 1/2

Consider the sequence defined by $a_n = \left(\frac{2}{3}\right)^n$. ($n$ starts at 1)

a. Write the first five terms of the sequence.

b. Determine the limit of the sequence.

c. Let $b_n = \frac{a_{n+1}}{a_n}$. Write the first five terms of this sequence.

d. Determine the limit of b_n.

Consider the sequence defined by $a_n = \left(\frac{-3}{2}\right)^n$. ($n$ starts at 1)

a. Write the first five terms of the sequence.

b. Determine the limit of the sequence.

c. Let $b_n = \frac{a_{n+1}}{a_n}$. Write the first five terms of this sequence.

d. Determine the limit of b_n.

Are the following increasing, decreasing, or not monotonic?

$$a_n = \frac{3n+4}{2n+5}$$

$$a_n = \frac{3+(-1)^n}{n}$$

$$a_n = \frac{\sqrt{n+1}}{5n+3}$$

Give an upper bound for the set of negative real numbers.

Give a lower bound for the set of negative real numbers.

Give the LUB and GLB for the set of negative real numbers.

Give the LUB and GLB of $\left\{ \dfrac{(-1)^n}{n} \right\}_{n=3}^{\infty}$

Determine whether $\left\{ \ln\left(\dfrac{2n-1}{3n+7} \right) \right\}_{n=1}^{\infty}$ is bounded.

Sequences can be defined *recursively*: one or more terms are given explicitly; the remaining ones are then defined in terms of their predecessors. Give the first six terms
of the sequence and then give the nth term.

$a_1 = 1$; $a_{n+1} = \frac{1}{2} a_n + 1$.

3. Give the limit (if it exists) of $\left\{ \dfrac{(-1)^n}{n} \right\}_{n=1}^{\infty}$.

a. -1

b. 0

c. 1

d. $1/2$

4. Give the limit (if it exists) of $\left\{\dfrac{1+sin(n)}{n}\right\}_{n=1}^{\infty}$

a. -1

b. 0

c. 1

d. 1/2

5. Give the LUB for $\{x \mid x^2 - 2x < 3\}$ $= (-1, 3)$

a. 3
b. –3
c. 1
d. –1
e. none of these

$x^2 - 2x - 3 < 0$

$(x-3)(x+1) < 0$

Pop Quiz 2

$\int \sec^4(x)\tan^2(x)\,dx$ - trig powers

$\int \frac{1}{x\sqrt{x^2+3}}\,dx$ - trig substitution (θ)

$\int \ln x\,dx$ - IBP

$\int \frac{3}{x^2-6x+9}\,dx$ - PFD

$\int x\tan(x^2)\,dx$ - u-sub

6. Give the **form** of the partial fraction decomposition:

$$\frac{2x+3}{x^4-1} = \frac{2x+3}{(x-1)(x+1)(x^2+1)}$$

$$\frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$$

$$\frac{4-x}{x^3-8x^2+16x} = \frac{4-x}{x(x^2-8x+16)} = \frac{4-x}{x(x-4)^2}$$

$$\frac{A}{x} + \frac{B}{x-4} + \frac{C}{(x-4)^2}$$

$$\frac{2x+3}{x^4-x^2} = \frac{2x+3}{x^2(x^2-1)}$$

$$\frac{2x+3}{x^2(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{x+1}$$

$$\frac{x+x^2}{(x^2-8x+16)(x^2-4)} = \frac{x+x^2}{(x-4)^2(x-2)(x+2)}$$

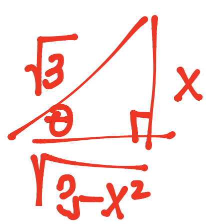
$$\frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{x-2} + \frac{D}{x+2}$$

7. Re-write the integral in terms of θ using trig-substitution.

$$\int \frac{2}{x^2\sqrt{3-x^2}}\,dx$$

$x = \sqrt{3}\sin\theta$

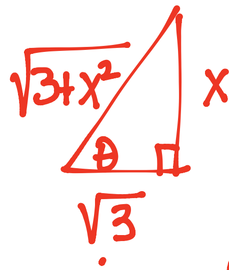
$dx = \sqrt{3}\cos\theta\,d\theta$



$\sqrt{3-x^2} = \sqrt{3}\cos\theta$

$$\int \frac{2\sqrt{3}\cos\theta\,d\theta}{3\sin^2\theta\cdot\sqrt{3}\cos\theta}$$

$$\int \frac{1}{x^2\sqrt{3+x^2}}\,dx$$



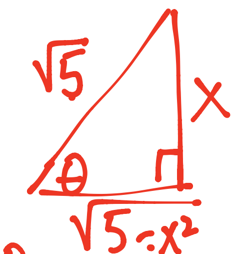
$x = \sqrt{3}\tan\theta$

$dx = \sqrt{3}\sec^2\theta\, d\theta$

$\sqrt{3+x^2} = \sqrt{3}\sec\theta$

$$\int \frac{\sqrt{3}\sec^2\theta\, d\theta}{3\tan^2\theta \cdot \sqrt{3}\sec\theta}$$

$$\int \frac{x^2}{\sqrt{5-x^2}}\,dx$$



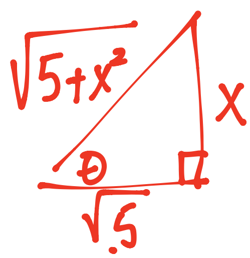
$x = \sqrt{5}\sin\theta$

$dx = \sqrt{5}\cos\theta\, d\theta$

$\sqrt{5-x^2} = \sqrt{5}\cos\theta$

$$\int \frac{5\sin^2\theta}{\sqrt{5}\cos\theta}\sqrt{5}\cos\theta\, d\theta$$

$$\int \frac{1}{x\sqrt{5+x^2}}\,dx$$



$x = \sqrt{5}\tan\theta$

$dx = \sqrt{5}\sec^2\theta\, d\theta$

$\sqrt{5+x^2} = \sqrt{5}\sec\theta$

$$\int \frac{\sqrt{5}\sec^2\theta\, d\theta}{\sqrt{5}\tan\theta \cdot \sqrt{5}\sec\theta}$$