Math 1432

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Office Hours:

Mondays 1-2pm,
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Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

$n = \quad 1 \quad 2 \quad 3 \quad 4$

Find a formula for the general term a_n of $\left\{\frac{3}{2}, -\frac{9}{4}, \frac{27}{8}, -\frac{81}{16}, \ldots\right\}$ assuming the pattern of the first few terms continues.

$$a_n = \frac{(-1)^{n+1}\, 3^n}{2^n}$$

$2^n/3^n$

Consider the sequence defined by $a_n = \left(\frac{2}{3}\right)^n$. ($n$ starts at 1)

a. Write the first five terms of the sequence.

$\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \frac{32}{243}$

b. Determine the limit of the sequence.

0

c. Let $b_n = \frac{a_{n+1}}{a_n}$. Write the first five terms of this sequence.

$b_1 = a_2/a_1 = \frac{4/9}{2/3} = 2/3 \quad b_2 = a_3/a_2 = \frac{8/27}{4/9} = 2/3$

d. Determine the limit of b_n. $2/3$ (common ratio)

Consider the sequence defined by $a_n = \left(\frac{-3}{2}\right)^n$. ($n$ starts at 1)

a. Write the first five terms of the sequence.

$-3/2,\ 9/4,\ -27/8,\ 81/16,\ -243/32$

b. Determine the limit of the sequence.

DNE

c. Let $b_n = \frac{a_{n+1}}{a_n}$. Write the first five terms of this sequence.

$b_1 = \frac{9/4}{-3/2} = -3/2 \qquad b_2 = \frac{-27/8}{9/4} = -3/2$

d. Determine the limit of b_n. $-3/2$

Are the following increasing, decreasing, or not monotonic?

$$a_n = \frac{3n+4}{2n+5}$$

$a_1 = \frac{7}{7}$ $a_2 = \frac{10}{9}$ $a_3 = \frac{13}{11}$

increasing

$$a_n = \frac{3+(-1)^n}{n}$$

not monotonic

$$a_n = \frac{\sqrt{n+1}}{5n+3}$$

$a_1 = \frac{\sqrt{2}}{8}$ $a_2 = \frac{\sqrt{3}}{13}$ $a_3 = \frac{2}{18}$

decreasing

Give an upper bound for the set of negative real numbers. 10, 1000

$(-\infty, 0)$

Give a lower bound for the set of negative real numbers.

None

Give the LUB and GLB for the set of negative real numbers.

no GLB

LUB = 0

Give the LUB and GLB of $\left\{\dfrac{(-1)^n}{n}\right\}_{n=3}^{\infty} = \dfrac{-1}{3}, \dfrac{1}{4}, \dfrac{-1}{5}, \dfrac{1}{6}, \ldots \to 0$

LUB = $1/4$

GLB = $-1/3$

Determine whether $\left\{\ln\left(\dfrac{2n-1}{3n+7}\right)\right\}_{n=1}^{\infty}$ is bounded. yes

$$\lim_{n\to\infty} \ln\left(\frac{2n-1}{3n+7}\right) = \ln 2/3$$

Sequences can be defined *recursively*: one or more terms are given explicitly; the remaining ones are then defined in terms of their predecessors. Give the first six terms of the sequence and then give the *n*th term.

$\underline{a_1 = 1}$; $a_{n+1} = ½\, a_n + 1$.

$a_2 = \frac{1}{2} a_1 + 1 = \frac{1}{2}(1) + 1 = \frac{3}{2}$

$a_3 = \frac{1}{2} a_2 + 1 = \frac{1}{2}\left(\frac{3}{2}\right) + 1 = \frac{7}{4}$

$a_4 = \frac{15}{8}$

$a_5 = \frac{31}{16}$

$a_6 = \frac{63}{32}$

$\frac{2^n - 1}{2^{n-1}}$

$\frac{2(2^{n-1}) - 1}{2^{n-1}}$

With sequences, we are concerned with limits of sequences as n approaches infinity.

A sequence that has a limit is said to be **convergent**.

A sequence that has no limit is said to be **divergent**.

Every convergent sequence is bounded and every unbounded sequence is divergent.

"The sequence converges"
means
"The sequence has a limit".

"The sequence diverges"
means
"The sequence does not have a limit".

Important Limits:

$\frac{1}{n^2}$ $\frac{1}{n}$ $\frac{1}{\sqrt{n}}$

A. For each $a > 0$, $\frac{1}{n^a} \to 0$ as $n \to \infty$

B. For each real x, $\frac{x^n}{n!} \to 0$ as $n \to \infty$

$\frac{100^n}{n!} \to 0$

C. If $|x| < 1$, then $x^n \to 0$, as $n \to \infty$

$(2/3)^n \to 0$ $(-6/7)^n \to 0$ $(3/2)^n \not\to 0$ diverges

D. $\frac{\ln n}{n} \to 0$ as $n \to \infty$

E. If $x > 0$, then $x^{\frac{1}{n}} \to 1$ as $n \to \infty$

$\frac{1}{n} \to 0$

F. $n^{\frac{1}{n}} \to 1 \quad as \;\; n \to \infty$

G. For each real x, $\left(1+\frac{x}{n}\right)^n \to e^x \quad as \;\; n \to \infty$

Examples:

Give the limit of $\left\{(-1)^n\right\}_{n=1}^{\infty}$ diverges

Give the limit of $\left\{\frac{2n-6}{3n^2+2}\right\}_{n=1}^{\infty}$ $\to 0$

Give the limit (if it exists) of $\left\{ \ln(n+1) - \ln(n) \right\}_{n=1}^{\infty}$

$$\left\{ \ln\left(\frac{n+1}{n} \right) \right\} \to \ln(1) = 0$$

$$\frac{n+1}{n} \to 1$$

Give the limit (if it exists) of $\left\{ \ln(2n+1) - \ln(n) \right\}_{n=1}^{\infty}$

$$= \left\{ \ln\left(\frac{2n+1}{n} \right) \right\} \to \ln 2$$

Give the limit (if it exists) of $\left\{ \dfrac{\ln(n+1)}{n} \right\}_{n=1}^{\infty} \to 0$

similar to $\dfrac{\ln n}{n} \to 0$

Popper18

1. Give the limit of the sequence: $\left\{ \dfrac{ln(n+2)}{\sqrt{2}+n} \right\}_{n=1}^{\infty}$

a. DNE
b. 1
c. 0
d. 3
e. e

Give the limit (if it exists) of $\left\{ \frac{3^n}{4^n} \right\}_{n=1}^{\infty}$ $= \left\{ \left(\frac{3}{4}\right)^n \right\} \to 0$

Give the limit (if it exists) of $\left\{ n^{\frac{1}{n+2}} \right\}_{n=1}^{\infty} \to 1$

behaves like

$n^{1/n}$

Note: $\left\{ \left(e^n \right)^{\frac{1}{n}} \right\}_{n=1}^{\infty} \to e$

$$\left\{ \left(stuff \right)^{\frac{1}{n}} \right\}_{n=1}^{\infty}$$

may not go to 1 as n approaches infinity if "*stuff*" overpowers the exponent. Be careful!

1^{∞}

Give the limit (if it exists) of $\left\{\left(1-\frac{1}{n}\right)^{n}\right\}_{n=1}^{\infty}$

-1

$$\lim_{n\to\infty} e^{\ln(1-1/n)^n} = \lim_{n\to\infty} e^{\boxed{[n\cdot\ln(1-1/n)]}} = e^{-1}$$

$$\lim_{n\to\infty} \frac{\ln(1-1/n)}{1/n}$$

as $n\to\infty$ $1/n \to 0$

$$\frac{\ln(1)}{0} = \frac{0}{0} \checkmark$$

$$= \lim_{n\to\infty} \frac{\frac{1/n^2}{1-1/n}}{-1/n^2}$$

$$= \lim_{n\to\infty} \frac{-1}{1-1/n} = -1$$

$$\lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n \to e^x$$

$$\lim_{n \to \infty} \left(1 - \frac{1}{n}\right)^n = \lim_{n \to \infty} \left(1 + \frac{-1}{n}\right)^n = e^{-1}$$

note $\displaystyle \lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^{an} = \lim_{n \to \infty} \left[\left(1 + \frac{x}{n}\right)^n\right]^a$

$$\downarrow$$

$$\left(e^x\right)^a$$

$$= e^{a \cdot x}$$

2. Give the limit of $\left\{\left(1-\dfrac{2}{n}\right)^n\right\}_{n=1}^{\infty}$

a. 1
b. e^2
c. e
d. e^{-2}
e. none of these

$$\left(1+\frac{\cdot}{n}\right)^n$$

$$\left(1+\frac{x}{n}\right)^n \to e^x$$

$$\star \quad \left(\frac{n}{1+n}\right)^n$$

$$= \left(\frac{1+n}{n}\right)^{-n} = \left(1/n+1\right)^{-n}$$

$$= \left[\left(1+1/n\right)^n\right]^{-1} \to e^{-1}$$

3. Give the limit of $\left\{ \dfrac{3n^3 - 2n + 1}{1000n^2 - n^3 + 3} \right\}_{n=1}^{\infty}$

a. -3
b. 3
c. 3/1000
d. DNE

4. Give the limit of $\left\{ \cos(n\pi) \right\}_{n=1}^{\infty}$

a. -1
b. 1
c. 0
d. DNE

5. Give the limit (if it exists) of the sequence $\left\{ 3^{\frac{2}{n}} \right\}_1^{\infty}$

a. 3
b. 1
c. 0
d. 9
e. DNE