Math 1432

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Office Hours:

Mondays 1-2pm,
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Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

Test 3 : Ch 8 & 9.1–9.5

Due Friday: 10 pts (HW)

Write a free response question over

- Choose something challenging to You
- Write up a solution (your problem is out of 10 pts)
  include point breakdown

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State whether the **sequence** converges as n → ∞.

C. Converges D. Diverges

1. $e^{\frac{-7}{n}}$.

$$\lim_{n\to\infty} e^{-7/n} = e^0 = 1 \quad \boxed{\text{converges to } 1}$$

2. $\frac{8\ln n}{n}$ $= 8\left(\frac{\ln n}{n}\right) \to 0$

3. $\frac{5^{375n}}{n!}$ $= \frac{(5^{375})^n}{n!} \to 0$

$\frac{x^n}{n!}$ conv. to 0

$\frac{n^n}{n!} = \frac{n \cdot n \cdot n \cdot}{n(n-1)(n-2)}$ div.

$n!/n^n$ conv to 0

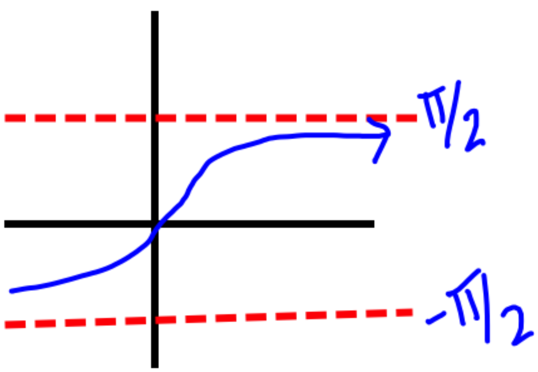
4. $\frac{8^{n+1}}{9^{n-1}} = \frac{8^n \cdot 8}{9^n \cdot 9^{-1}} = 72\boxed{\left(\frac{8}{9}\right)^n} \to 0$

$\to x^n$ if $\underline{|x| < 1}$ then conv. to 0

5. $\int_0^n e^{-8x}\,dx = -\frac{1}{8}e^{-8x}\Big|_0^n = -\frac{1}{8}e^{-8n} - -\frac{1}{8}e^0$

$$\lim_{n\to\infty}\left(-\frac{1}{8e^{8n}} + \frac{1}{8}\right) = \frac{1}{8}$$

6. $\int_{-n}^n \frac{8}{1+x^2}\,dx = 8\arctan(x)\Big|_{-n}^n$



$= 8\arctan(n) - 8\arctan(-n)$

$\to 8\left(\frac{\pi}{2}\right) - 8\left(-\frac{\pi}{2}\right) = 8\pi$

Sequences: Let $\{a_n\}$ be a sequence of real numbers.

Possibilities:

1) If $\lim_{n\to\infty} a_n = \infty$ then $\{a_n\}$ diverges to infinity.

2) If $\lim_{n\to\infty} a_n = -\infty$, then $\{a_n\}$ diverges to negative infinity.

3) If $\lim_{n\to\infty} a_n = c$, a finite real number, then $\{a_n\}$ converges to c.

4) If $\lim_{n\to\infty} a_n$ oscillates between two numbers, then $\{a_n\}$ diverges by oscillation.

If a sequence has a finite limit as n approaches infinity, we say that the sequence **converges**.
If a sequence does not have a finite limit as n approaches infinity, then it **diverges**.

9.3 Infinite **Series**

Series vs. Sequence

First and most important – a sequence is a LIST and a series is a SUM.

Sequence: $\{a_n\} = a_1, a_2, a_3, \ldots, a_n, \ldots$

Series: $\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_n$

Notation for series:

Σ Sigma – means summation

$\sum_{k=0}^{n} a_k = a_0 + a_2 + a_3 + \ldots + a_n$ $\quad \sum_{n=0}^{\infty} a_n = a_0 + a_1 + a_2 + \ldots\ldots$

Examples:

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots\cdots$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots$$

Properties:

$$\sum_{k=0}^{n} \alpha a_k = \alpha \sum_{k=0}^{n} a_k$$

$$\sum_{k=0}^{n} \left(a_k \pm b_k \right) = \sum_{k=0}^{n} a_k \pm \sum_{k=0}^{n} b_k$$

$$\sum_{k=0}^{m} a_k + \sum_{k=m+1}^{n} a_k = \sum_{k=0}^{n} a_k$$

$a \le b \le c$

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

$$= \int_a^c f(x)\,dx$$

The sum of a finite series is denoted by S_n where $S_n = \sum_{k=0}^{n} a_k$

For an *infinite* series, we are taking an infinite sequence $(a_0, a_1, a_2, \ldots.)$ and adding all the terms together. But since there are an infinite amount of terms, doing this (literally) would be impossible so we will begin by looking at the *partial sums:*

$$S_0 = \sum_{k=0}^{0} a_k = a_0$$

$$S_1 = \sum_{k=0}^{1} a_k = a_0 + a_1$$

$$S_2 = \sum_{k=0}^{2} a_k = a_0 + a_1 + a_2$$

$$\vdots$$

$$S_n = \sum_{k=0}^{n} a_k = a_0 + a_1 + a_2 + \ldots + a_n$$

$$\vdots$$

If we determine that the sequence of these partial sums has a limit, then that limit would be the sum of the infinite series $\sum_{k=0}^{\infty} a_k$. In other words, $\sum_{k=0}^{\infty} a_k = \lim_{n\to\infty}\{S_n\}$

What is the sequence of partial sums for each?

$$\sum_{k=0}^{\infty} 1 = \underbrace{\{1, 2, 3, 4, \dots\}}_{\text{seq. of partial sums}}$$

seq. of partial sums

& it diverges

$S_0 = 1$

$S_1 = \sum_{k=0}^{1} 1 = 1 + 1 = 2$

$S_2 = 1 + 1 + 1 = 3$

$S_3 = 1 + 1 + 1 + 1 = 4$

$$\sum_{k=0}^{\infty} r =$$

const.

$S_0 = r$

$S_1 = r + r = 2r$

$S_2 = r + r + r = 3r$

$S_3 = r + r + r + r = 4r$

$$\sum_{k=0}^{\infty} \frac{1}{2^k} = 2$$

$S_0 = 1$

$S_1 = 1 + 1/2 = 3/2$

$S_2 = 1 + 1/2 + 1/4 = 7/4$

$S_3 = 1 + 1/2 + 1/4 + 1/8 = 15/8$

$S_4 = 31/16$

$$S_n = \frac{2(2^n) - 1}{2^n} = 2 - \frac{1}{2^n} \to 2$$

If the sequence of partial sums converges to a finite limit L, we write $\sum_{k=0}^{\infty} a_k = L$ and say that the series $\sum_{k=0}^{\infty} a_k$ converges to L. We call L the sum of the series. If the sequence of partial sums diverges, we say that the series $\sum_{k=0}^{\infty} a_k$ diverges.

THEOREM: The *kth term* of a convergent series tends to 0; namely,

$$\text{if } \sum_{k=0}^{\infty} a_k \text{ converges, then } a_k \to 0 \text{ as } k \to \infty$$

THEOREM: BASIC DIVERGENCE TEST (BDT)

$$\text{If } a_k \not\to 0 \text{ as } k \to \infty, \text{ then } \sum_{k=0}^{\infty} a_k \text{ diverges.}$$

terms don't go to 0

****The Basic Divergence Test only proves divergence.****

More General Properties:

- If $\sum_{k=0}^{\infty} a_k$ converges and $\sum_{k=0}^{\infty} b_k$ converges,

  then $\sum_{k=0}^{\infty} (a_k + b_k)$ converges.

- If $\sum_{k=0}^{\infty} a_k$ converges, then $\sum_{k=0}^{\infty} \alpha a_k$ converges.

Ex. 1 Does $\sum_{n=1}^{\infty}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$ converge or diverge?

$n=1 \qquad n=2 \qquad n=3 \qquad n=4$

$$=\left(1-\cancel{\frac{1}{3}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{5}}\right)+\left(\cancel{\frac{1}{5}}-\cancel{\frac{1}{7}}\right)+\left(\cancel{\frac{1}{7}}-\frac{1}{9}\right)+\cdots$$

$$=1$$

Telescoping Series:

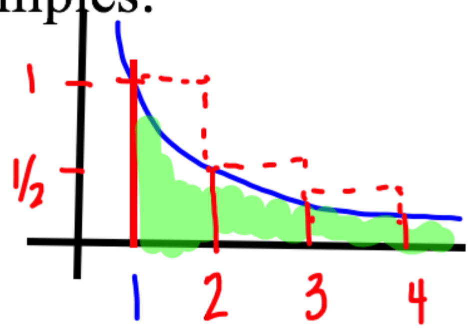
A series such as $\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\ldots$ is called a telescoping series because it collapses to one term or a few terms. If a series collapses to a finite sum, then it converges.

Ex. 2. Does the series 1 – 1 + 1 – 1 + 1 … converge or diverge?
How would we write the series using sigma notation?

$$\sum_{n=0}^{\infty}(-1)^n$$

Important examples:

$$\int_1^\infty \frac{dx}{x}$$



$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \dots$$

diverges

$$\lim_{b\to\infty} \int_1^b \frac{dx}{x}$$

$$= \lim_{b\to\infty} \ln|x| \Big|_1^b$$

$$= \lim_{b\to\infty} \ln|b| \to \infty$$

diverges

$$\int_1^\infty \frac{dx}{x^2}$$

$$\sum_{n=1}^\infty \frac{1}{n^2} = \text{conv.}$$

$$\lim_{b \to \infty} \int_1^b \frac{dx}{x^2}$$

$$= \lim_{b \to \infty} -\frac{1}{x} \Big|_1^b$$

$$= \lim_{b \to \infty} \left(-\frac{1}{b} + 1\right) \to 1$$

Series (SUM)

$$\sum_{k=1}^{4} \frac{1}{k^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} = \_\_\_\_\_\_$$

$$\sum_{n=1}^{6} \frac{(-1)^n}{5n+1} = \frac{-1}{6} + \frac{1}{11} - \frac{1}{16} + \frac{1}{21} - \frac{1}{26} + \frac{1}{31}$$

$$\sum_{n=0}^{5} \left(\frac{1}{2}\right)^n = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$$

$$\sum_{n=1}^{\infty} \left(\frac{-1}{2}\right)^n = \frac{-1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} + \dots\dots$$

$$\sum_{n=1}^{\infty} (-1)^n = -1 + 1 - 1 + 1 + \dots$$

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if $a_n \not\to 0$ then $\sum_{n=1}^{\infty} a_n$ diverges

7. Which of the following diverge by the BDT?

a. $\sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^n$ $\qquad \lim_{n\to\infty}(-1/2)^n = 0$

b. $\sum_{n=1}^{\infty}\frac{1}{n}$ $\qquad \lim_{n\to\infty}\frac{1}{n} = 0$

c. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^n$ $\qquad \lim_{n\to\infty}(1+1/n)^n = e$

d. $\sum_{n=1}^{\infty}\left(\frac{6}{11}\right)^n$ $\qquad \lim_{n\to\infty}\left(\frac{6}{11}\right)^n = 0$

$\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x$