# Math 1432

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Office Hours:

Mondays 1-2pm,
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(also available by appointment)

## Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

* Write one question  Due Friday
  Write up solution
  with point values - total 10 pts
  8.1 - 9.4

Does the series $\sum_{n=1}^{\infty}\left(\frac{1}{n+2}-\frac{1}{n+4}\right)$ converge or diverge?

Telescoping

$n=1$ $\quad$ $n=2$ $\quad$ $n=3$ $\quad$ $n=4$

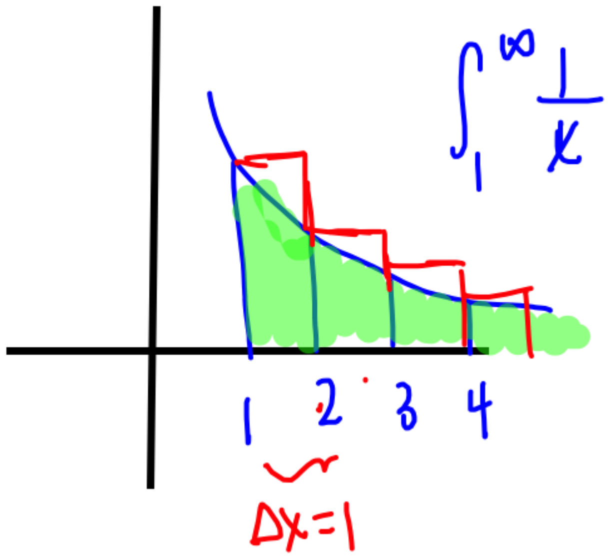
$$\left(\boxed{\frac{1}{3}}-\cancel{\frac{1}{5}}\right)+\left(\boxed{\frac{1}{4}}-\cancel{\frac{1}{6}}\right)+\left(\cancel{\frac{1}{5}}-\cancel{\frac{1}{7}}\right)+\left(\cancel{\frac{1}{6}}-\frac{1}{8}\right)$$

$n=5$

$$+\left(\cancel{\frac{1}{7}}-\frac{1}{9}\right)+\ldots$$

$$\text{Sum}=\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$$

★ Does the series $\sum_{n=1}^{\infty} \frac{1}{n}$ converge or diverge?




$\int_1^{\infty} \frac{1}{x}\,dx = \quad = \lim_{b\to\infty} \ln|b| = \infty$

$L = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n}$

★ Does the sequence $\left\{\frac{1}{n}\right\}_1^{\infty}$ converge or diverge?

$\lim_{n\to\infty} \frac{1}{n} = \underline{0}$

Important reminders:

- A sequence is a LIST { }
  - A sequence converges if it has a limit as $n -> \infty$

- A series is a SUM $\Sigma$
  - Converges if the sequence of partial sums $\{S_0, S_1, S_2, \ldots\}$ converges
  - If terms do not approach 0 then it diverges

## Geometric Series Test

A geometric series is in the form $\sum_{n=0}^{\infty} \underline{a_1} r^n$, $a_1 \neq 0$.

ex: $\sum_{n=1}^{\infty} \frac{1}{2^n}$

$\left(\frac{1}{2}\right)^n$

$$\sum_{n=0}^{\infty} a_1 r^n = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \ldots + a_1 r^n + \ldots$$

The geometric series **diverges** if $|r| \geq 1$.

The geometric series **converges** if $|r| < 1$.

1st term

If $|r| < 1$, then the series **converges** to the sum $S = \dfrac{a_1}{1-r}$.

Infinite

Finite

The sum of the first n terms of a geometric series is

$$S_n = \frac{a_1\left(1-r^n\right)}{1-r}.$$

Examples: Determine whether the following infinite series converge or diverge. If they converge, what is the sum of the series?

1) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{2^{n}} = \sum_{n=1}^{\infty} 3 \cdot \left(\frac{1}{2}\right)^{n}$ $\quad r = 1/2 \quad |1/2| < 1 \Rightarrow$ conv.

$$S = \frac{3/2}{1 - 1/2} = \frac{3/2}{1/2} = \boxed{3}$$

2) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^{n}$ $\quad r = 3/2 \quad |3/2| \geq 1 \Rightarrow$ diverges

3) 0.080808....

$.08 + .0008$

$\rightarrow 8/100 + 8/10000 + 8/1000000 + \ldots$

$8/10^2 + 8/10^4 + 8/10^6 + \ldots$

$\sum_{n=1}^{\infty} 8\left(1/100\right)^n$ $\quad r = 1/100 \quad |1/100| < 1$

Conv.

$$S = \frac{8/100}{1 - 1/100} = \frac{8/100}{99/100} = \boxed{8/99}$$

4) $\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{n-1}$

$$\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^n \left(1/2\right)^{-1} = \sum_{n=1}^{\infty} 6\left(1/2\right)^n$$

$r = 1/2 \quad < 1$

Conv.

$$S = \frac{6/2}{1 - 1/2} = \frac{3}{1/2} = \boxed{6}$$

5) $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots+\left(\frac{-1}{2}\right)^{n-1}+\ldots = \sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^{n-1} = \sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^{n}\left(-\frac{1}{2}\right)^{-1}$

$= \sum_{n=1}^{\infty}(-2)\left(-\frac{1}{2}\right)^{n}$

$r = -\frac{1}{2}$

$\left|-\frac{1}{2}\right| < 1$

Conv.

$S = \frac{1}{1 - -\frac{1}{2}} = \boxed{\frac{2}{3}}$

6) $\frac{\pi}{2}+\frac{\pi^2}{4}+\frac{\pi^3}{8}+\ldots = \sum_{n=1}^{\infty}\left(\frac{\pi}{2}\right)^{n}$

$r = \frac{\pi}{2}$

$\left|\frac{\pi}{2}\right| \geq 1$ diverges

We can typically only determine whether an infinite series (sum) converges or diverges. Quite often, we cannot find the actual sum.

**Basic Divergence Test**

If $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.

Note: This does **NOT** say that if $\lim_{n\to\infty} a_n = 0$, the series converges.

****The Basic Divergence Test only proves **divergence**.****

If $\lim_{n\to\infty} a_n = 0$, then the test doesn't tell us anything, and we need to use another test.

Examples:

1) $\sum_{n=1}^{\infty} \frac{2n+3}{3n-5}$ does $\frac{2n+3}{3n-5} \to 0$ ? No $\left(2/3\right)$

diverges by BDT

2) $\sum_{n=1}^{\infty} \frac{n}{2n+1}$ does $\frac{n}{2n+1} \to 0$ ? No $\left(1/2\right)$

div. by BDT

3) $\sum_{n=1}^{\infty} \frac{3^n - 2}{3^n} = \sum_{n=1}^{\infty} \left( 1 - \frac{2}{3^n} \right)$ div. by BDT

$\frac{2}{3^n} \to 0$

4) $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$ div. by BDT

Note: (2n)! is not the same as 2n!.

$0! = 1$

$1! = 1$

$2! = 2 \cdot 1 = 2$

$3! = 3 \cdot 2 \cdot 1 = 6$

$\vdots$

$5! \quad (n=5) \qquad 2 \cdot 5! \neq 10!$

$\vdots$

$n! = n \cdot (n-1) \cdot (n-2) \cdots 1$

$\vdots$

$(2n)! = (2n)(2n-1)(2n-2) \cdots n \cdots 1$

**Popper 20**

**1.** $$\sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^{n}$$

a. converges

b. diverges

**2.** $$\sum_{n=1}^{\infty}\frac{1}{n}$$

a. converges

b. diverges

**3.** $\sum_{n=1}^{\infty}\left(\frac{6}{7}\right)^{n}$

a. converges

b. diverges

**4.** $\sum_{n=1}^{\infty}(-1)^{n}$

a. converges

b. diverges

**5.** $$\sum_{n=1}^{\infty}\left(\frac{8.0001}{8}\right)^{n}$$

a. converges

b. diverges

**6.** Find $\lim\limits_{x\to\infty}\dfrac{x^{25}}{3^{x}}$.

a. 0

b. $\infty$

c. 1/3

d. –2

e. None of these

**7.** Give the value of $\sum_{n=1}^{\infty} \frac{1}{3^n}$ $= \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$ $r = \frac{1}{3}$

a. 1/2

b. 1/3

c. 364/729

d. 728/729

Some more examples:

Find the sequence of partial sums for $\sum_{n=1}^{\infty} \frac{2n}{n^2+1}$

$S_1 = \frac{2}{2} = 1$

$S_2 = 1 + \frac{4}{5} = 9/5$

$S_3 = 1 + 4/5 + 6/10 = 12/5$

$S_4 = 1 + 4/5 + 3/5 + 8/17$

$= 12/5 + 8/17$

$= \frac{204+40}{85} = \frac{244}{85}$

$S_5 = 3.255$

Looking at the sequence of partial sums for $\sum_{n=1}^{\infty} \frac{2n}{n^2+1}$, we can say that this series

diverges

$\{1, 9/5, 12/5, 244/85, 3.255,$ gets bigger... $\}$

notice $\lim_{n\to\infty} \frac{2n}{n^2+1} = 0$

looks like $\frac{2n}{n^2} = \frac{2}{n} = 2\left(1/n\right)$

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

$$\left(1+\frac{x}{n}\right)^n \to e^x$$

**Popper 20**

**8.** Determine whether the following *sequence* converges or diverges. If it converges, find its limit.

$$a_n = \left(1 + \frac{2}{5n}\right)^n = \left(1 + \frac{2/5}{n}\right)^n$$

a. Diverges

b. Converges, L = e

c. Converges, $L = e^{2/5}$

d. Converges, $L = e^2$

**9.** Determine whether the following *series* converges or diverges.

$$\sum_{n=1}^{\infty} \left(1 + \frac{2}{5n}\right)^n$$

a. Converges

b. Diverges