Math 1432

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Office Hours:

Mondays 1-2pm,
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(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

7) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^3+2n+1}}$

$$\sqrt{\frac{n}{n^3+2n+1}}$$

$$\sqrt{\frac{n}{n^3}} = \sqrt{\frac{1}{n^2}} = \frac{1}{n}$$

compare to $\sum \frac{1}{n}$ $\boxed{\text{Divergent}}$

$$\lim_{n\to\infty} \frac{\sqrt{\frac{n}{n^3+2n+1}}}{\frac{1}{n}}$$

$$= \lim_{n\to\infty} \sqrt{\frac{n}{n^3+2n+1}} \cdot \sqrt{\frac{n^2}{1}} = \lim_{n\to\infty} \sqrt{\frac{n^3}{n^3+2n+1}} = \sqrt{1}$$

$$= \underline{\underline{1}}$$

13) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

$$\lim_{n\to\infty} \frac{\sin(1/n)}{(1/n)} = 1$$

Compare to $\underline{\underline{\sum \frac{1}{n}}}$

$\frac{\sin(x)}{x} \to 1$ as $x \to 0$

Divrges

$y = 1/n$

as $n \to \infty$, $y \to 0$

$$\lim_{y\to 0} \frac{\sin(y)}{y} = \underline{\underline{1}}$$

Σ

Review of test for series convergence:

$\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n \neq 0$ BDT

$\sum_{n=1}^{\infty} \frac{1}{n}$ Harmonic Series – diverges

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ P-Series – converges if p>1, diverges otherwise

1st term

$\sum_{n=0}^{\infty} (r)^n$ Geometric – converges if $|r| < 1$ to $\frac{a_1}{1-r}$ and diverges if $|r| \geq 1$

Basic Comparison Test: $\sum_{n=1}^{\infty} a_n, a_n > 0$

1. If $a_n \geq b_n$ and $\sum_{n=1}^{\infty} b_n, b_n > 0$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges

2. If $a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n, b_n > 0$ converges, then $\sum_{n=1}^{\infty} a_n$ converges

Limit Comparison Test: $\sum_{n=1}^{\infty} a_n, a_n \geq 0$

If you know $\sum_{n=1}^{\infty} b_n, b_n \geq 0$

1. If $\sum_{n=1}^{\infty} b_n$ converges and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ (L is any finite number),then $\sum_{n=1}^{\infty} a_n$ converges

2. If $\sum_{n=1}^{\infty} b_n$ diverges and $\lim_{n\to\infty} \frac{a_n}{b_n} > 0$, then $\sum_{n=1}^{\infty} a_n$ diverges

The Integral Test:

If f is positive, continuous and decreasing for $x \geq 1$ and $a_n = f(n)$, then $\sum_{n=1}^{\infty} a_n$ *and* $\int_1^{\infty} f(x)dx$ either both converge or both diverge.

The Root Test:

Let $\sum a_k$ be a series with nonnegative terms. Suppose $(a_k)^{1/k} \to \rho$, then

1. $\sum a_k$ converges if $\rho < 1$.
2. $\sum a_k$ diverges if $\rho > 1$.
3. The test is inconclusive if $\rho = 1$.

The Ratio Test:

Let $\sum a_k$ be a series with positive terms. Suppose $\frac{a_{k+1}}{a_k} \to \lambda$, then

1. $\sum a_k$ converges if $\lambda < 1$.
2. $\sum a_k$ diverges if $\lambda > 1$.
3. The test is inconclusive if $\lambda = 1$.

More Examples:

1. $\sum \frac{n}{3n+1}$ $\qquad \frac{n}{3n+1} \to 0?$ NO $\left(\frac{1}{3}\right)$

Diverges by BDT

2. $\sum 2\left(\frac{4}{5}\right)^n = 2 \underbrace{\sum \left(\frac{4}{5}\right)^n}_{\text{geometric}}$

geometric $r = 4/5$ $\quad |4/5| < 1 \Rightarrow$ Converges

3. $\sum \frac{\sqrt{n}}{n} = \sum \frac{1}{\sqrt{n}}$ p series w/ $p = 1/2 \Rightarrow$ diverge

4. $\sum \frac{1}{n^{1.1}}$ p series $p=1.1 \Rightarrow$ converges

5. $\sum \frac{5n}{3n^2-6n+2}$ comp to $\sum \frac{1}{n}$ diverges

for BCT, must show $\frac{5n}{3n^2-6n+2} > \frac{1}{n}$

use LCT: $\lim_{n\to\infty} \frac{5n}{3n^2-6n+2} \cdot \frac{n}{1} = \lim_{n\to\infty} \frac{5n^2}{3n^2-6n+2} = \frac{5}{3}$

$$\lim_{K\to\infty} K^{1/K} = 1$$

6. $\sum \frac{k \cdot 2^k}{3^k}$

Root: $\lim_{K\to\infty} \left(\frac{K\,2^K}{3^K}\right)^{1/K} = \lim_{K\to\infty} \frac{K^{1/K} \cdot 2}{3} = \frac{2}{3} < 1$

<u>converges</u>

Ratio: $\lim_{K\to\infty} \frac{(K+1)\,2^{K+1}}{3^{K+1}} \cdot \frac{3^K}{K\,2^K}$

$$= \lim_{K\to\infty} \underbrace{\frac{(K+1)}{K}}_{\to 1} \cdot \frac{\cancel{2^K} \cdot 2 \cdot \cancel{3^K}}{\cancel{3^K} \cdot 3 \cdot \cancel{2^K}} = \frac{2}{3} < 1$$

7. $\sum \frac{3^n}{(n+1)!}$

Ratio: $\lim_{n\to\infty} \frac{3^{n+1}}{(n+2)!} \cdot \frac{(n+1)!}{3^n}$

$$= \lim_{n\to\infty} \frac{3^n \cdot 3 \cdot (n+1)!}{(n+2)(n+1)!\, 3^n} = \lim_{n\to\infty} \frac{3}{n+2} = 0 < 1$$

converges

$\sum \frac{3^n}{(2n)!}$

$(2(n+1))!$

Ratio:

$$\lim_{n\to\infty} \frac{3^{n+1}}{(2n+2)!} \cdot \frac{(2n)!}{3^n} = \lim_{n\to\infty} \frac{\cancel{3^n} \cdot 3 \cdot \cancel{(2n)!}}{(2n+2)(2n+1)\cancel{(2n)!}\, \cancel{3^n}}$$

$$\lim_{n\to\infty} \frac{3}{(2n+2)(2n+1)} \to 0 < 1 \text{ conv.}$$

8. $\sum \frac{(n+1)!}{(n+4)!} = \sum \frac{(n+1)!}{(n+4)(n+3)(n+2)(n+1)!}$

$= \sum \frac{1}{(n+4)(n+3)(n+2)}$ comp to $\sum \frac{1}{n^3}$

conv pseries

$\frac{1}{(n+4)(n+3)(n+2)} < \frac{1}{n^3}$

converges by BCT

1 c. $\int \frac{x^2+5x+2}{(x+1)(x^2+1)}\,dx$

$$\frac{A}{x+1} + \frac{Bx+C}{x^2+1} = \frac{x^2+5x+2}{(x+1)(x^2+1)}$$

$$A(x^2+1) + (Bx+C)(x+1) = x^2+5x+2$$

$x=-1$: $2A = 1-5+2 = -2$ $\boxed{A=-1}$

$x=0$: $A + C = 2$

$-1 + C = 2$

$\boxed{C=3}$

$x=1$: $2A + 2B + 2C = 1+5+2$

$-2 + 2B + 6 = 8$

$2B = 4$

$\boxed{B=2}$

$$\int \left(\frac{-1}{x+1} + \frac{2x}{x^2+1} + \frac{3}{x^2+1} \right) dx$$

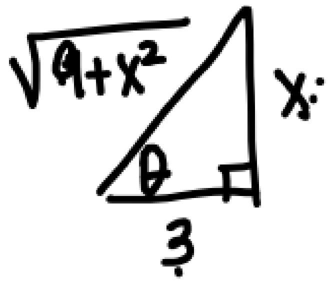
$u = x^2+1$

$du = 2x\,dx$

$\int \frac{1}{u}\,du$

$$-\ln|x+1| + \ln(x^2+1) + 3\arctan(x) + C$$

e. $\int \frac{2}{x\sqrt{9+x^2}}\,dx$



$x = 3\tan\theta \leftarrow$

$dx = 3\sec^2\theta \, d\theta$

$\frac{\sqrt{9+x^2}}{3}$ $\frac{H}{a}$ $= \sec\theta$

$\sqrt{9+x^2} = 3\sec\theta$

$$\sqrt{9+9\tan^2\theta} = \sqrt{9(1+\tan^2\theta)} = \sqrt{9\sec^2\theta} = 3\sec\theta$$

i. $\displaystyle\int_0^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x)\Big|_0^{\sqrt{3}/2}$

1. $\int \cot^3 x\,dx = \int \cot^2(x)\cot(x)\,dx$

$$\int \left(\csc^2(x)-1\right)\cot(x)\,dx$$

$$\int \csc^2(x)\,dx = -\cot(x) + C$$

$$\int \sec^2(x)\,dx = \tan(x) + C$$

$$\int \csc(x)\cot(x)\,dx = -\csc(x) + C$$

$$\vdots$$

o. $\int \frac{5}{36+(x-1)^2}\,dx$

$\frac{1}{a}\arctan u/a + C$

$a=6$

$u=(x-1)$

q. $\int 2x\sec\left(4x^2\right)dx$

u

$\int \sec u \, du = \ln\left|\sec u + \tan u\right| + C$

2. Write an expression for the nth term of the sequence:

a. 1, 4, 7, 10, ...

b. 2, -1, $\frac{1}{2}$, $-\frac{1}{4}$, $\frac{1}{8}$, ...

1 2 3 4 5

$\cancel{\sum_{n=1}^{\infty} \frac{1}{2^n}}$

$\cancel{\sum_{n=1}^{\infty}} \frac{(-1)^{n+1}}{2^{n-2}}$

b. $\sum_{k=2}^{\infty}\left(\frac{2}{3}\right)^{k}$

$r = 2/3$ $\quad a_1 = ?$

Popper 23

1. Use the Root test to determine if the following are convergent or divergent (or if test is inconclusive).

$$\sum \frac{k^6}{e^{3k}}$$

$$\lim_{K\to\infty} \frac{K^{6/K}}{e^3} = \frac{1}{e^3} < 1$$

a. Converges b. Diverges c. Inconclusive

2-12 State whether each series $\boxed{\text{converges (C)}}$ or $\boxed{\text{diverges (D):}}$

2. $\sum_{n=0}^{\infty}\left(\frac{1}{3^n}-\frac{5}{6^n}\right) = \sum\left(\frac{1}{3}\right)^n - 5\sum\left(\frac{1}{6}\right)^n$

3. $\sum_{n=1}^{\infty}\frac{n-1}{n!}$

$$\lim_{n\to\infty} \frac{n}{(n+1)!}\cdot\frac{n!}{(n-1)} = \lim_{n\to\infty} \frac{n\cdot \cancel{n!}}{(n+1)\cancel{n!}\,(n-1)}$$

$$= 0 < 1$$

4. $\sum_{n=1}^{\infty} \frac{n+3}{n}$

5. $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$

6. $\sum_{n=1}^{\infty} \frac{1}{n^2+4n+3}$

7. $\sum_{n=1}^{\infty} \frac{1}{n^5}$

8. $\sum_{n=1}^{\infty} \frac{1}{n}$

9. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{5^{k-1}}$ $\quad = \sum \frac{1}{5^k \cdot 5^{-1}} = \sum \frac{5}{5^k} = 5 \sum \left(\frac{1}{5}\right)^k$

10. $\displaystyle\sum_{n=2}^{\infty} \frac{3n^2 - 1}{10n + 5n^2}$

11. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$ $\quad$ compare to $\sum \frac{1}{n}$

12. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$

13. Choose the series that converges:

a) $\sum_{n=1}^{\infty} \frac{n}{2n+3}$

b) $\sum_{n=0}^{\infty} 5\left(\frac{3}{2}\right)^{n}$

c) $\sum_{n=1}^{\infty} \frac{2^{n+3}}{5^{n+1}}$

d) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$

EMAIL ME ANY TEST REVIEW QUESTIONS BY 5PM THURSDAY!