Math 1432

Bekki George
bekki@math.uh.edu
639 PGH

Office Hours:

Mondays 1-2pm,
Fridays noon-1pm
(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

TEST 3 REVIEW!!

10 questions 5 m/c 5 f/r

6pts 14pts

Integration
(IBP, trigPowers, trigsub, PFD)

5. Determine the values of n which guarantee a theoretical error less than ε if the integral is estimated by the trapezoidal rule and then by Simpson's rule if $\varepsilon = 0.01$.

a. $\int_1^3 \left[\frac{1}{4}x^2 + 3x - 2\right] dx$

→ b. $\int_1^3 \cos(5x)dx$

$a=1,\ b=3$

$f(x) = \cos(5x)$

$f'(x) = -5\sin(5x)$

$f''(x) = -25\cos(5x) \Leftarrow$

$f'''(x) = 125\sin(5x)$

$f^{4}(x) = 625\cos(5x) \Leftarrow$

Trap $E_n^T \le \frac{(3-1)^3}{12n^2} M < .01$

$\frac{2^3}{12n^2}(25) \le \frac{1}{100}$

$\frac{8 \cdot 2500}{12} \le n^2$

Simp: $E_n^S \le \frac{(3-1)^5}{180n^4} M < .01$

$\frac{2^5}{180n^4}(625) < \frac{1}{100}$

Sequence - list

converges if a_n has a limit as $n \to \infty$

* review important limits *

diverges if limit DNE $(\to \infty)$

NO Σ

$\{a_n\}$

Series – sum $\sum a_n = a_1 + a_2 + a_3 + \dots$

BDT: if $a_n \not\to 0$ then $\sum a_n$ diverges

Geometric

$\sum (r)^n$

conv. if $|r| < 1$

to $\frac{a_1}{1-r}$ ← 1st term

Harmonic

$\sum \frac{1}{n}$

diverges

P-Series

$\sum \frac{1}{n^p}$

if $p > 1$ conv.

if $p \leq 1$ div.

Does it "look like" ↑

Basic Comparison or Limit Comparison

Root test → $\lim_{n\to\infty} (|a_n|)^{1/n}$ if < 1 conv., > 1 div., $= 1$??

Ratio test → $\lim_{n\to\infty} a_{n+1}/a_n$ " " "

Integral test → $\int_1^\infty a_x \, dx$ if conv. to pos#, $\sum a_n$ conv.

8.

g. $\sum\left(1+\frac{1}{n}\right)^{n}$ $\rightarrow$ diverges by the BDT

$\left(1+\frac{1}{n}\right)^{n} \rightarrow e^{1} \neq 0$

h. $\sum\frac{n^{3}}{3^{n}}$ Root test: $\lim_{n\to\infty}\frac{n^{3/n}}{3} = \frac{1}{3} < 1 \Rightarrow$ conv.

i. $\sum\frac{1}{\sqrt[4]{n^{3}}} = \sum\frac{1}{n^{3/4}}$ P series, $p = 3/4 \Rightarrow$ div.

2 4 8

3. Determine if the following sequences are monotonic. Also indicate if the sequence is bounded and if it is give the least upper bound and/or greatest lower bound.

   a. $a_n = \dfrac{2n}{1+n}$

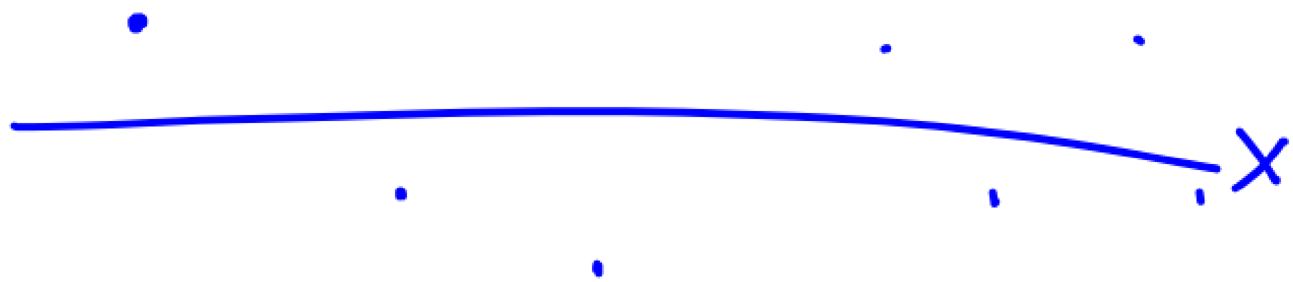
   b. $a_n = \dfrac{\cos n}{n}$ ← not monot.

LUB GLB

$$\boxed{\frac{\cos(1)}{1}}, \frac{\cos(2)}{2}, \boxed{\frac{\cos(3)}{3}}, \frac{\cos(4)}{4}, \dots \to 0$$

$\pi \approx 3.14$

$\pi/2 \approx 1.5.$



Question 4

Find the sum of the series: $\displaystyle\sum_{\substack{k=4}}^{\infty} \frac{2}{\underset{K(K-1)}{k^2-k}}$ $= \displaystyle\sum_{K=4}^{\infty} \frac{A}{K} + \frac{B}{K-1}$

$A(K-1) + BK = 2$

$K=1 \qquad B=2$

$K=0 \qquad A=-2$

$= \displaystyle\sum_{K=4}^{\infty} \left(\frac{2}{K-1} - \frac{2}{K}\right)$

$n=4 \qquad n=5 \qquad n=6 \qquad n=7$

$= \left(\boxed{\frac{2}{3}} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{2}{5}}\right) + \left(\cancel{\frac{2}{5}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \frac{2}{7}\right) + \dots$

Geom $\boxed{\dfrac{\text{1st term}}{1-r}}$

Question 6

Find the sum of the series: $\sum_{k=3}^{\infty} \frac{3}{4^k}$ $= \sum_{K=3}^{\infty} 3\left(\frac{1}{4}\right)^K$ $r = 1/4$

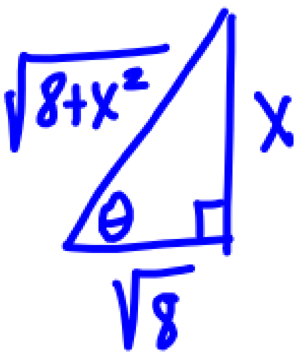
first term "a_1" $= 3\left(1/4\right)^3 = \frac{3}{64}$

$$\text{Sum} = \frac{3/64}{1-1/4} = \frac{3/64}{3/4} = \frac{3}{64}\cdot\frac{4}{3}$$

$$= \frac{1}{16}$$

$\frac{K}{4^K}$ not geom

$$\int \frac{5}{x^2\sqrt{8+x^2}}\,dx \rightarrow \int \frac{5}{8\tan^2\theta \cdot \cancel{\sqrt{8}}\,\cancel{\sec\theta}} \cdot \cancel{\sqrt{8}}\,\sec^{\cancel{2}}\theta\, d\theta$$



$x = \sqrt{8}\tan\theta$

$dx = \sqrt{8}\sec^2\theta\, d\theta$

$\sqrt{8+x^2} = \sqrt{8}\sec\theta$

$\dfrac{\sqrt{8+x^2}}{\sqrt{8}}$ $\dfrac{H}{A}$

$$= \frac{5}{8}\int \frac{\sec\theta}{\tan^2\theta}\, d\theta$$

$$= \frac{5}{8}\int \frac{1}{\cos\theta} \cdot \frac{\cos^2\theta}{\sin^2\theta}\, d\theta$$

$$= \frac{5}{8}\int \frac{\cos\theta}{\sin^2\theta}\, d\theta \qquad \int \frac{1}{u^2}\,du$$

$$= \boxed{\frac{-5}{8\sin\theta} + C}$$

$$= -\frac{5}{8} \cdot \csc\theta + C$$

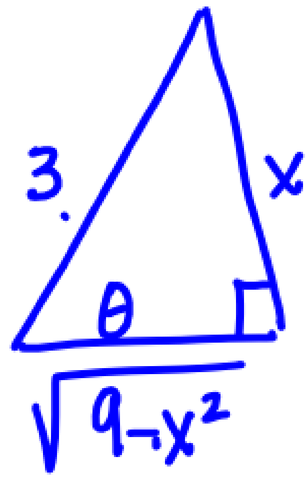
$$= -\frac{5}{8}\left(\frac{\sqrt{8+x^2}}{x}\right) + C$$

$$\frac{1}{2}\theta - \frac{1}{2}\cos\theta\sin\theta + C$$

$$x = 3\sin\theta$$

$$\rightarrow \frac{x}{3} = \sin\theta$$

$$\theta = \sin^{-1}(x/3)$$



$$\frac{1}{2}\sin^{-1}(x/3) - 1/2\left(\frac{\sqrt{9-x^2}}{3}\right)\left(\frac{x}{3}\right) + C$$

$$\int \underset{A}{\underline{x}}\ \underset{L}{\underline{\ln x}}\ dx$$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{x^2}{2}$

$uv - \int v\,du$

$$\frac{x^2}{2}\ln(x) - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx$$

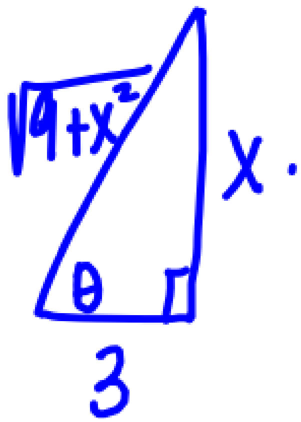
$$\frac{x^2}{2}\ln x - \int \frac{x}{2}\,dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$$

ILATE

popper 24

1-10 = "A"

1e $\displaystyle\int \frac{2}{x\sqrt{9+x^2}}\,dx \rightarrow \int \frac{2}{3\tan\theta \cdot 3\sec\theta}\cdot 3\sec^2\theta\, d\theta$



$x = 3\tan\theta$

$dx = 3\sec^2\theta\, d\theta$

$\sqrt{9+x^2} = 3\sec\theta$

$$\frac{2}{3}\int \frac{\sec\theta}{\tan\theta}\,d\theta$$

$$\frac{2}{3}\int \frac{1}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}\,d\theta$$

$$\frac{2}{3}\int \csc\theta\, d\theta$$

$$\frac{2}{3}\ln\left|\underset{\frac{H}{O}}{\csc\theta} - \underset{\frac{A}{O}}{\cot\theta}\right| + C$$

$\dfrac{\sqrt{9+x^2}}{3}$ $\dfrac{H}{A}$

$$\frac{2}{3}\ln\left|\frac{\sqrt{9+x^2}}{x} - \frac{3}{x}\right| + C$$

n. $\int \underset{A}{2x} \underset{T}{\sin(3x)}\, dx$

$u = 2x \qquad dv = \sin(3x)\,dx$

$du = 2dx \qquad v = -\frac{1}{3}\cos(3x)$

$uv - \int v\,du$

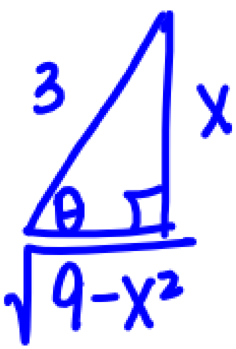
$$-\frac{2}{3}x\cos(3x) - \int -\frac{1}{3}\cos(3x)\cdot 2\cdot dx$$

$$-\frac{2}{3}x\cos(3x) + \frac{2}{3}\int \cos(3x)\,dx$$

$$-\frac{2}{3}x\cos(3x) + \frac{2}{9}\sin(3x) + C$$

d. $\displaystyle\int \frac{2x^2}{\sqrt{9-x^2}}\,dx$

$x = 3\sin \cancel{x}\theta$



$dx = 3\cos\theta\, d\theta$

$\sqrt{9-x^2} = 3\cos\theta$

$\displaystyle\int \frac{2\cdot 9\sin^2\theta}{3\cos\theta}\cdot 3\cos\theta\, d\theta$

$\displaystyle 18\int \sin^2\theta\, d\theta$

$18\left(\frac{1}{2}\theta - \frac{1}{2}\sin\theta\cos\theta\right) + C$

h. $\displaystyle\int \frac{5x+14}{(x+1)(x^2-4)}\,dx$

$(x+2)(x-2)$

K. $\int \cos^5 x \sin^2 x \, dx$

$\int \cos^4(x) \sin^2 x \cdot \cos x \, dx$

$\int (1-\sin^2 x)^2 \sin^2 x \cos x \, dx$

$\int (1-2\sin^2 x+\sin^4 x) \sin^2 x \cos x \, dx$

$\int (\sin^2 x-2\sin^4 x+\sin^6 x) \underbrace{\cos x \, dx}_{du}$

$u=\sin x$