Math 1432

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Office Hours:

Mondays 1-2pm,
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(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

Expand in powers of $(x+1)$ $\leftarrow$ centered @ $x = -1$

$(x-c)$

do a table for $f(x)$ centered @ c

Recall $\left|R_n(x)\right| \le \left(max\left|f^{(n+1)}(c)\right|\right)\dfrac{|x|^{n+1}}{(n+1)!}$ for c between 0 and x.

Assume that f is a function such that $\left|f^{(n)}(x)\right| \le 1$ for all n and x. Estimate the error if $\underset{n}{P_7}(-2)$ is used to approximate $f(\underset{x}{-2})$

$$R_7 \le \frac{|f^{(8)}(c)|}{8!}|x|^8 \le \underbrace{\frac{1}{8!}|-2|^8}_{} = \frac{256}{40320} \approx .006$$

Use the Lagrange formula to find the smallest value of n so that the Taylor polynomial of degree n for $f(x) = \cos(x)$ centered at $x = 0$ can be used to approximate $f(x)$ within 10^{-4} at $x = 1$.

$$|f^{(n)}(x)| \le 1$$

R must be less than

$$R_n \le \frac{|f^{(n+1)}(c)|}{(n+1)!} |x|^{n+1} \le 10^{-4}$$

$$\frac{1}{(n+1)!} |1|^{n+1} \le \frac{1}{10{,}000}$$

$7! = 5040$

$8! = 40320$

$$\frac{1}{(n+1)!} \le \frac{1}{10{,}000}$$

$n+1 = 8$

$$\boxed{n = 7}$$

$f(x) = \ln x$ $\quad 0 < x \leq 1$

$f'(x) = \frac{1}{x}$

$f''(x) = \frac{-1}{x^2}$

$f'''(x) = \frac{-1 \cdot -2}{x^3}$

$f^{(4)}(x) = \frac{-1 \cdot -2 \cdot -3}{x^4}$

alt $\cdot$!

$f^{(5)}(x) = \frac{-1 \cdot -2 \cdot -3 \cdot -4}{x^5}$

$$\frac{(-1)^{n-1}(n-1)!}{x^n}$$

plug in where centered

Given $f(x) = x\cos x^2$, find $f^9(0)$.

for $f(x) = \cos x$, $P_n(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$

for $f(x) = x\cos x^2$, $P_n(x) = x\left(1 - \frac{(x^2)^2}{2!} + \frac{(x^2)^4}{4!} - \frac{(x^2)^6}{6!} + \dots\right)$

$$= x - \frac{x^5}{2!} + \frac{x^9}{4!} - \frac{x^{13}}{6!} + \dots$$

$$\underbrace{\frac{f^{(n)}(0)}{n!}}_{\text{coeff of the } x^n \text{ term}}$$

$$= \frac{f^{(9)}(0)}{9!}x^9$$

$$\frac{1}{4!} = \frac{f^{(9)}(0)}{9!}$$

$$f^{(9)}(0) = 9!/4! = 15120$$

Taylor Series of the Exponential $f(x) = e^x$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \quad \text{for all real } x$$

Taylor Series of the Sine $f(x) = \sin x$

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \quad \text{for all real } x$$

Taylor Series of the Cosine $f(x) = \cos x$

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \quad \text{for all real } x$$

Taylor Series of the Logarithm $f(x) = \ln(1+x)$

$$\ln(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} x^k = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \quad \text{for } -1 < x \leq 1$$

$(-1, 1]$

Find the interval of convergence for $f(x) = e^x$ centered at $x = 0$.

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$\cancel{|x|^n} \cdot |x|$$

Ratio test: $\lim_{n \to \infty} \dfrac{|x|^{n+1}}{\underset{(n+1)\cdot \cancel{n!}}{(n+1)!}} \cdot \dfrac{\cancel{n!}}{\cancel{|x|^n}}$

$$= \lim_{n \to \infty} \frac{|x|}{n+1} = 0 < 1 \text{ for all } x$$

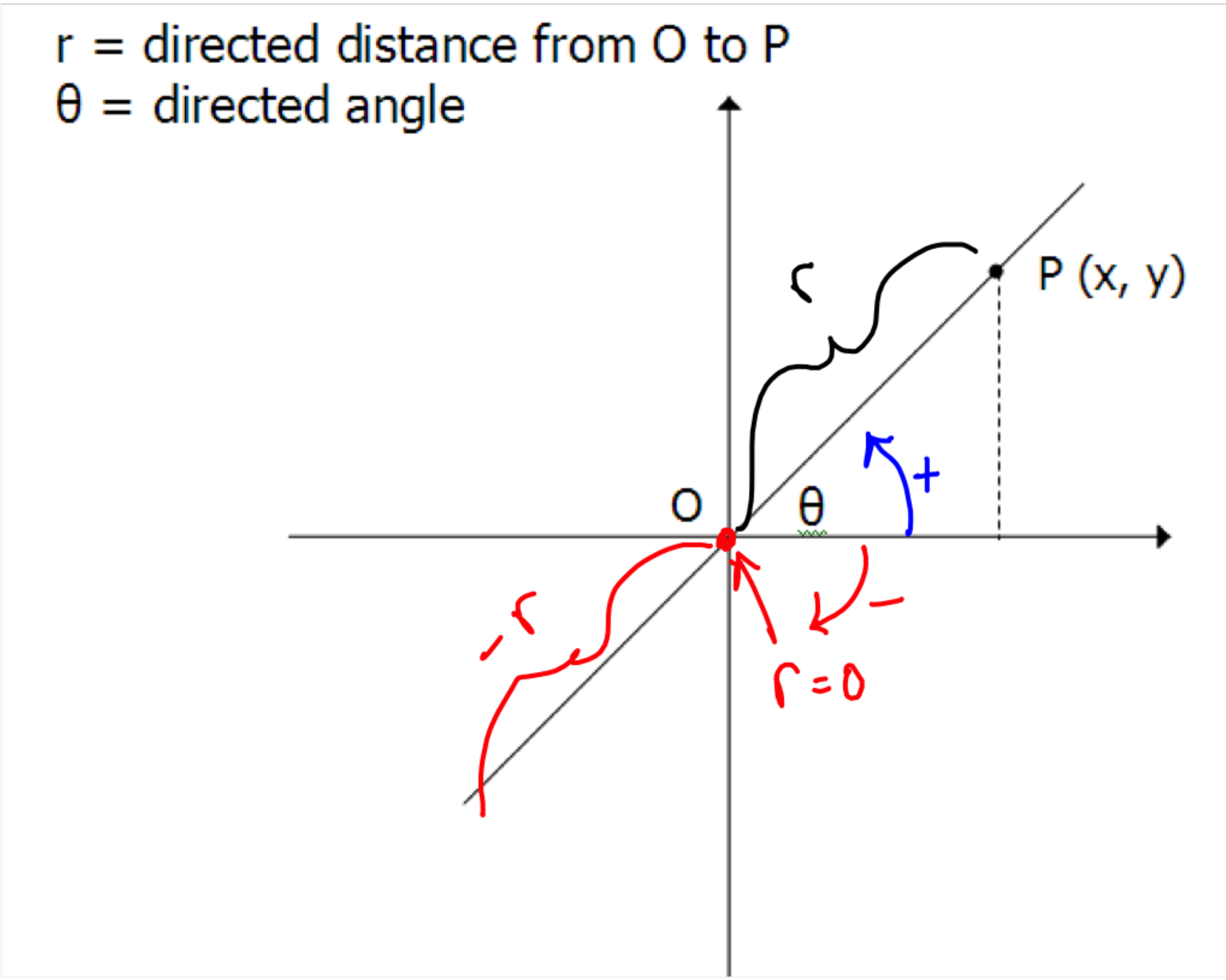
always converges no matter what x is.

$R = \infty$

Int of conv. $(-\infty, \infty)$

10.1 Polar Coordinates

How do you describe the position of a point in a plane using distance and angle rather than x- and y-coordinates?



Plot the points.

plot θ 1st

A. $\left[\overset{r}{2}, \overset{\theta}{\frac{\pi}{3}}\right]$

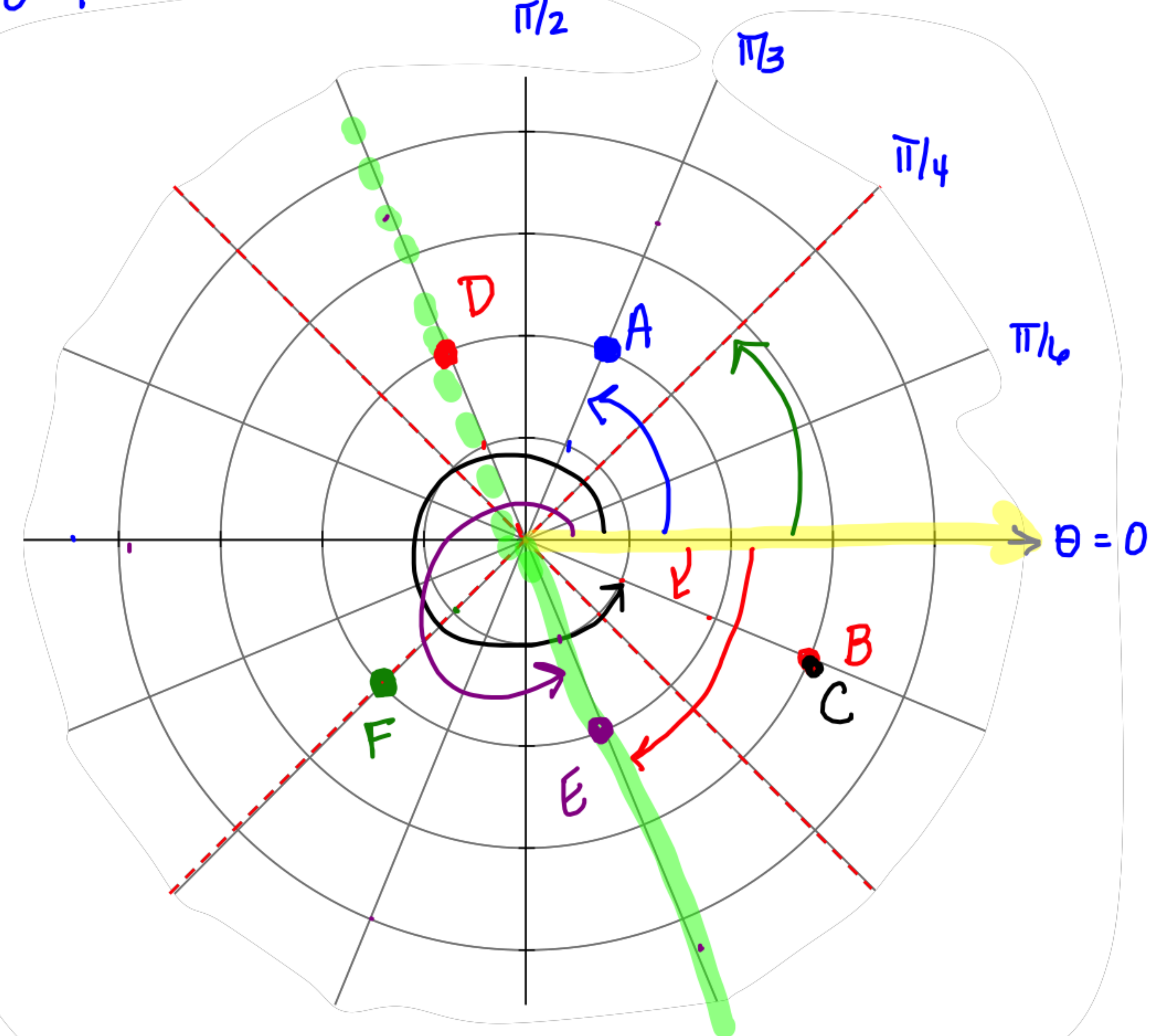
B. $\left[3, -\frac{\pi}{6}\right]$

C. $\left[3, \frac{11\pi}{6}\right]$

D. $\left[-2, \frac{-\pi}{3}\right]$

E. $\left[2, \frac{5\pi}{3}\right]$

F. $\left[-2, \frac{\pi}{4}\right]$



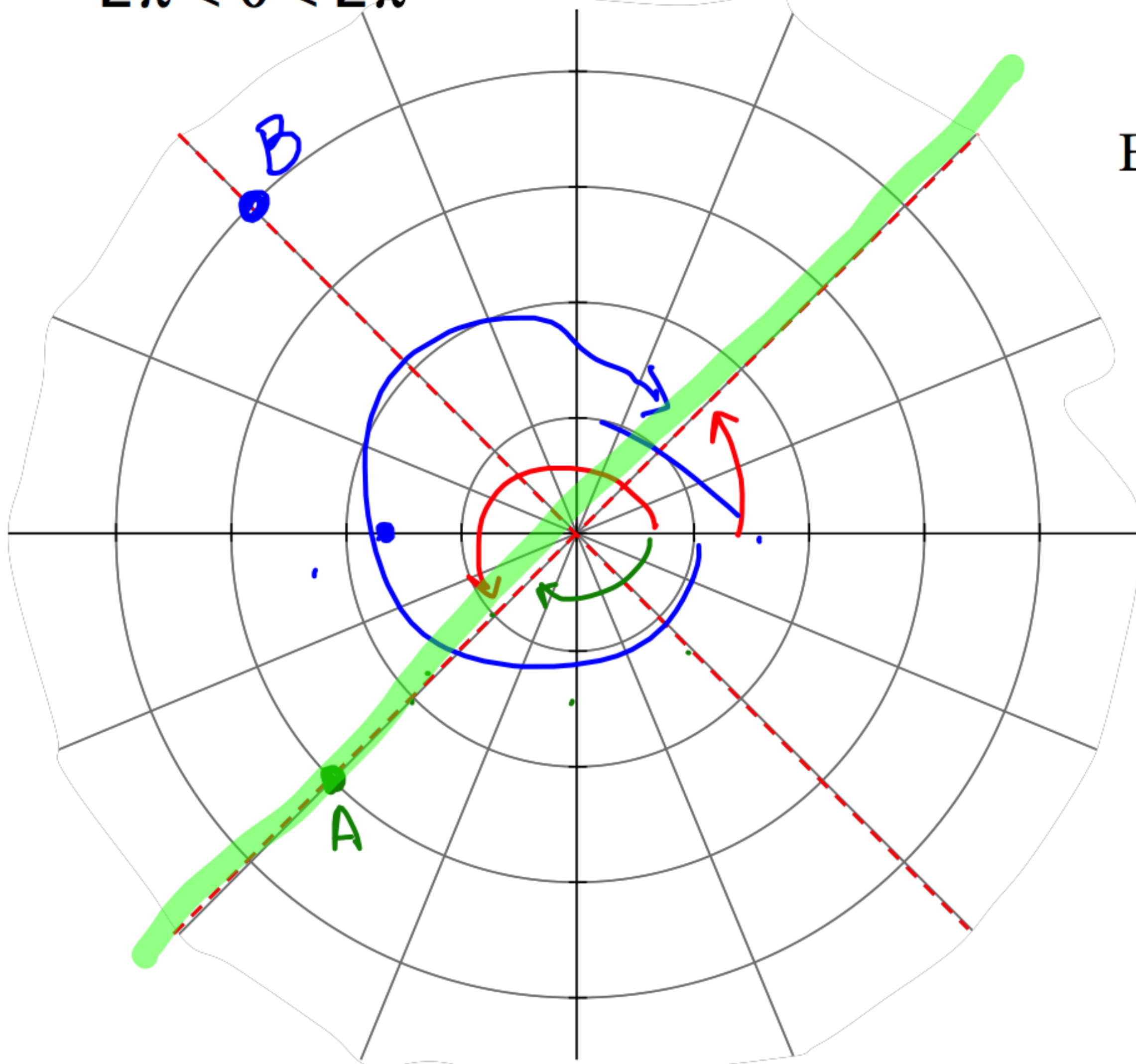
Plot the points and find three additional polar representations of each point using $-2\pi < \theta < 2\pi$

A $\left[3, \frac{-3\pi}{4}\right]$

$\left[3, \frac{5\pi}{4}\right]$

$\left[-3, \frac{\pi}{4}\right]$

$\left[-3, \frac{-7\pi}{4}\right]$



B $\left[4, \frac{3\pi}{4}\right]$

$\left[4, \frac{-5\pi}{4}\right]$

$\left[-4, \frac{-\pi}{4}\right]$

$\left[-4, \frac{7\pi}{4}\right]$

Note: $[r,\theta] = [r,\theta \pm 2n\pi] = [-r,\theta \pm (2n+1)\pi]$ and $[r,\theta+\pi] = [-r,\theta]$

Changing from polar form to rectangular form:

Formulas: $\boxed{x = r\cos\theta \quad y = r\sin\theta}$

Example : Change the following to rectangular form

A. $\left[2, \frac{\pi}{3}\right]$

$r \quad \theta$

$(1, \sqrt{3})$

$x = 2\cos \pi/3 = 2(1/2) = 1$

$y = 2\sin \pi/3 = 2(\sqrt{3}/2) = \sqrt{3}$

B. $\left[\sqrt{3}, \pi\right]$

$r \quad \theta$

$x = \sqrt{3}\cos\pi = -\sqrt{3}$

$y = \sqrt{3}\sin\pi = 0$

$(-\sqrt{3}, 0)$

Changing from rectangular to polar form:

Formulas: $\boxed{x^2 + y^2 = r^2}$ For θ, can use formulas above or $\boxed{\theta = \arctan\frac{y}{x},\ x \neq 0}$

Example: Change the following to polar form:

$* \quad x = r\cos\theta,\ y = r\sin\theta$

A. $(1, -\sqrt{3})$

$x \quad y$

$[2, -\pi/3]$

$(1)^2 + (-\sqrt{3})^2 = r^2$

$1 + 3 = 4 = r^2$

$r = \pm 2$

$r = 2:$

$\theta = \arctan\left(\frac{-\sqrt{3}}{1}\right) = \frac{-\pi}{3}$

$1 = 2\cos\theta \qquad -\sqrt{3} = 2\sin\theta$

$\underbrace{\frac{1}{2} = \cos\theta \qquad -\frac{\sqrt{3}}{2} = \sin\theta}_{\theta = -\pi/3}$

B. $(2, -2)$

$[2\sqrt{2}, -\pi/4]$

$(2)^2 + (-2)^2 = r^2$

$4 + 4 = 8 = r^2$

$r = 2\sqrt{2}$

$2 = 2\sqrt{2}\cos\theta \qquad -2 = 2\sqrt{2}\sin\theta$

$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} = \cos\theta \qquad -\frac{1}{\sqrt{2}} = \sin\theta$

More examples: $x^2+y^2=r^2$, $x=r\cos\theta$, $y=r\sin\theta$

1. Write the following in polar form.

A. $x^2 - y^2 = 4$

$$r^2\cos^2\theta - r^2\sin^2\theta = 4$$

$$r^2(\cos^2\theta - \sin^2\theta) = 4$$

$$r^2\cos(2\theta) = 4$$

B. y = 4

$r\sin\theta = 4$ $\qquad r = \dfrac{4}{\sin\theta}$ $\qquad r = 4\csc\theta$

C. y = x

$$r\sin\theta = r\cos\theta$$

$$\sin\theta = \cos\theta$$

$$\tan\theta = 1$$

$\theta = \pi/4$

2. Write in rectangular form and describe the graph.

A. $r\sin\theta = 4$

$y = 4$ line

B. $\theta = \frac{1}{3}\pi$ line

$\frac{y}{x} = \frac{r\sin\theta}{r\cos\theta} = \tan\theta$

$\tan(\theta) = \tan(\pi/3)$

$\frac{y}{x} = \sqrt{3}$

$y = \sqrt{3}\,x$

C. $r = 3\cos\theta$

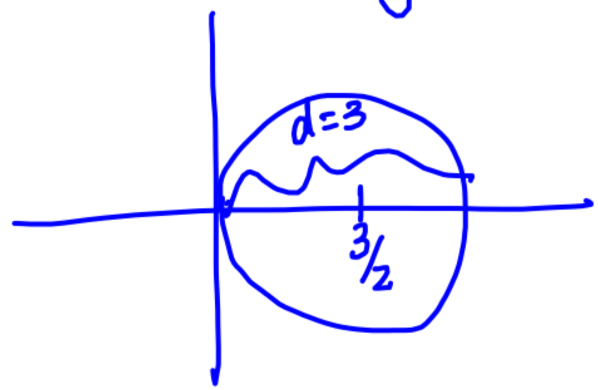
$$r^2 = 3r\cos\theta$$

$$x^2 + y^2 = 3x$$

circle

$$x^2 - 3x + \frac{9}{4} + y^2 = \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 + y^2 = \frac{9}{4}$$



D. $r = \csc\theta$

$$r = \frac{1}{\sin\theta}$$

$$r\sin\theta = 1$$

$$y = 1$$

line

E. $r = \dfrac{1}{1 - \cos\theta}$

$r - r\cos\theta = 1$

$\underline{\underline{r}} - x = 1$

$\sqrt{x^2+y^2} - x = 1$

$\sqrt{x^2+y^2} = x + 1$

$x^2 + y^2 = (x+1)^2$

$x^2 + y^2 = x^2 + 2x + 1 \rightarrow y^2 = 2x + 1$

$\underline{\underline{\text{parabola}}}$

Popper 30

1. The graph of $y^2 + x^2 = 4$ is a(n)

a. circle
b. ellipse
c. horizontal line
d. vertical line
e. hyperbola

2. The graph of $r = 2$ is a(n)

a. circle
b. ellipse
c. horizontal line
d. vertical line
e. hyperbola

3. The polar graph of r = 2 *sec*(θ) is a

$r = \dfrac{2}{\cos\theta}$

a. vertical line x = 2

b. horizontal line y = 2

#5-7 = B

c. vertical line y = 2

d. horizontal line x = 2

4. The polar graph of r = *sec*(θ)*tan*(θ) is

a. an ellipse

$r = \dfrac{1}{\cos\theta} \dfrac{\sin\theta}{\cos\theta}$

b. a hyperbola

$r\cos\theta = \dfrac{r\sin\theta}{r\cos\theta}$

c. a parabola

d. I have no clue

$x = \dfrac{y}{x}$