Math 1432

Bekki George
bekki@math.uh.edu
639 PGH

Office Hours:

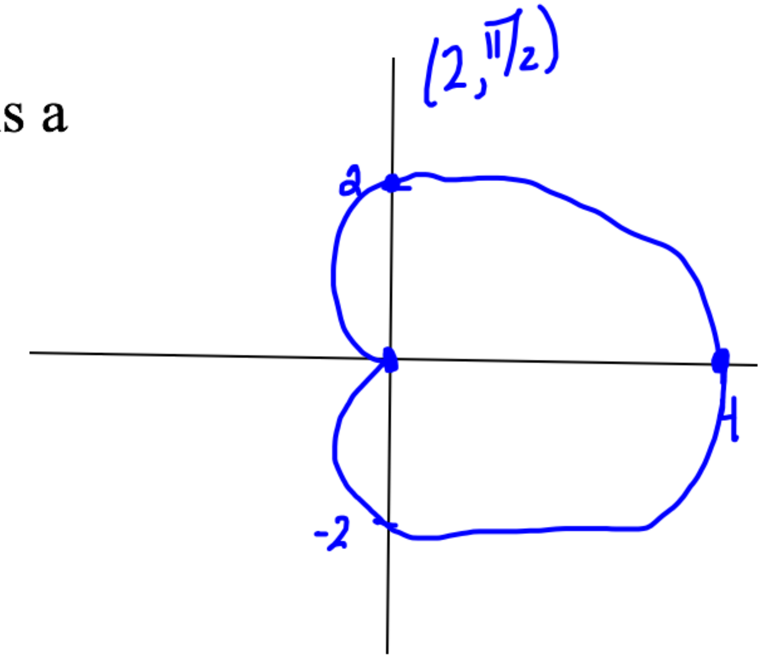
12 -1
Mondays ~~1-2pm,~~
Fridays noon-1pm
(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

POPPER 33

1. The polar plot of r = 2 + 2 *cos* θ is a



a. flower

b. line

c. cardioid

d. limaçon with loop

e. limaçon with dent (dimple)

2. The polar plot of $r = 5 - 2\,\mathit{cos}\,\theta$ is a

a. flower

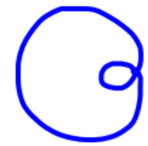
b. line

c. cardioid

d. limaçon with loop

e. limaçon with dent (dimple)

3. The polar plot of r = 7 – 12 *cos* θ is a



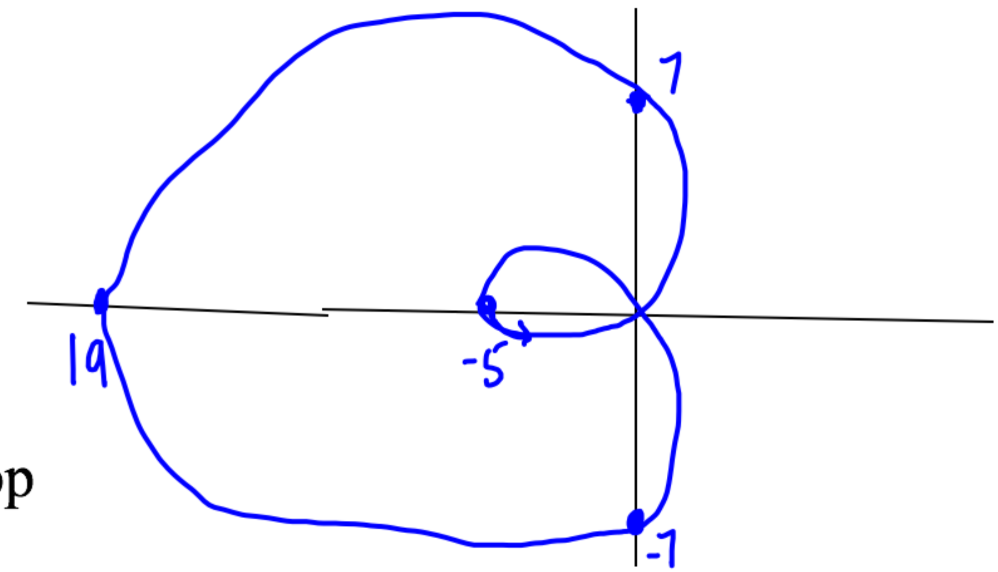
a. flower

b. line

c. cardioid

d. limaçon with loop

e. limaçon with dent (dimple)



4. The polar plot of $r = 2\ \mathit{cos}\ 5\theta$ is a

a. flower with 5 petals

b. flower with 2 petals

c. flower with 10 petals

d. circle with radius 5

e. circle with diameter 2

5. The polar plot of $r = \overset{\text{diam}}{4} \, cos \, \theta$ is a

$r = \underline{2a} \cos\theta$

~~a.~~ circle centered at (0, 0) $r = a$

~~b.~~ flower with 4 petals

c. circle with radius 4, centered at (4, 0)

d. circle with radius 2, centered at (2, 0)

e. circle with radius 1, centered at (1, 0)

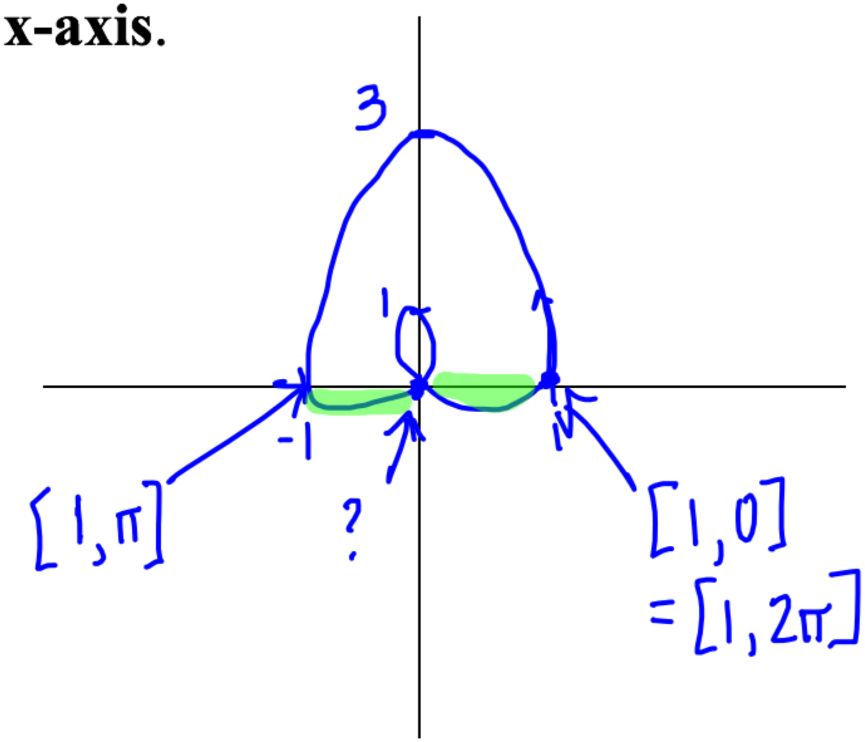
6. Give the formula for the area of the region that is enclosed by the polar curve r = 1 + 2sin(θ) and lies **below the x-axis**.

a. $\int_{-\pi/6}^{0} (1+2\sin\theta)^2 \, d\theta$

b. $\int_{11\pi/6}^{2\pi} (1+2\sin\theta)^2 \, d\theta$

c. $\int_{\pi}^{7\pi/6} (1+2\sin\theta)^2 \, d\theta$

d. all of these will give the area of the region



$0 = 1 + 2\sin\theta$

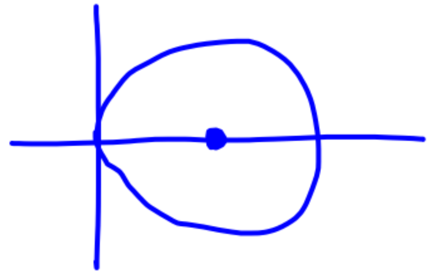
$-\frac{1}{2} = \sin\theta$

$\theta = 7\pi/6, \; 11\pi/6 = -\pi/6$

★ $x = r\cos\theta$
$y = r\sin\theta$ }

$x^2 + y^2 = r^2$

7. Re-write $(x-3)^2 + y^2 = 9$ in polar form



a. $r = 3$

b. $r^2 = 6\cos\theta$

c. $r = 6\cos\theta$

d. $r^2 = 6\sin\theta$

$$x^2 - 6x + 9 + y^2 = 9$$

$$x^2 + y^2 = 6x$$

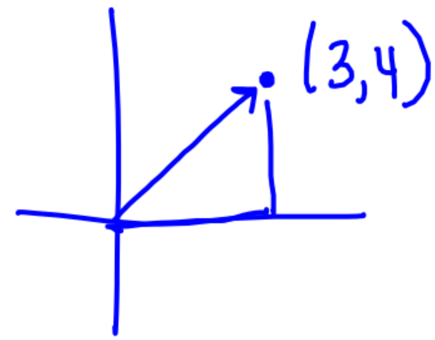
$$r^2 = 6r\cos\theta$$

$$r = 6\cos\theta$$

Parametric Curves

Parametric equations are sets of equations that are used to express quantities explicitly in terms of another variable.

So, instead of using y = f(x) (defining y in terms of x), we let x(t) and y(t) be functions where t is the parameter.



Then (x(t), y(t)) is the point that traces out the curve.

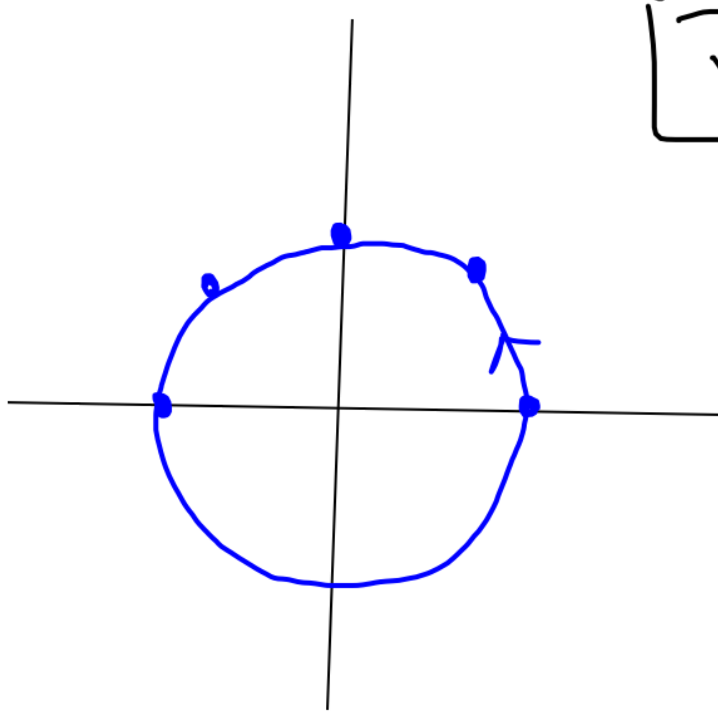
If t is restricted to lie on an interval [a, b] then x(t) and y(t) would have an initial point (x(a), y(a)) and a terminal point (x(b), y(b)). So a parametric curve has an orientation given by the parameterizing variable.

Ex. 1: Plot (cos(t), sin(t)) for $0 \le t \le 2\pi$ and express the curve by an equation in x and y.

$x(t) = \cos t$

$y(t) = \sin t$

t	x	y
0	1	0
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/2$	0	1
$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
π	-1	0



$\cos^2 t + \sin^2 t = 1$

$\boxed{x^2 + y^2 = 1}$

Ex. 2: Sketch the curve and eliminate the parameter.

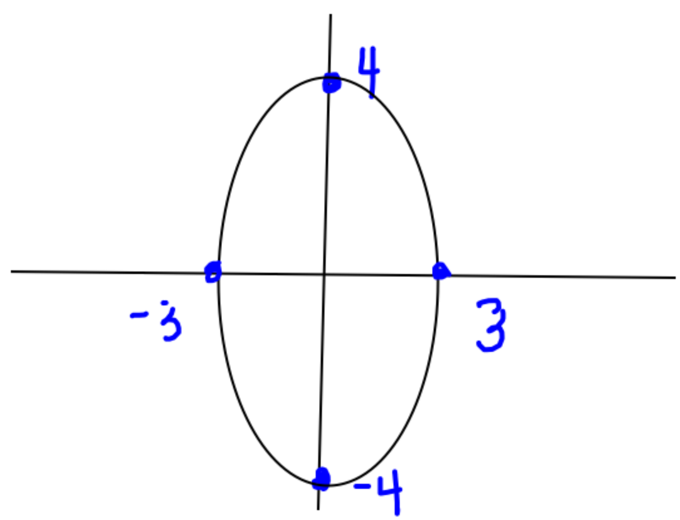
$x(\theta) = 3\cos(\theta) \qquad y(\theta) = 4\sin(\theta) \qquad 0 \le \theta \le 2\pi$

$\cos^2\theta + \sin^2\theta = 1$

$\cos\theta = \frac{x}{3} \qquad \sin\theta = \frac{y}{4}$

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1 \rightarrow \frac{x^2}{9} + \frac{y^2}{16} = 1$

θ	x	y
0	3	0
$\pi/2$	0	4
π	-3	0
$3\pi/2$	0	-4
2π	3	0



if $y = f(x)$ let $x = t$ $y = f(t)$

Ex. 3: Give a parameterization of the PORTION of the line $y = -2x + 5$ between $(1, 3)$ and $(-2, 9)$

$x(t) = t$

$y(t) = -2t + 5$

~~$1 \le t \le -2$~~

$-2 \le t \le 1$

from $(1,3)$ to $(-2,9)$

$x(t) = 1 + t(-2-1) = 1 - 3t$

$y(t) = 3 + t(9-3) = 3 + 6t$

$0 \le t \le 1$

To parameterize a line SEGMENT from (x_0, y_0) to (x_1, y_1):

$$\left.\begin{array}{l} x(t) = \overset{\text{start}}{x_0} + t\left(\overset{\text{end}}{x_1} - \overset{\text{start}}{x_0}\right) \\ y(t) = y_0 + t\left(y_1 - y_0\right) \\ 0 \le t \le 1 \end{array}\right\} \star$$

For a LINE: $-\infty < t < \infty$

Ex. 4: Parameterize the **line segment** from $(\overset{x_0}{3}, \overset{y_0}{6})$ to $(\overset{x_1}{-2}, \overset{y_1}{5})$.

$$x(t) = 3 + t(-2-3) = 3 - 5t$$

$$y(t) = 6 + t(5-6) = 6 - t$$

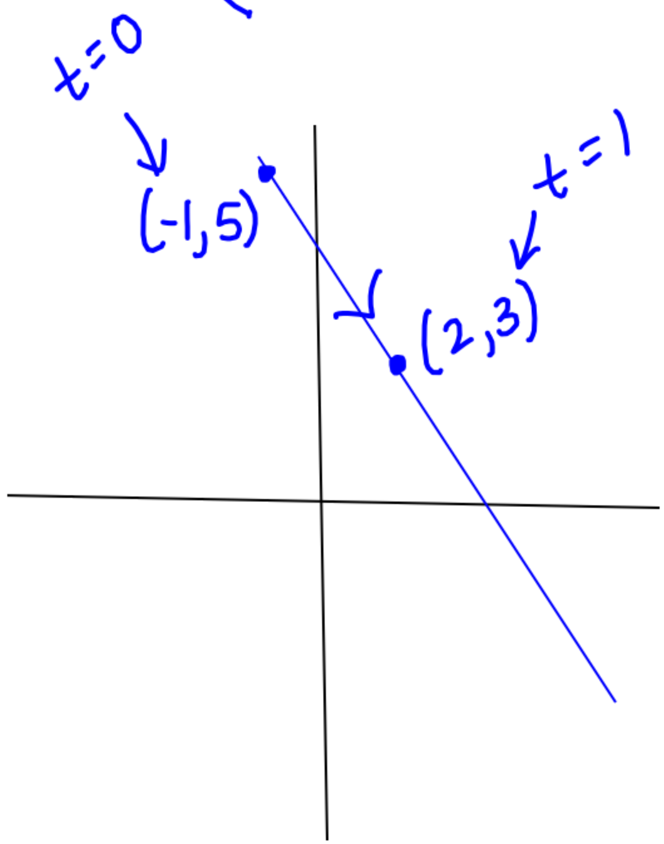
$$0 \le t \le 1$$

Ex. 5: Express the curve by an equation in x and y; then sketch the curve. $x(t) = 3t - 1 \quad y(t) = 5 - 2t \quad t \in (-\infty, \infty)$

$$\left(\frac{x+1}{3}\right) = t$$

$$y = 5 - 2\left(\frac{x+1}{3}\right)$$

$$\boxed{y = \frac{-2}{3}x + \frac{13}{3}}$$



Ex. 6: Express the curve by an equation in x and y

$$x(t) = 3\tan t \quad y(t) = 5 - \sec^2 t$$

$x/3 = \tan t$ $\sec^2 t = 5 - y$

$$\tan^2 t + 1 = \sec^2 t$$

$$\left(x/3\right)^2 + 1 = 5 - y$$

Ex. 7: Express the curve by an equation in x and y

$$x(t) = 4 + e^t \quad y(t) = 2e^{2t} = 2(\underline{e^t})^2$$

$$e^t = (x - 4)$$

$$y = 2(x-4)^2$$

Common parameterizations:

line segment: $x(t) = x_0 + t(x_1 - x_0)$ $\quad 0 \le t \le 1$

$y(t) = y_0 + t(y_1 - y_0)$

circle centered at $(0,0)$ radius r

$x(t) = r\cos t$ $\quad 0 \le t \le 2\pi$

$y(t) = r\sin t$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\left(\frac{x-h}{r}\right)^2 + \left(\frac{y-k}{r}\right)^2 = 1$$

$$\cos^2 t + \sin^2 t = 1$$

$\frac{x-h}{r} = \cos t$

$\frac{y-k}{r} = \sin t$

centered at (h,k), radius r

$$x(t) = r\cos t + h$$
$$y(t) = r\sin t + k$$

$0 \le t \le 2\pi$

ellipse centered at $(0,0)$

$$x(t) = a\cos t$$
$$y(t) = b\sin t$$

$0 \le t \le 2\pi$

8. The parametric curve given by ($2\cos(t)$, $2\sin(t)$) is a(n)

a. hyperbola

b. parabola

c. ellipse

d. circle

e. line

9. The parametric curve given by ($3\cos(t)$, $5\sin(t)$) is a(n)

a. hyperbola

b. parabola

c. ellipse

d. circle

e. line

10. Eliminate the parameter and find a corresponding rectangular equation: $x = 3t^2$ and $y = 2t + 1$

solve for t

a. $2x^2 + 3y^2 - 1 = 0$

b. $3y^2 - 4x - 6y + 3 = 0$

c. $3y^2 - 4x + 1 = 0$

d. $2x - 3y + 3 = 0$

e. none of these