Math 1432

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Office Hours:

Mondays 1-2pm,
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Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

Test 4 Review

- Exam covers sections 9.3-10.3
- Be able to tell quickly if a series converges or diverges and be able to justify your answer
- Absolute vs. conditional convergence
- Intervals of convergence
- Derivatives and integrals of power series
- Taylor polynomials and Taylor series (including remainders)
- Conversion from rectangular form to polar form (and vice versa)
- Polar graphing
- Polar area
- ~~Polar arc length~~
- Parametric curves

Converge or diverge?

a. $\displaystyle\sum_{n=2}^{\infty} \frac{4n^2+5n-2}{n^5-3n-1}$ $\sim \sum \frac{1}{n^3}$ ☆ p series w/ $p=3>1$ ☆

converges

(on F.R. do limit comp w/ $\sum \frac{1}{n^3}$)

b. $\displaystyle\sum_{n=1}^{\infty} \frac{5n^2+3n-2}{\sqrt{2n^6+n-10}}$ $\sum \frac{n^2}{\sqrt{n^6}} = \sum \frac{n^2}{n^3} = \sum \frac{1}{n}$ div.

c. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^{-n}$ $= \sum \left(\frac{3}{2}\right)^n$ div. geom $r = 3/2$

or BDT

d. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}$ $= \sum \frac{1}{\sqrt{n^2+n}}$ $\sum \frac{1}{\sqrt{n^2}} = \sum \frac{1}{n}$

div.

e. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{3}{2}\right)^{-n} = \sum \left(\frac{2}{3}\right)^{n}$ converges geom $r = 2/3$

$|2/3| < 1$

f. $\displaystyle\sum_{n=1}^{\infty}\frac{n}{\sqrt{n^3+2n}}$ $\quad \sum \frac{n}{n^{3/2}} = \sum \frac{1}{n^{1/2}}$ div. $\quad p = 1/2 \le 1$

div.

g. $\displaystyle\sum_{n=0}^{\infty}\frac{2}{7^n} = 2\sum\left(\frac{1}{7}\right)^n$ conv. geom $r = 1/7$

h. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ conv. by AST or p-series

i. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right) = \sum \frac{n+1-n}{n(n+1)} = \sum \frac{1}{n^2+n}$ conv.

comp. to $\sum \frac{1}{n^2}$

j. $\displaystyle\sum_{n=1}^{\infty}\frac{5}{2n-1} \sim \sum \frac{1}{n}$ diverges

k. $\displaystyle\sum_{n=1}^{\infty}\frac{3^{2n}}{n!}$ Ratio: $\displaystyle\lim_{n\to\infty} \frac{3^{2(n+1)}}{(n+1)!}\cdot\frac{n!}{3^{2n}} = \lim_{n\to\infty}\frac{\cancel{3^{2n}}\cdot 3^2\cdot \cancel{n!}}{(n+1)\cancel{n!}\,\cancel{3^{2n}}}$

$\displaystyle = \lim_{n\to\infty}\frac{9}{n+1} = 0 < 1$ conv.

l. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2n}{5n-1}\right)^{n}$ Root: $\displaystyle\lim_{n\to\infty}\left[\left(\frac{2n}{5n-1}\right)^n\right]^{1/n} = \lim_{n\to\infty}\frac{2n}{5n-1} = \frac{2}{5}$

< 1

conv.

m. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{3n^3+1}$ $\quad \frac{n^2}{3n^3+1} \to 0$ $\quad$ conv. by AST

n. $\displaystyle\sum_{n=0}^{\infty} 3\left(-\frac{5}{2}\right)^n$ $\quad$ div. $\quad$ (geom $r = -5/2$) or BDT

$= \sum 3(-1)^n (5/2)^n$ $\quad |-5/2| > 1$

o. $\displaystyle\sum_{n=1}^{\infty} n\left(\frac{5}{6}\right)^n$ $\quad$ Root: $\lim_{n\to\infty} n^{1/n}(5/6) = 5/6 < 1 \Rightarrow$ conv.

p. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1+e^{-n}}$ $\quad$ Div. by BDT $\quad \lim_{n\to\infty} \frac{1}{1+e^{-n}} = 1 \neq 0$

DIV

q. $\sum_{n=1}^{\infty} \frac{5^n}{n^3}$

Ratio: $\lim_{n\to\infty} \frac{5^{n+1}}{(n+1)^3} \cdot \frac{n^3}{5^n} = \lim_{n\to\infty} \frac{5\,n^3}{n^3+\cdots} = 5 > 1$

Root: $\lim_{n\to\infty} \frac{5}{n^{3/n}} = 5$

r. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)}$ DIV

Integral: $\int_2^{\infty} \frac{1}{x\ln x}\,dx = \lim_{b\to\infty} \left[\ln(\ln x)\right]_2^b \to \infty$

$u = \ln x$

$du = \frac{1}{x}dx \quad \int \frac{1}{u}\,du$

$\boxed{\text{DIV}}$

s. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ conv

Integral: $\int_2^{\infty} \frac{1}{x(\ln x)^2}\,dx \quad \int \frac{1}{u^2}\,du = \frac{-1}{u} + C$

$\lim_{b\to\infty} \left.\frac{-1}{\ln x}\right|_2^b = \lim_{b\to\infty} \frac{-1}{\ln b} + \frac{1}{\ln 2}$

t. $\sum_{n=1}^{\infty} n e^{-n^3} = \sum \frac{n}{e^{n^3}}$ conv

Root: $\lim_{n\to\infty} \frac{n^{1/n}}{e^{n^2}} = \frac{1}{\infty} \to 0 < 1$

u. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n}$ BDT <u>Diverges</u> $\left(\frac{n+1}{n}\right)^{-n}=\left(1+\frac{1}{n}\right)^{-n}\to e^{-1}\neq 0$

v. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n(n+1)(n+2)}$ $\underset{<}{\sim}$ $\sum\frac{1}{n^3}$ <u>conv.</u>

w. $\displaystyle\sum_{n=1}^{\infty}\frac{n!}{e^{n}}$ <u>DIV</u>

Ratio: $\displaystyle\lim_{n\to\infty}\frac{(n+1)!}{e^{n+1}}\cdot\frac{e^{n}}{n!}$

$\displaystyle=\lim_{n\to\infty}\frac{(n+1)\cancel{n!}\,\cancel{e^{n}}}{\cancel{e^{n}}\cdot e\cdot\cancel{n!}}\to\infty$

$n^n > n! > x^n$

Converge absolutely or conditionally or diverge?

a. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n!}{n^n}$

ABS. CONV

$\frac{n!}{n^n} \to 0?$ converges: $\sum \frac{n!}{n^n}$

Ratio: $\lim_{n\to\infty} \frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!}$

$\longrightarrow$

b. $\displaystyle\sum_{n=2}^{\infty} \frac{25n!}{(n+3)!}$ ← not alternating so only choices are abs. conv. or diverge

ABS CONV

$\sum \frac{\cancel{n!}}{(n+3)(n+2)(n+1)\cancel{n!}}$ 1

c. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n!}{n(n+1)!}$

$\frac{\cancel{n!}}{n\cdot(n+1)\cdot\cancel{n!}}$ 1

ABS. CONV.

d. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n (n-1)}{5n^2+2n-1}$ $\sim$ $\sum \frac{(-1)^n}{n}$

Cond. conv

$$\lim_{n\to\infty} \frac{\cancel{(n+1)}\,\cancel{n!}\;n^n}{(n+1)^n\,\cancel{(n+1)}\,\cancel{n!}} = \lim_{n\to\infty} \frac{n^n}{(n+1)^n}$$

$$= \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^{-n}$$

$$= \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{-n} = e^{-1} = \frac{1}{e} < 1$$

e. $\sum_{n=2}^{\infty} \frac{\cos(\pi n)}{n+7} = \sum \frac{(-1)^n}{n+7}$ Cond. conv.

f. $\sum_{n=2}^{\infty} \frac{(-1)^n 2^n}{2^n+1}$ $\frac{2^n}{2^n+1} \to 1 \neq 0$ div. by BDT

g. $\sum_{n=2}^{\infty} n(2)^n$ div. by BDT

Find the 5th degree Taylor polynomials centered at 0 for the following:

a) $f(x) = e^{5x^2}$

$$e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{5x^2} = \underbrace{1 + 5x^2 + \frac{(5x^2)^2}{2!}}_{P_4(x) = P_5(x)} + \frac{(5x^2)^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{(5x^2)^n}{n!}$$

b) $f(x) = \cos\left(2x^3\right)$

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c) the function with the following properties:

$k=$ 0	1	2	3	4	5
$f(0)$	$f'(0)$	$f''(0)$	$f'''(0)$	$f^{(4)}(0)$	$f^{(5)}(0)$
-1 $/0!$	3 $/1!$	12 $/2!$	-6 $/3!$	5 $/4!$	-10 $/5!$

$\div k!$

k $\quad f^{k}(x) \quad f^{k}(0) \quad \dfrac{f^{k}(0)}{k!}$ coeff

$$P_5(x) = -1 + 3x + 6x^2 - x^3 + \frac{5}{24}x^4 - \frac{10}{5!}x^5$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\cos(2x^3) = \underbrace{\underset{P_5}{\underset{\uparrow}{1}} - \frac{(2x^3)^2}{2!}}_{P_6} + \frac{(2x^3)^4}{4!} - \cdots = \underbrace{\sum_{n=0}^{\infty} \frac{(-1)^n (2x^3)^{2n}}{(2n)!}}_{\text{Series}}$$

Assume that $|f^{(n)}(x)| \leq 10$ for all x in the interval (0, 1). If you estimated $f(0.1)$ using a 5$^{\text{th}}$ degree Taylor polynomial, what is the maximum possible error? $n=5$

$$\frac{|f^{(n+1)}(c)|}{(n+1)!} x^{n+1}$$

$$\boxed{\frac{10}{6!}(.1)^6}$$

For this same function, what is the smallest value of n for which $P_n(0.1)$ will approximate $f(0.1)$ within 0.0001?

$$\frac{|f^{(n+1)}(c)|}{(n+1)!} x^{n+1} < .0001$$

$$\frac{10}{(n+1)!}\left(\frac{1}{10}\right)^{n+1} < \frac{1}{10{,}000} \qquad \frac{1}{10^4}$$

$$\frac{1}{(n+1)!} \frac{1}{10^n}$$

$n=2$: $\frac{1}{3!} \frac{1}{100} = \frac{1}{600}$

$\boxed{n=3}$: $\frac{1}{4!} \cdot \frac{1}{1000} = \frac{1}{24000}$

Find the radius of convergence and interval of convergence for the following Power series:

a. $\displaystyle\sum_{n=0}^{\infty} \frac{3^n (x-1)^n}{n!}$

-5 -3 -1

$[-5, -1]$

b. $\displaystyle\sum \frac{(-1)^k}{k^2 2^k}(x+3)^k$

$x = -5$: $\displaystyle\sum \frac{(-1)^k (-2)^k}{k^2\, 2^k}$

$= \displaystyle\sum \frac{\boxed{(-1)^k (-1)^k}^{=1}\, \cancel{2^k}}{k^2\, \cancel{2^k}}$ Conv.

$x = -1$: $\displaystyle\sum \frac{(-1)^k (\cancel{2})^k}{k^2\, \cancel{2^k}}$ conv

$\displaystyle\lim_{k\to\infty} \left(\frac{|x+3|^k}{k^2\, 2^k} \right)^{1/k}$

$= \displaystyle\lim_{k\to\infty} \frac{|x+3|}{k^{2/k} \cdot 2} = \frac{|x+3|}{2} < 1$

$|x+3| < 2$ R

Give the derivative of each power series below, and
give the antiderivative F of the power series so that F(0)=0.

a. $$\sum_{n=0}^{\infty} \frac{x^n}{n+2}$$

b. $$\sum_{n=0}^{\infty} \frac{n^2 x^n}{2n^3+1}$$

Write the following in polar coordinate form:

a. $x^2 + (y+3)^2 = 9$

b. $y = \frac{1}{3}x$

Write the following in rectangular coordinate form:

a. $r = 3\cos\theta$

b. $r = 5$

Find the area inside the inner loop for $r = 3 - 6\cos\theta$

Find the area inside $r = 2$ and outside $r = 4\cos\theta$

Find the length of the curve $r = 1 + \cos\theta$

Find a parameterization for the ellipse $\frac{(x-1)^2}{4}+\frac{(y+2)^2}{9}=1$.

Write as an equation of x and y: