Math 1432

Bekki George
bekki@math.uh.edu
639 PGH

Office Hours:

Mondays 1-2pm,
Fridays noon-1pm
(also available by appointment)

Class webpage:
http://www.math.uh.edu/~bekki/Math1432.html

May 1	2	3	4	5	6	7
Quiz 25 help: **Video** **Completed Notes** Quiz 26 help: **Video** **Completed Notes**	Exam 4 **Blank Slides** **Completed Notes** **Video**	Exam 4	Online Review Session 10:00am - noon **CLICK THIS LINK TO ENTER THE ONLINE CLASSROOM** **Completed Notes** **Video** Quiz 25 (10.4) closes	Final Exam Review by Dylan Domel-White CBB 104 3-5pm Kayla's online review **online classroom** 4-6pm		Final Exam Quiz 26 (10.5) closes
8	9	10	11	12	13	14
Final Exam	Final Exam	Final Exam				

~ 20 questions

7.1 – 10.4

T/F, M/C, F/R

watch class webpage & CASA discussion board

Conceptual questions

converge or diverge

$a \leftarrow + \quad \rightarrow b$

1. Evaluate each improper integral and tell why it is improper:

a. $\displaystyle\int_0^{27} x^{-2/3}\,dx = \lim_{a\to 0+}\int_a^{27} x^{-2/3}\,dx = \lim_{a\to 0+} 3x^{1/3}\Big|_a^{27}$

$0 \qquad 27$

$$= \lim_{a\to 0+}\left(3\sqrt[3]{27} - 3\sqrt[3]{a}\right) = 9-0 = 9$$

b. $\displaystyle\int_0^{4} \frac{1}{\sqrt{4-x}}\,dx = \lim_{b\to 4-}\int_0^{b} (4-x)^{-1/2}\,dx = \lim_{b\to 4-}\left[-2(4-x)^{1/2}\right]_0^{b}$

4

$$= \lim_{b\to 4-}\left(-2\sqrt{4-b} - -2\sqrt{4}\right) = 4$$

c. $\displaystyle\int_1^{9} (x-1)^{-2/3}\,dx$

$$\lim_{a\to 1+}\int_a^{9} (x-1)^{-2/3}\,dx = \lim_{a\to 1+} 3(x-1)^{1/3}\Big|_a^{9} = \lim_{a\to 1+} 3\sqrt[3]{8} - 3\sqrt[3]{a-1}$$

$$= 6$$

d. $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx = \int_{-\infty}^{0} \frac{1}{1+x^2} dx + \int_{0}^{\infty} \frac{1}{1+x^2} dx$

$\lim_{a\to-\infty} \int_a^0 \frac{1}{1+x^2} dx + \lim_{b\to\infty} \int_0^b \frac{1}{1+x^2} dx = \lim_{a\to-\infty} \arctan x \Big|_a^0 + \lim_{b\to\infty} \arctan x \Big|_0^b$

$= \lim_{a\to\infty} 0 - \tan^{-1} a + \lim_{b\to\infty} \tan^{-1} b$

$= -\left(-\pi/2\right) + \pi/2 = \pi$

2. Integrate (Some may be improper integrals!)

a. $-\frac{1}{3}\int \frac{-3\sin(3x)}{16+\cos^2(3x)} dx = -\frac{1}{3}\int \frac{du}{4^2+u^2}$

$u = \cos(3x)$

$du = -3\sin(3x)\,dx$

$-\frac{1}{3}\left(\frac{1}{4}\right)\tan^{-1}\left(\frac{u}{4}\right) + C$

$-\frac{1}{12}\arctan\left(\frac{\cos(3x)}{4}\right) + C$

b. $\int \frac{3\cdot 2\, x}{4+x^4} dx = 3\int \frac{du}{2^2+u^2}$

$u = x^2$

$du = 2x\,dx$

$3\left(\frac{1}{2}\right)\arctan\left(\frac{u}{2}\right) + C = \frac{3}{2}\tan^{-1}\left(\frac{x^2}{2}\right) + C$

$$\boxed{\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C}$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right) + C$$

c. $\displaystyle\int_0^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}}\,dx$ $\quad \frac{\sqrt{3}}{2} < \frac{2}{2}$

not improper $\quad x \neq 1$

$$= \arcsin(x)\Big|_0^{\sqrt{3}/2} = \arcsin\left(\frac{\sqrt{3}}{2}\right) - \arcsin(0)$$

$$\pi/3 - 0 = \boxed{\pi/3}$$

d. $\displaystyle\frac{1}{2}\int \frac{\cancel{1}\,2}{\sqrt{9-4x^2}}\,dx$ $\quad = \boxed{\frac{1}{2}\arcsin\left(\frac{2x}{3}\right) + C}$

$a = 3 \quad u = 2x$

$du = 2dx$

e. $\displaystyle\int \frac{5}{36+(x-1)^2}\,dx$ $\quad = 5 \cdot \frac{1}{6}\arctan\left(\frac{x-1}{6}\right) + C$

$a = 6 \quad u = x-1$

$du = dx$

IBP $\quad uv - \int v\,du \quad$ + be able to do definite

3. Integrate:

a) $\int 2x\cos(10x)\,dx$

A T

$u = 2x \qquad dv = \cos(10x)\,dx$

$du = 2\,dx \qquad v = \frac{1}{10}\sin(10x)$

$$\frac{x}{5}\sin(10x) - \int \frac{1}{5}\sin(10x)\,dx = \frac{x}{5}\sin(10x) + \frac{1}{50}\cos(10x) + C$$

b) $\int 10xe^{4x}\,dx$

$u = 10x \qquad dv = e^{4x}\,dx$

$du = 10\,dx \qquad v = \frac{1}{4}e^{4x}$

$$\frac{5}{2}xe^{4x} - \int \frac{5}{2}e^{4x}\,dx$$

$$= \frac{5}{2}xe^{4x} - \frac{5}{8}e^{4x} + C$$

c) $\int x\ln(x)\,dx$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{x^2}{2}$

$$\frac{x^2}{2}\ln x - \int \frac{x^{\cancel{2}}}{2}\cdot\frac{1}{\cancel{x}}\,dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$$

$$\int_a^b g(x)\cdot f(x)\,dx$$

"dv" needs to be integrable

$u = g(x) \qquad dv = f(x)\,dx$

$du = g'(x)\,dx \qquad v = \int_a^b f(x)\,dx$

Suppose area from a to b for f(x) is 10 (f(x) > 0)

$$g(x)\cdot\underbrace{\int_a^b f(x)\,dx}_{10} - \int_a^b \boxed{10}\, g'(x)\,dx$$

$\cos^2 x + \sin^2 x = 1 \quad \tan^2 x + 1 = \sec^2 x \quad \cot^2 x + 1 = \csc^2 x$

4. Integrate:

a) $\int \cos^3 2\theta \, d\theta = \int \cos^2 2\theta \cdot \underline{\cos 2\theta \, d\theta}$

$\frac{1}{2} \int (1 - \sin^2(2\theta)) \, 2\cos 2\theta \, d\theta$

$u = \sin(2\theta)$

$du = 2\underline{\cos(2\theta) \, d\theta}$

$\frac{1}{2} \int (1 - u^2) \, du = \frac{1}{2}\left(u - \frac{u^3}{3}\right) + C$

$\frac{1}{2} \sin(2\theta) - \frac{1}{6} \sin^3(2\theta) + C$

b) $\int \cos^3 \theta \sin^2 \theta \, d\theta$

$\int \underbrace{\cos^2\theta}_{(1-\sin^2\theta)} \sin^2\theta \, \underline{\cos\theta \, d\theta} \rightarrow \int (\sin^2\theta - \sin^4\theta) \cos\theta \, d\theta$

$u = \sin\theta$

$du = \cos\theta \, d\theta$

$\int (u^2 - u^4) \, du = \frac{u^3}{3} - \frac{u^5}{5} +$

$= \frac{1}{3} \sin^3\theta - \frac{1}{5} \sin^5\theta + C$

c) $\int \underline{\sec^4} \theta \tan^2 \theta \, d\theta$

$\int \underbrace{\sec^2\theta}_{(1+\tan^2\theta)} \tan^2\theta \, \underbrace{\sec^2\theta \, d\theta}_{du}$

$\int (\tan^2\theta + \tan^4\theta) \underbrace{\sec^2\theta \, d\theta}_{du} = \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta + C$

$u = \tan\theta$

5. Integrate:

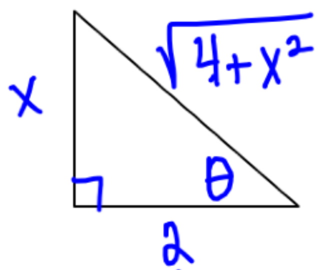
$$-\frac{1}{2}\int \frac{-2x}{\sqrt{4-x^2}}\,dx \qquad -\frac{1}{2}\int \frac{du}{\sqrt{u}} = -\frac{1}{2}\int u^{-1/2}\,du = -\frac{1}{2}\cdot 2u^{1/2} + C$$

$$= \underline{-\sqrt{4-x^2} + C}$$

$u = 4 - x^2$

$du = -2x\,dx$

$$\int \frac{x^2}{\sqrt{4+x^2}}\,dx$$



$x = 2\tan\theta \qquad \left(\frac{x}{2} = \tan\theta\right)$

$dx = 2\sec^2\theta\,d\theta$

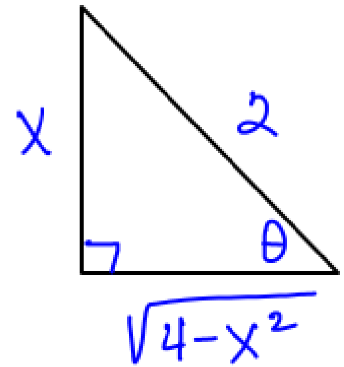
$\sqrt{4+x^2} = 2\sec\theta$

$$\int \frac{(2\tan\theta)^2}{2\sec\theta}\,2\sec^2\theta\,d\theta = \int 4\tan^2\theta\sec\theta\,d\theta = 4\int \frac{\sin^2\theta}{\cos^2\theta}\cdot\frac{1}{\cos\theta}\,d\theta$$

$$\int 4(\sec^2\theta - 1)\sec\theta\,d\theta$$

$$= 4\int \sec^3\theta - \sec\theta\,d\theta$$

$$\int \frac{x}{\sqrt{4-x^2}}\,dx$$



$$\frac{x}{2} = \sin\theta$$

$$x = 2\sin\theta$$

$$dx = 2\cos\theta\, d\theta$$

$$\sqrt{4-x^2} = 2\cos\theta$$

$$\int \frac{2\sin\theta}{2\cos\theta} \cdot 2\cos\theta\, d\theta$$

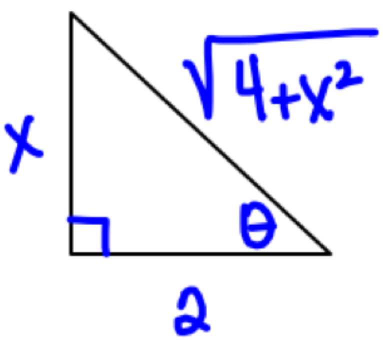
$$\int 2\sin\theta\, d\theta = -2\cos\theta + C$$

$$-2\left(\frac{\sqrt{4-x^2}}{2}\right) + C$$

$$-\sqrt{4-x^2} + C$$

$$4\left(\frac{1}{2}\sec\theta\tan\theta+\frac{1}{2}\ln|\sec\theta+\tan\theta|-\ln|\sec\theta+\tan\theta|\right)+C$$

$$2\sec\theta\tan\theta-2\ln|\sec\theta+\tan\theta|+C$$

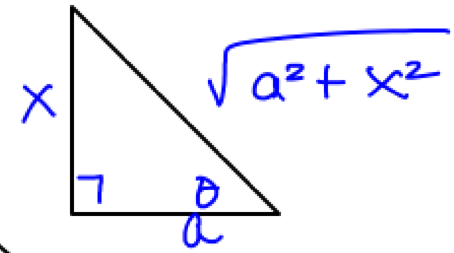


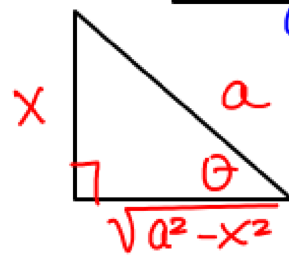
$$2\cdot\left(\frac{\sqrt{4+x^2}}{2}\right)\left(\frac{x}{2}\right)-2\ln\left|\frac{\sqrt{4+x^2}}{2}+\frac{x}{2}\right|+C$$

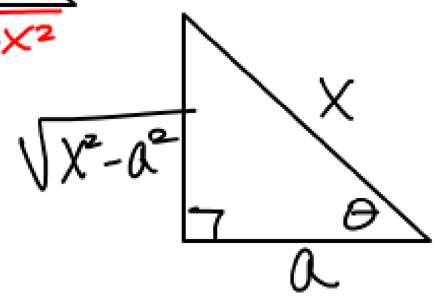
$$\sqrt{a^2+x^2} \qquad x = a\tan\theta$$

$$\sqrt{a^2-x^2} \qquad x = a\,\mathrm{sen}\,\theta$$

$$\sqrt{x^2-a^2} \qquad x = a\sec\theta$$







6. Integrate

PFD Know forms & how to work out.

$$\int \frac{x^2+5x+2}{(x+1)\left(x^2+1\right)}\,dx$$

Form: $\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+1}=\dfrac{x^2+5x+2}{(x+1)(x^2+1)}$

$A(x^2+1)+(Bx+C)(x+1)=x^2+5x+2$

$x=-1:\quad 2A=1-5+2 \rightarrow \underline{A=-1}$

$x=0:\quad \overset{-1}{\cancel{A}}+C=2 \rightarrow \underline{C=3}$

$x=1:\quad 2A+2B+2C=4+10+2 \rightarrow \underline{B=6}$

$-2+2B+6=16$

$$\int \frac{-1}{x+1}+\frac{6x}{x^2+1}+\frac{3}{x^2+1}\,dx$$

$$\boxed{-\ln|x+1|+3\ln(x^2+1)+3\arctan(x)+C}$$

$$\int \frac{2x^2}{(x+1)^2(x-2)}\,dx$$

FORM: $\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}+\dfrac{C}{x-2}$

$A(x+1)(x-2)+B(x-2)+C(x+1)^2=2x^2$

$x=-1:\quad -3B=2 \qquad B=-2/3$

$x=2:\quad 9C=8 \qquad C=8/9$

$x=0:\quad -2A-2B+C=0$

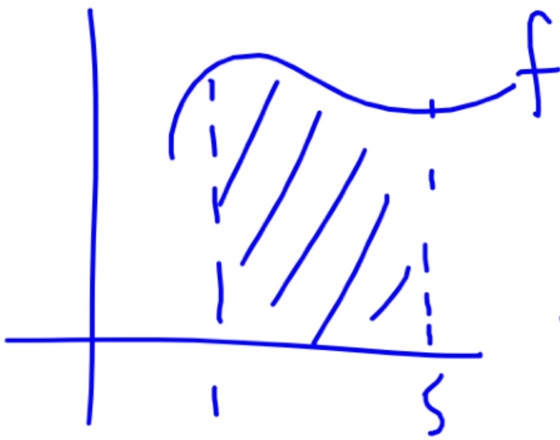
$-2A+4/3+8/9=0$

$-2A=-20/9 \qquad A=10/9$

$$\int \frac{10/9}{x+1}-\frac{2/3}{(x+1)^2}+\frac{8/9}{x-2}\,dx$$

$$\frac{10}{9}\ln|x+1|+\frac{2/3}{x+1}+\frac{8}{9}\ln|x-2|+C$$

7. Let f be a positive function. The area bounded by f(x) and the x-axis from x=1 to x=5 is 21/5. Find the average value of this function.



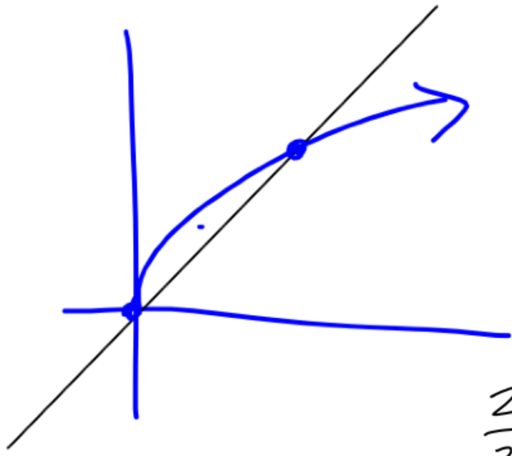
$$\boxed{AV = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

$$\int_1^5 f(x)\,dx = 21/5$$

$$AV = \frac{1}{5-1}\left(\frac{21}{5}\right) = \frac{21}{20}$$

$$\int_a^b f'(x)\,dx = f(b) - f(a) \quad (FTOC)$$

8. Find the area of the region bounded by $f(x)=\sqrt{x}$ and $g(x)=2x$.



$$\int_0^{1/4} (\sqrt{x}-2x)\,dx$$

$$\sqrt{x}=2x$$
$$x=4x^2$$
$$4x^2-x=0$$
$$x(4x-1)=0$$
$$x=0,\ 1/4$$

$$\frac{2}{3}x^{3/2}-x^2\Big|_0^{1/4}$$

$$\frac{2}{3}(1/4)^{3/2}-(1/4)^2-0$$

$$=\frac{1}{12}-\frac{1}{16}$$

$$=\boxed{1/48}$$

b) Find the centroid of the region in (a).

$$\bar{x}=\frac{\int_0^{1/4} x\cdot(\sqrt{x}-2x)\,dx}{1/48}$$

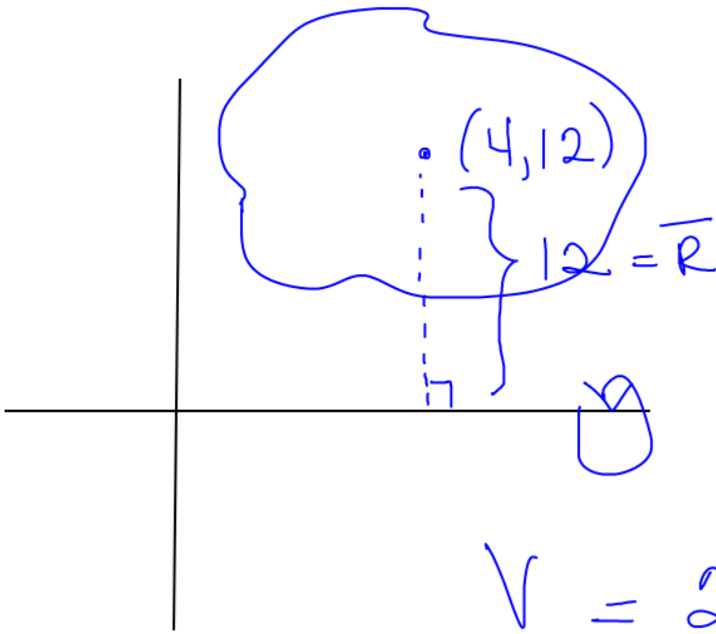
$$\bar{y}=\frac{\int_0^{1/4} \frac{1}{2}\left(\sqrt{x}^{\,2}-(2x)^2\right)dx}{1/48}$$

The centroid $(\bar{x},\bar{y})$ of a region R can be obtained by:

$$\bar{x}=\frac{\int_a^b x[f(x)-g(x)]\,dx}{A} \quad \text{and} \quad \bar{y}=\frac{\int_a^b \frac{1}{2}\left([f(x)]^2-[g(x)]^2\right)dx}{A}$$

Where A is the area of the region.

9. Given that the centroid of a region is (4,12) and its area is 5, find the volume of the solid formed when this region is rotated about the x-axis.



Theorem 7.5.1: Pappus's Theorem on Volumes:
Suppose a solid is created by revolving region Ω in the plane around any axis, such that Ω does not cross this axis. Then the volume of the solid is given by:

$$V = 2\pi \bar{R} A$$

Where $\bar{R}$ is the distance from the centroid of Ω to the axis of revolution and A is the area of the region Ω.

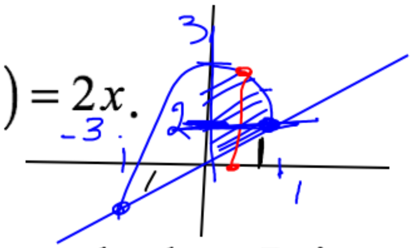
$$V = 2\pi(12)(5) = 120\pi$$

1st Quadrant

$y = 3 - x^2$

$x^2 = 3 - y$

10. Let R be the region bounded by $f(x) = 3 - x^2$ and $g(x) = 2x$.



$3 - x^2 = 2x$

$0 = x^2 + 2x - 3 = (x+3)(x-1)$

Set up the formulas that will give the volume of the solid formed when R is rotated a) about the x-axis.

$$\underbrace{\int_0^1 \pi\left[(3-x^2)^2 - (2x)^2\right]dx}_{\text{disc.}}$$

$$\int_0^3 2\pi y\,[\text{Rt} - \text{left}]\,dy$$

$$\boxed{\int_0^2 2\pi y\left(\tfrac{y}{2}\right)dy + \int_0^3 2\pi y\sqrt{3-y}\,dy}$$

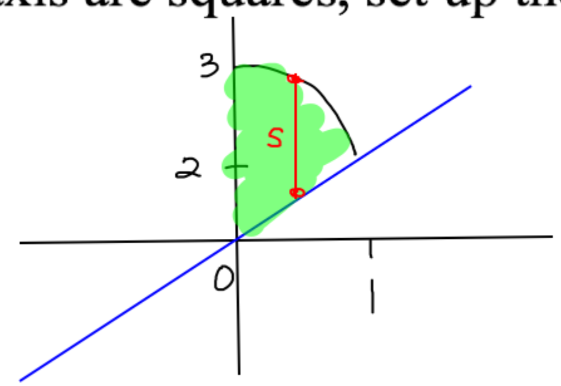
shell

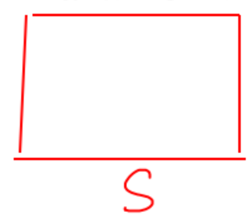
b) about the y-axis.

$$\int_0^2 \pi\left(\tfrac{y}{2}\right)^2 dy + \int_2^3 \pi\left(\sqrt{3-y}\right)^2 dy$$

$$\int_0^1 2\pi x\,(3 - x^2 - 2x)\,dx$$

c) If R is the base of a solid such that the cross sections perpendicular to x-axis are squares, set up the formula for the volume of that solid.





$s = 3 - x^2 - 2x$

$A = s^2 = (3 - x^2 - 2x)^2$

$$V = \int_0^1 (3 - x^2 - 2x)^2\,dx$$

11. Set up the formula that gives the arc length of the following curve:

a) $f(x)=\frac{2}{3}(x-1)^{3/2}, \quad x\in[1,2]$

$$L=\int_1^2\sqrt{1+\left[(x-1)^{1/2}\right]^2}\,dx$$

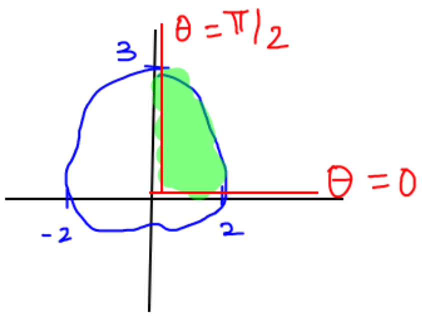
$f'(x)=(x-1)^{1/2}$

$$=\int_1^2\sqrt{x}\,dx$$

b) $r=2+\sin\theta$, in the first quadrant.

$r'=\cos\theta$

$$\int_0^{\pi/2}\sqrt{(2+\sin\theta)^2+\cos^2\theta}\,d\theta$$



12. Solve

$y'=e^{2x}\left(1+y^2\right)$

$$\frac{dy}{dx}=e^{2x}(1+y^2)$$

$$y=\tan\left[\tfrac{1}{2}e^{2x}+C\right]$$

$$\int\frac{dy}{1+y^2}=\int e^{2x}\,dx$$

$$\Rightarrow \arctan(y)=\tfrac{1}{2}e^{2x}+C$$

$$A(t) = A_0 e^{kt}$$

13. Given that 10% of a radioactive substance decays in 5 years, give a formula for the amount of substance in terms of t if the initial amount is 100 grams.

$$90 = 100e^{k(5)}$$
$$.9 = e^{5k}$$
$$\ln(.9) = 5k$$
$$k = \frac{\ln(.9)}{5}$$

$$\underline{\underline{A(t) = 100\, e^{\frac{\ln(.9)}{5} t}}}$$

14. The population of a bacteria culture increases by 20% in 10 hours. What is the doubling time? What is the population in 24 hours if the initial population is 1000?

$$1.2\cancel{x} = \cancel{x} e^{k(10)}$$
$$\ln(1.2) = 10k$$
$$k = \frac{\ln(1.2)}{10}$$
$$A(t) = A_0 e^{\frac{\ln(1.2)}{10} t}$$

$$A(24) = 1000\, e^{\frac{\ln(1.2)}{10}(24)}$$
$$= 1000\, e^{\ln(1.2)^{12/5}}$$
$$= \underline{1000(1.2)^{12/5}}$$

converge if have a finite limit

15. Determine if the following sequences converge or diverge. If they converge, give the limit.

a. $\left\{(-1)^n\left(\frac{n}{n+1}\right)\right\}$ $\longrightarrow$ diverges by oscillation

b. $\left\{\frac{6n^2-2n+1}{4n^2-1}\right\}$ $\rightarrow$ $\frac{6}{4}=\frac{3}{2}$ conv.

c. $\left\{\frac{(n+2)!}{n!}\right\}$ $\frac{(n+2)(n+1)\cancel{n!}}{\cancel{n!}} = \{(n+2)(n+1)\} \longrightarrow \infty$

diverges

d. $\left\{\frac{3}{e^n}\right\}$ conv. to 0

e. $\left\{\frac{4n+1}{n^2-3n}\right\}$ conv. to 0

f. $\left\{\dfrac{e^n}{n^3}\right\}$ faster

diverges

*g. $\left\{\dfrac{2n^2+1}{3n^3+4n^2+6}\right\}$ conv. to 0

*h. $\left\{\dfrac{1}{n\ln(n)}\right\}$ conv. to 0

$\infty \cdot 0$

as $n \to \infty$ $\frac{1}{n} \to 0$

$y = 1/n$

conv. to 1

*i. $\{n\sin(1/n)\}$ $\lim_{n\to\infty} \cancel{n} \cdot \dfrac{\sin(1/n)}{1/n}$ $\dfrac{0}{0}$ $\lim_{y\to 0} \dfrac{\sin y}{y} = 1$

*j. $\left\{\left(\dfrac{n-1}{n}\right)^n\right\}$ $\lim_{n\to\infty} \left(1+\frac{-1}{n}\right)^n = e^{-1}$

conv. to e^{-1}

16. Determine if the following series (A) converge absolutely, (B) converge conditionally or (C) diverge.

alt + $\frac{\sqrt{n}}{n+3} \to 0 \to$ conv. by AST

a. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+3}$ abs: $\sum \frac{\sqrt{n}}{n+3} \sim \sum \frac{1}{\sqrt{n}}$ (limit comp) $\to$ diverges

Conditionally convergent

b. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{n^2} = \sum \frac{(-1)^n}{n^2}$ $\sum \frac{1}{n^2}$ conv.

absolutely convergent

c. $\displaystyle\sum_{n=0}^{\infty} \frac{4n(-1)^n}{3n^2+2n+1}$ Cond. convergent

$\sum \frac{4n}{3n^2+2n+1} \sim \sum \frac{1}{n}$

d. $\displaystyle\sum_{n=0}^{\infty} \frac{3(-1)^n}{\sqrt{3n^2+2n+1}}$ cond. convergent

e. $\displaystyle\sum_{n=0}^{\infty} \frac{3n(-1)^n}{\sqrt{3n^2+2n+1}}$ diverges

$$\frac{3\sqrt{n^2}}{\sqrt{3n^2+2n+1}} \to \frac{3}{\sqrt{3}} \neq 0$$

BDT

f. $\displaystyle\sum_{n=0}^{\infty} \left(4(-1)^n \left(\frac{n}{n+3}\right)^n\right)$

$$\left(\frac{n}{n+3}\right)^n \to 0 \,?$$

$$\left(\frac{n+3}{n}\right)^{-n} = \left(1+\frac{3}{n}\right)^{-n} \to e^{-3} \neq 0$$

div. by BDT

g. $\displaystyle\sum_{n=0}^{\infty}\left(\frac{2(-1)^n \arctan n}{3+n^2+n^3}\right) < \sum \frac{\pi(-1)^n}{3+n^2+n^3}$

$\arctan n \to < \pi/2$

abs. convergent

h. $\displaystyle\sum_{n=0}^{\infty}\left(\frac{(-1)^n 3^n}{4^n+3n}\right)$

$$\boxed{\sum \frac{3^n}{4^n+3n} < \sum \left(\frac{3}{4}\right)^n \text{ Conv.}}$$

abs. convergent

alt + $\dfrac{n!}{(n+1)!} \to 0$

*j. $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n n!}{(n+1)!} = \sum \frac{(-1)^n \cancel{n!}}{(n+1)\cancel{n!}}$

$\sum \frac{1}{n+1}$ div.

Conditionally convergent.

$n^n > n! > x^n > n^x$

$n^{1/n} \to 1$

*k. $\sum_{n=2}^{\infty} \frac{(-1)^n}{3n+2}$

$\sum \frac{1}{3n+2}$ div.

conditionally conv.

*l. $\sum_{n=0}^{\infty} \frac{(-1)^n 10n^2}{3^n}$

$\frac{10n^2}{3^n} \to 0$

$\sum \frac{10n^2}{3^n}$

Root: $\lim_{n\to\infty} \frac{10^{1/n} n^{2/n}}{3} = 1/3 < 1$

conv. by root test

abs. convergent

*m. $\sum_{n=2}^{\infty} \frac{(-1)^n 3^n}{n!}$

$\frac{3^n}{n!} \to 0$

Ratio: $\lim_{n\to\infty} \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n}$

$= \lim_{n\to\infty} \frac{3}{n+1} = 0 < 1$

abs. convergent

*o. $\sum_{n=2}^{\infty} \frac{\cos(\pi n) n^n}{n!}$.

divergent (BDT)

17. Find the sum of the following convergent series:

geom $S = \frac{1\text{st term}}{1 - r}$

$|r| < 1$

a. $\sum_{n=0}^{\infty} 2\left(-\frac{4}{9}\right)^n$. $S = \frac{2(-4/9)^0}{1-(-4/9)} = \frac{2}{13/9}$

$= \boxed{18/13}$

b. $\sum_{n=0}^{\infty}\left(\frac{1}{3^n} - \frac{5}{6^n}\right)$ $= \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n - 5\sum_{n=0}^{\infty}\left(\frac{1}{6}\right)^n$

$\frac{1}{1-1/3} - 5\left(\frac{1}{1-1/6}\right)$

$\frac{3}{2} - 5\left(\frac{6}{5}\right)$

$\frac{3}{2} - 6 = \boxed{-9/2}$

look over w/s posted on Seq + Series

18. Find the radius of convergence and interval of convergence for the following Power series:

a. $\sum_{n=0}^{\infty} \frac{1}{3^n}(x-1)^n$

Root: $\lim_{n\to\infty}\left[\frac{|x-1|^n}{3^n}\right]^{1/n} = \lim_{n\to\infty}\frac{|x-1|}{3} = \frac{|x-1|}{3}$

$x=-2$: $\sum \frac{1}{3^n}(-3)^n = \sum (-1)^n$ div.

$-2 \quad 1 \quad 4$

$\frac{|x-1|}{3} < 1$

$x=4$: $\sum \frac{1}{3^n}3^n = \sum 1$ div.

$\boxed{(-2,4)}$

$|x-1| < \underline{\underline{3}}$ R

b. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{4^n}$

$-4 \quad 0 \quad 4$

Root: $\lim_{n\to\infty}\left[\frac{|x|^n}{4^n}\right]^{1/n} \to \frac{|x|}{4} < 1$

$|x| < \underline{\underline{4}}$

Int. of conv. of $\sum a_n x^n$ is $[-4, 4)$ conv. for any x in R

$x=-4$: $\sum \frac{(-1)^{n+1}(-4)^n}{4^n} = \sum (-1)$

$x=4$: $\sum (-1)^{n+1}$ } div.

$\boxed{(-4,4)}$

19. Give the derivative of each power series below, and for each series, give the antiderivative F of the power series so that F(0)=0.

$$\int \sum \frac{(n+1)x^n}{n^2+2}\,dx = \sum_{n=0}^{\infty} \frac{\cancel{(n+1)}\,x^{n+1}}{(n^2+2)\cancel{(n+1)}} + C$$

$$0 + C = 0$$

a. $\displaystyle\sum_{n=0}^{\infty} \frac{(n+1)x^n}{n^2+2}$

deriv: $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)n \cdot x^{n-1}}{n^2+2}$

Int: $\displaystyle\sum_{n=0}^{\infty} \frac{x^{n+1}}{n^2+2}$

b. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{2n+1}$

deriv: $\displaystyle\sum_{n=1}^{\infty} \frac{n x^n}{2n+1}$

$F(0)=0$

$0 + C = 0$

Int: $\displaystyle\int \sum_{n=0}^{\infty} \frac{x^n}{2n+1}\,dx = \boxed{\sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)(2n+1)}} + \underbrace{C}_{=0}$

20. Determine the convergence or divergence for each series

Series	Converge or Diverge?	Test used
$\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$	div.	p series $p = 3/4$
$\sum_{n=1}^{\infty} \frac{2^n}{n^3}$	div.	BDT
$\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n} \right)$	conv.	telesc.
$\sum_{n=1}^{\infty} \frac{3^{2n}}{n!}$	conv.	Ratio: $\lim_{n\to\infty} \frac{3^{2n+2}}{(n+1)!} \cdot \frac{n!}{3^{2n}}$ $= \lim_{n\to\infty} \frac{9}{n+1} \to 0 < 1$
$\sum_{n=1}^{\infty} \cos(\pi n)$	div	BDT

$\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n}$	div.	p series $p=\frac{1}{2}$
$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}n^2}{3n^3+1}$	conv.	AST
$\sum_{n=0}^{\infty}3\left(-\frac{1}{2}\right)^n$	conv.	geom $r=-\frac{1}{2}$
$\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^2}$	conv	Integral test $\int_2^{\infty}\frac{1}{x(\ln x)^2}dx$
$\sum_{n=1}^{\infty}ne^{-n^3}$	conv.	Root: $\lim_{n\to\infty} n^{1/n}e^{-n^2}\to 0<1$
$\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^n$	div.	BDT

$\sum_{n=1}^{\infty} \frac{1}{n^3+1}$	conv.	$p=3$ Basic comp $\sum \frac{1}{n^3}$
$\sum_{n=0}^{\infty} \left(\frac{2}{9}\right)^n$	conv.	geom $r = 2/9$
$\sum_{n=1}^{\infty} \frac{n^2}{2^n}$	conv.	Root $\lim_{n\to\infty} \frac{n^{2/n}}{2} = \frac{1}{2} < 1$
$\sum_{n=2}^{\infty} \frac{10n^2+n-2}{2n^6+7n-1}$	conv.	limit comp to $\sum \frac{1}{n^4}$
$\sum_{n=1}^{\infty} \frac{n^2+3n-2}{\sqrt{4n^9+n-1}}$	conv.	$\sum \frac{n^2}{n^{9/2}} = \sum \frac{1}{n^{5/2}}$ limit comp

$\rightarrow$ to get x^{13} we need $n=7$

21. Suppose $f(x)=\sum_{n=0}^{\infty}\frac{x^{2n-1}}{(2n)!}$. Give the 13$^{\text{th}}$ derivative of f at x = 0.

$\cdot\ f^{(13)}(0)$

$$\boxed{\frac{f^{(13)}(0)}{13!}x^{13}}$$

$$\frac{x^{13}}{14!}$$

$$\frac{f^{(13)}(0)}{13!}\cancel{x^{13}}=\frac{\cancel{x^{13}}}{14!}$$

$$f^{(13)}(0)=\frac{13!}{14!}=\frac{1}{14}$$

22. Give the Taylor series expansion for $f(x)=e^{-x}$ centered at 0.

$$e^{x}=\sum_{n=0}^{\infty}\frac{1}{n!}x^{n}$$

$$e^{-x}=\sum_{n=0}^{\infty}\frac{1}{n!}(-x)^{n}=\sum_{n=0}^{\infty}\frac{(-1)^{n}x^{n}}{n!}$$

23. Give Taylor series expansion for $f(x)=\ln(x)$ centered at 1.

k	$f^k(x)$	$f^k(1)$	$f^k(1)/k!$	$(x-1)^k$
0	$\ln x$	0	0	
1	$1/x$	1	1	
2	$-1/x^2$	-1	$-1/2$	
3	$2/x^3$	2	$1/3$	
4	$-6/x^4$	-6	$-1/4$	

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}(x-1)^k$$

24. Give Taylor series expansion for $f(x)=\sin(3x)$ centered at 0.

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = \frac{x}{1!} - \frac{x^3}{3!} + $$

$$\sin(3x) = \sum_{n=0}^{\infty} \frac{(-1)^n (3x)^{2n+1}}{(2n+1)!}$$

$$\int \cos(x^2)\,dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n (x^2)^{2n}}{(2n)!}\,dx$$

Taylor Series of the Exponential $f(x) = e^x$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \quad \text{for all real } x$$

Taylor Series of the Sine $f(x) = \sin x$

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \quad \text{for all real } x$$

Taylor Series of the Cosine $f(x) = \cos x$

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \quad \text{for all real } x$$

Taylor Series of the Logarithm $f(x) = \ln(1+x)$

$$\ln(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} x^k = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \quad \text{for } -1 < x \leq 1$$

$$f^{(k)}(c)$$

$$\underbrace{k=0 \qquad k=1 \qquad k=2}$$

25. $f(1)=-1$, $f'(1)=2$, $f''(1)=-1$. Give the 2^{nd} degree Taylor polynomial for f centered at 1.

$$\frac{f^{(k)}(c)}{k!}(x-c)^k$$

$$\frac{-1}{0!}(x-1)^0+\frac{2}{1!}(x-1)^1+\frac{-1}{2!}(x-1)^2$$

$$\boxed{-1+2(x-1)-\frac{1}{2}(x-1)^2}$$

$$\frac{f^{(n+1)}(c)}{(n+1)!}(x-c)^{n+1} \le \frac{M}{(n+1)!}(x-c)^{n+1}$$

26. Give a value of n so that the Taylor polynomial of degree n for $f(x)=\sin(x)$ centered at 0 can be used to approximate $f(\underline{0.5})$ within 10^{-4}.

x

$\left(\frac{1}{10000}\right)$

$5! = 120$

$6! = 720$

$7! = 5040$

$$\max |f^{(k)}(c)| \le 1$$

$$\frac{1}{(n+1)!}x^{n+1} < \frac{1}{10000}$$

$n+1=6$

$\boxed{n=5}$

$$\frac{1}{(n+1)!}\left(\frac{1}{2}\right)^{n+1} < \frac{1}{10000}$$

$$\frac{1}{5!}\cdot\frac{1}{2^5}\ \frac{1}{(120)(32)} = \frac{1}{3840}$$

$$\frac{1}{7!}\left(\frac{1}{2}\right)^7 = \frac{1}{(5040)(128)}$$

$$\frac{1}{6!}\cdot\frac{1}{2^6} = \frac{1}{(720)(64)}$$

$$\frac{1}{46080}$$

$$f(x) = \ln(x+1)$$

$$f'(x) = \frac{1}{x+1}$$

$$f''(x) = \frac{-1}{(x+1)^2}$$

$$f'''(x) = \frac{2}{(x+1)^3}$$

$$f^{(4)}(x) = \frac{-6}{(x+1)^4}$$

$$\vdots$$

$$f^{(n)}(x) = \frac{(-1)^{n-1}(n-1)!}{(x+1)^n}$$

max $f^{(n+1)}(x) = n!$

$$\frac{(n+1-1)!}{(x+1)^{n+1}} = \frac{n!}{(x+1)^{n+1}}$$

$x=0$

Know – Graphing polar curves. Converting polar form to rectangular form and vice versa.

$x = r\cos\theta,\ y = r\sin\theta,\ x^2+y^2=r^2$

Set up the area:

a) Inside one petal of $r = 2\sin 4\theta$.

$$\int_0^{\pi/4} \frac{1}{2}(2\sin 4\theta)^2\, d\theta$$

$0 = 2\sin 4\theta \quad 4\theta = 0, \pi, 2\pi, \ldots$

$\sin 4\theta = 0 \quad \theta = \underbrace{0, \pi/4}, \pi/2, \ldots$

b) Inside one petal of $r = 4\cos 3\theta$.

$0 = \cos 3\theta$

$3\theta = \pi/2, 3\pi/2, \ldots \quad \theta = \underbrace{\pi/6, \pi/2}, \ldots$

$$\int_{\pi/6}^{\pi/2} \frac{1}{2}(4\cos 3\theta)^2\, d\theta$$

c) Inside the inner loop of $r = 1+2\sin\theta$

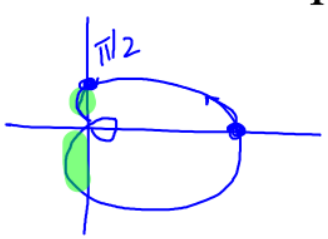
$0 = 1+2\sin\theta$

$\sin\theta = -1/2$

$\theta = 7\pi/6, 11\pi/6$

$$\int_{7\pi/6}^{11\pi/6} \frac{1}{2}(1+2\sin\theta)^2\, d\theta$$

d) Inside the outer loop and to the left of the y-axis, $r = 4+8\cos\theta$



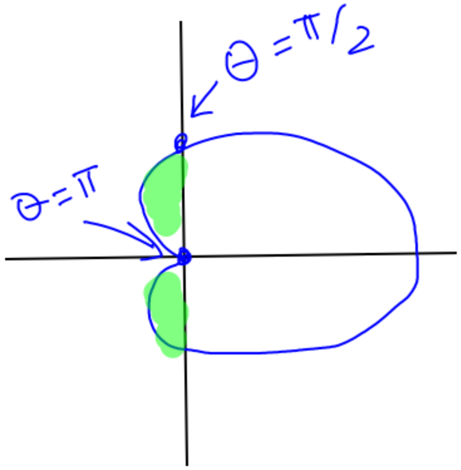
$0 = 4+8\cos\theta$

$\cos\theta = -1/2$

$\theta = \underline{2\pi/3}, 4\pi/3$

$$\boxed{2\int_{\pi/2}^{2\pi/3} \frac{1}{2}(4+8\cos\theta)^2\, d\theta}$$

e) Inside the curve and to left of the y-axis, $r = 4 + 4\cos\theta$.



$$2\int_{\pi/2}^{\pi} \frac{1}{2}(4+4\cos\theta)^2\, d\theta$$

27. Find the equation of the tangent and the normal lines to the parametric curves at the given points:

$3 = 3\cos(3t) + 2t$ $\quad$ $\boxed{t=0}$ $\quad$ $1 = 1 + 5t$ $\qquad$ $m = \left.\frac{dy}{dx}\right|_{pt} = \frac{y'(t)}{x'(t)}$

$x(t) = 3\cos(3t) + 2t,\ y(t) = 1 + 5t,$ at $(3,1)$

$x'(t) = -9\sin(3t) + 2 \qquad y' = 5$

$m = \frac{5}{2}$ $\qquad$ $y - 1 = \frac{5}{2}(x-3)$ tangent $\qquad$ $y - 1 = -\frac{2}{5}(x-3)$ normal

28. Find the point(s) where the curve has (a) horizontal (b) vertical tangent lines.

$y'(t)=0$ $x'(t)=0$

$$x(t)=t^2+2t, \quad y(t)=4t^2+t$$

$x'(t)=2t+2$

$0=2t+2$

$t=-1$

vert $\boxed{(-1,3)}$

$y'(t)=8t+1$

$8t+1=0$

$t=-1/8$

horiz. $\boxed{\left(\frac{-15}{64}, -\frac{1}{16}\right)}$

29. Give an equation relating x and y for the curve given parametrically by

a. $x(t)=-1+3\cos t \quad y(t)=1+2\sin t$

$\cos^2 t+\sin^2 t=1$

$$\boxed{\left(\frac{x+1}{3}\right)^2+\left(\frac{y-1}{2}\right)^2=1}$$

c. $x(t) = -1 + 4e^t \quad y(t) = 2 + 3e^{-t} = 2 + 3(\underline{e^t})^{-1}$

$e^t = \boxed{\dfrac{x+1}{4}}$

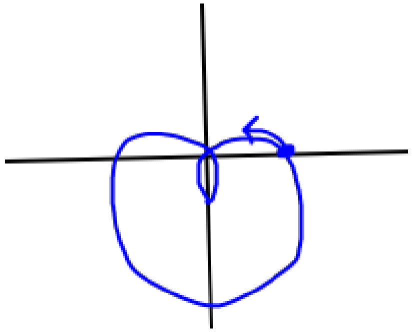
$$y = 2 + 3\left(\frac{x+1}{4}\right)^{-1} = 2 + \frac{12}{x+1}$$

30. Give a parameterization for the line segment from the point (1, 6) to the point (–3, 1).

$$x(t) = x_0 + (x_1 - x_0)t \qquad 0 \le t \le 1$$
$$y(t) = y_0 + (y_1 - y_0)t$$

$$\boxed{\begin{gathered} x(t) = 1 - 4t \\ y(t) = 6 - 5t \\ 0 \le t \le 1 \end{gathered}}$$

$r = 2 - 4\sin\theta$ area of inner loop



$0 = 2 - 4\sin\theta$

$\sin\theta = 1/2$

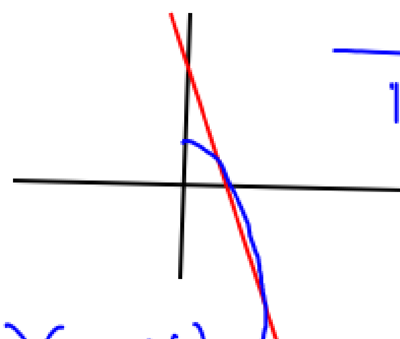
$\theta = \pi/6, 5\pi/6$

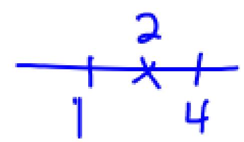
$$\int_{\pi/6}^{5\pi/6} \frac{1}{2}(2-4\sin\theta)^2\, d\theta$$

$$\int_0^a 9x\sqrt{a^2 - x^2}\, dx$$ u-sub $u = a^2 - x^2$

-2 * top $\qquad -4$

Area: $y = 2 - x^2 \qquad y = 6 - 5x$





$2 - x^2 = 6 - 5x$

$0 = x^2 - 5x + 4 = (x-1)(x-4)$

$$\int_1^4 2 - x^2 - (6 - 5x)\, dx$$

$$\frac{dy}{dx} = \frac{x-1}{y-1}$$

$$\int (y-1)\,dy = \int (x-1)\,dx$$

$$\frac{y^2}{2} - y = \frac{x^2}{2} - x + C$$

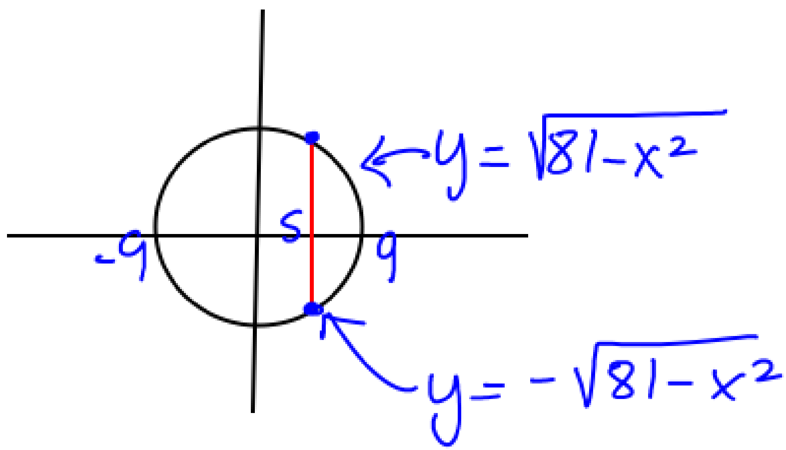
$$\int 3 \arctan x \, dx$$

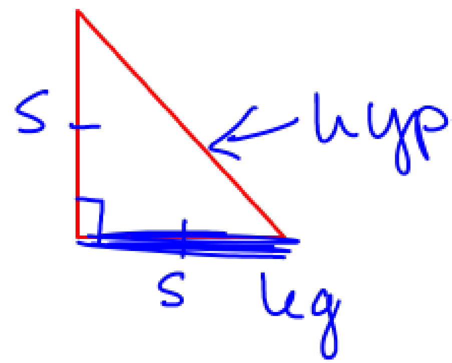
$u = 3\arctan x \qquad dv = dx$

$du = \frac{3}{1+x^2}\,dx \qquad v = x$

$$3x \arctan x - \underbrace{\int \frac{3x}{1+x^2}\,dx}_{\text{u-sub}}$$

Area of isos. rt Δ





$A = \frac{1}{2} s^2$

$$s = \sqrt{81-x^2} - \left(-\sqrt{81-x^2}\right)$$
$$= 2\sqrt{81-x^2}$$

$$V = \int_{-9}^{9} \frac{1}{2}\left(2\sqrt{81-x^2}\right)^2 dx$$

$$\int 2\sec^6(7x)\,dx = \int 2\underbrace{\sec^4(7x)}_{(1+\tan^2(7x))^2}\sec^2(7x)\,dx$$

$$= \int \frac{2}{7}\left(1 + 2\tan^2(7x) + \tan^4(7x)\right)\underline{7\sec^2(7x)\,dx}$$

$$u = \tan(7x)$$
$$du = 7\sec^2(7x)\,dx$$

$$\frac{2}{7}\int (1 + 2u^2 + u^4)\,du$$

$$\frac{2}{7}\left(u + \frac{2}{3}u^3 + \frac{1}{5}u^5\right) + C$$

$$\frac{2}{7}\tan(7x) + \frac{4}{21}\tan^3(7x) + \frac{2}{35}\tan^5(7x) + C$$

seq. $a_n = \dfrac{(4n+1)^2}{(6n-1)^2} = \dfrac{(16n^2+8n+1)}{(36n^2-12n+1)} \to \dfrac{16}{36}$

conv to $\dfrac{4}{9}$

Sum: $\displaystyle\sum_{k=0}^{\infty} \frac{1-4^k}{6^k} = \sum_{k=0}^{\infty} \left(\frac{1}{6}\right)^k - \sum_{k=0}^{\infty} \left(\frac{4}{6}\right)^k$

$S = \dfrac{1st}{1-r}$

Dr. Almus will have review tonight @ 6pm – see her webpage