6 m/c @ 7 pts each
4 f/R 2 @ 10 pts
1 @ 15 pts
1 @ 23 pts

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total: 100pts

1432 Test 2

50 min
no calculator

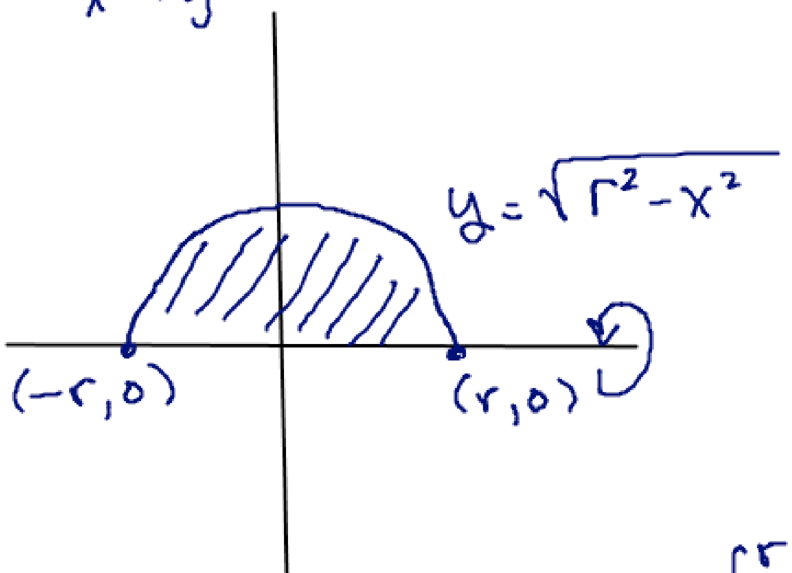
7.1 - 7.7

# Review Sheet

8. Derive the formula $V = \frac{4}{3}\pi r^3$ for the volume of a sphere of radius $r$ by revolving the region bounded by a circle of radius $r$, centered at the origin, around either the $x$ axis or the $y$ axis.

9. R is the region bounded by the given graphs and the given axis. Sketch each graph then find the area of

$x^2+y^2=r^2$




$$V = \int_{-r}^{r} \pi \left(\sqrt{r^2-x^2}\right)^2 dx$$

$$= \int_{-r}^{r} \pi \left(r^2-x^2\right) dx$$

$$= \pi\left(r^2x - \frac{x^3}{3}\right)\Big|_{-r}^{r}$$

$$= \pi\left[\left(r^2\cdot r - \frac{r^3}{3}\right) - \left(r^2(-r) - \frac{(-r)^3}{3}\right)\right] = \frac{4}{3}\pi r^3$$

3. Find the area for each region:

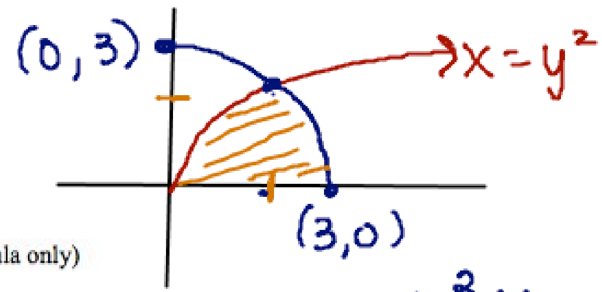
   a. between $f(x)=x^3-x^2$ and the $x$-axis on the interval [0,2].

   b. between $y=x^2-1$ and $y=3$

   c. between $f(x)=\sqrt{x}$ and $g(x)=\frac{x}{a}$

   d. between $x^2+y^2=9$, $x=y^2$ and the x-axis in the first quadrant (set up the formula only)

d.




$x^2+x=9$

$x^2+x-9=0$

$x=\frac{-1\pm\sqrt{1+36}}{2}$

$=\frac{-1\pm\sqrt{37}}{2}$

$A=\int_0^{\frac{-1+\sqrt{37}}{2}}\sqrt{x}\,dx$

$+\int_{\frac{-1+\sqrt{37}}{2}}^{3}\sqrt{9-x^2}\,dx$

OR find y for pt.

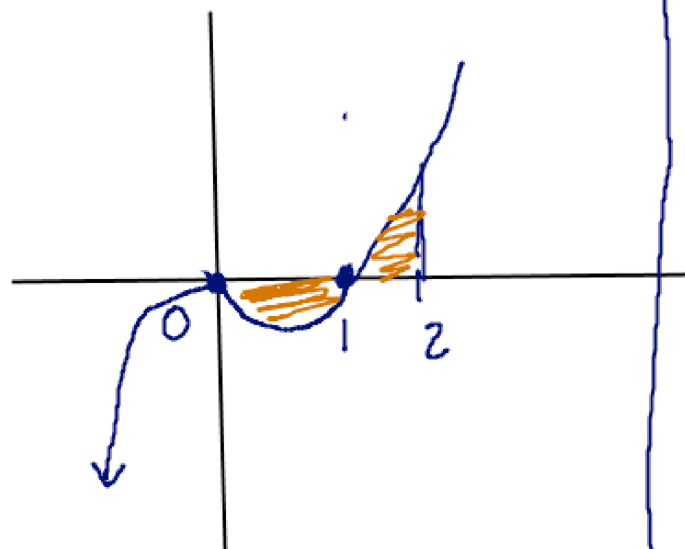
$\int_0^{pt.}\sqrt{9-y^2}-y^2\,dy$

a. $f(x)=x^3-x^2$ & x-axis [0,2]

$x^3-x^2=0$

$x^2(x-1)=0$

$x=0,1$




$A=\int_0^1 0-(x^3-x^2)\,dx$

$+\int_1^2 (x^3-x^2)-0\,dx$

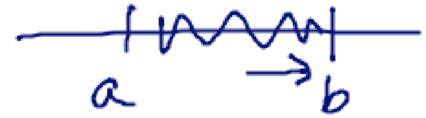
or $A=\left|\int_0^1 (x^3-x^2)\,dx\right|+\int_1^2 (x^3-x^2)\,dx$

Public Page 2

19.

c. $\int_0^4 \frac{1}{\sqrt{4-x}}\,dx$ imp. @ $x = 4$

d. $\int_e^\infty \frac{\ln x}{x}\,dx$ — imp @ $\infty$




c. $$\lim_{b\to 4^-}\int_0^b (4-x)^{-1/2}\,dx = \lim_{b\to 4^-}\left[-2(4-x)^{1/2}\right]_0^b$$

$$= \lim_{b\to 4^-}\left[-2(4-b)^{1/2} + 2(4)^{1/2}\right] = 2\cdot 2 = 4$$

d. $$\lim_{b\to\infty}\int_e^b \frac{\ln x}{x}\,dx$$

$u = \ln x \quad du = \frac{1}{x}\,dx$

$\int u\,du$

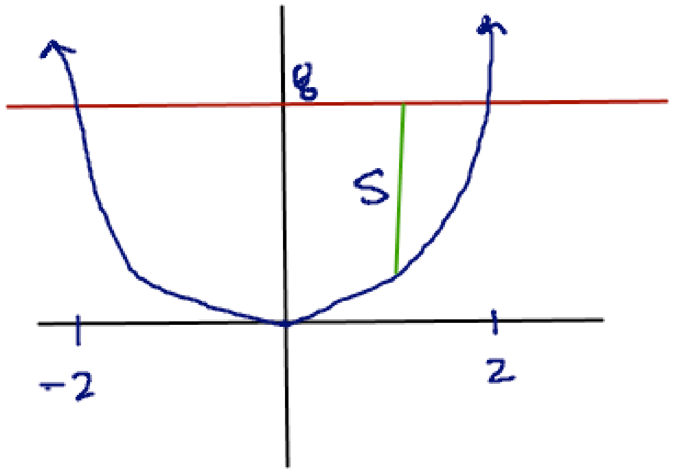
$$= \lim_{b\to\infty}\left[\frac{(\ln x)^2}{2}\right]_e^b = \lim_{b\to\infty}\left[\underbrace{\frac{(\ln b)^2}{2}}_{\to\infty} - \frac{(\ln e)^2}{2}\right]$$

diverges

12. The base of a solid is the region bounded by $y = 2x^2$ and $y = 8$. Find the volume of the solid given that the cross sections perpendicular to the $x$ – axis are:

a. Squares
b. Semicircles
c. Right triangles with leg on the xy-plane.

Isosc.

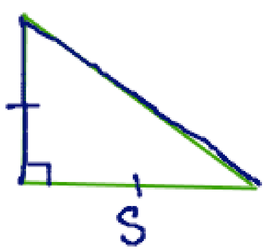



$$s = 8 - 2x^2$$

a) $A = s^2 = (8-2x^2)^2$

b) $A = \frac{\pi r^2}{2} = \frac{\pi}{2}\left(\frac{s}{2}\right)^2 = \frac{\pi}{8} s^2$

$= \frac{\pi}{8}(8-2x^2)^2$




c) $A = \frac{1}{2} s^2 = \frac{1}{2}(8-2x^2)^2$

$$V = \int_{-2}^{2} A \, dx$$

16. Suppose that the population of Zeegers grows at a rate proportional to itself, doubling every 12500 years. When the Zeeger population has reached 93 percent more than their current population, they plan to invade Earth. How many years will it be before the Zeegers attack Earth?

$$2A_0 = A_0 \, e^{K(12500)}$$

$$2 = e^{12500K}$$

$$\ln 2 = 12500K$$

$$K = \frac{\ln 2}{12500} \leftarrow$$

$$A(t) = A_0 \, e^{\left(\frac{\ln 2}{12500}\right)t} \leftarrow$$

$$1.93 \cancel{A_0} = \cancel{A_0} \, e^{\left(\frac{\ln 2}{12500}\right)t}$$

$$\ln(1.93) = \left(\frac{\ln 2}{12500}\right)t$$

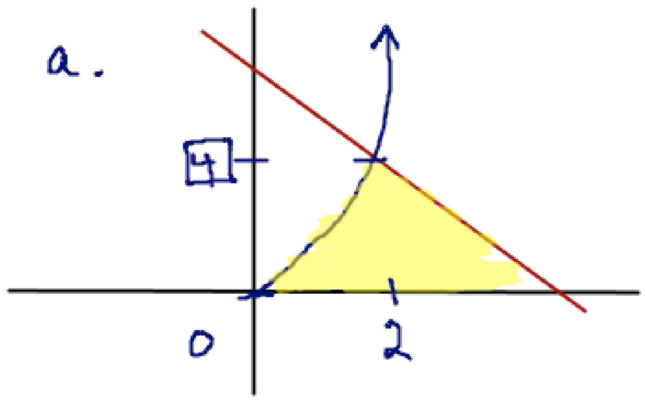
$$\boxed{\frac{12500 \ln(1.93)}{\ln 2}} \text{ yrs} \approx 11857.5 \text{ yrs} \leftarrow$$

9. R is the region bounded by the given graphs and the given axis. Sketch each graph then find the area of R, the volume when R is revolved about the x-axis and the volume when R is revolved about the y-axis

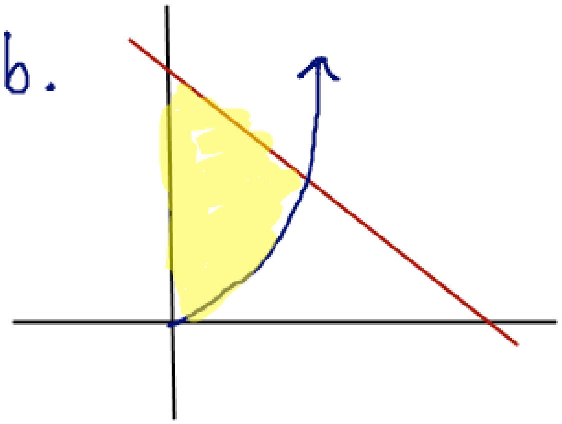
a. $y = x^2$, $y = 6 - x$, $x - axis$

b. $y = x^2$, $y = 6 - x$, $y - axis$

a.




b.



$$V_x(\text{shell}) = \int_0^{\boxed{\ }} 2\pi y\,(\text{Rt} - \text{left})\,dy$$

$$x^2 = 6 - x$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$x = -3, 2$$

$$V_x = \int_0^4 2\pi y\,(6 - y - \sqrt{y})\,dy$$

$$V_x(\text{disk}) = \int_0^2 \pi (x^2)^2\,dx + \int_2^6 \pi (6-x)^2\,dx$$

10b. $f(x) = \cosh(3x) \quad [0, \ln 2]$

$f'(x) = 3\sinh(3x)$

$$L = \int_0^{\ln 2} \sqrt{1 + (3\sinh(3x))^2}\, dx$$

---

13. b. $y' = e^x / y$

$$\frac{dy}{dx} = \frac{e^x}{y}$$

$$\int y\, dy = \int e^x dx$$

$$\frac{y^2}{2} = e^x + C$$

$$y^2 = 2e^x + C$$

c. $y' = y(x-1)$

$$\frac{dy}{dx} = y(x-1)$$

$$\int \frac{1}{y}\, dy = \int (x-1)\, dx$$

$$\ln|y| = \frac{x^2}{2} - x + C$$

d. $\frac{dy}{dx} = x^2 \sec y$

$\int \cos y \, dy = \int x^2 \, dx$

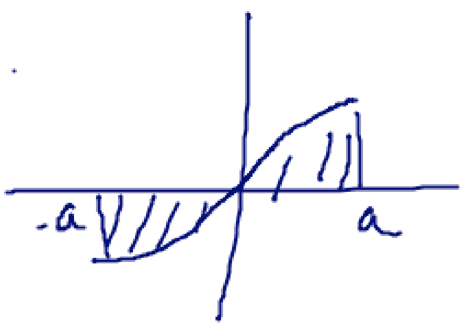
$\sin y = \frac{x^3}{3} + C$

e. $\frac{dy}{dx} = e^{2x}(1+y^2)$

$\int \frac{1}{1+y^2} dy = \int e^{2x} dx$

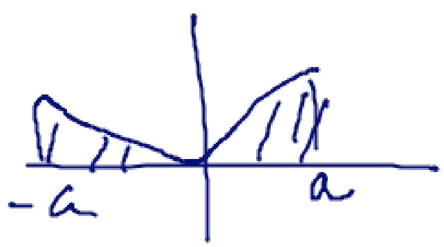
$\arctan y = \frac{1}{2} e^{2x} + C$

1d.




$\int_{-a}^{a} f(x) \, dx = 0$

even:




$\int_{-a}^{a} f(x) \, dx = 2\int_{0}^{a} f(x) dx$

Quiz 7

5) A particle moves along a line with acceleration $a(t) = -\frac{1}{(t+3)^2}$ ft/sec$^2$. Find the distance traveled by the particle during the time interval [0, 1], given that the initial velocity $v(0)$ is $\frac{1}{3}$ ft/sec.

$a(t) = v'(t)$

$v(t) = y'(t)$

dist along $y(t)$

$\int_a^b v(t)\,dt$

$a(t) = -(t+3)^{-2}$

$v(t) = \int -(t+3)^{-2}\,dt = (t+3)^{-1} + C$

$v(0) = \frac{1}{3}$ $\qquad \frac{1}{0+3} + C = 1/3 \Rightarrow C = 0$

$v(t) = \frac{1}{t+3}$

$L = \int_0^1 \frac{1}{t+3}\,dt = \ln|t+3|\Big|_0^1 = \ln 4 - \ln 3$

$2\ln 2 - \ln 3$

10) Calculate the given integral: $\int_{5}^{7} \frac{1}{4+(x-5)^2}\,dx$

$a^2 = 4 \qquad u = x-5$

$a = 2 \qquad du = dx$

$\frac{1}{2}\arctan\left(\frac{x-5}{2}\right)\Big|_5^7$

---

: $\int_{0}^{\ln(2)} \frac{e^{3x}}{1+e^{6x}}\,dx$

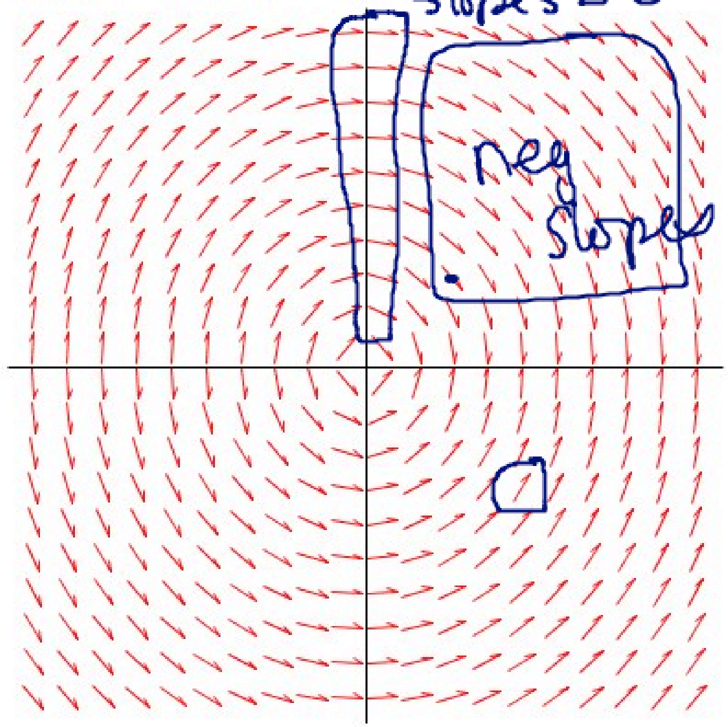
$u = e^{3x}$

$du = 3e^{3x}\,dx$

$\frac{1}{3}\arctan\frac{e^{3x}}{1}\Big|_0^{\ln 2}$

Q8 #5

Which of the following differential equations has the following slope field?




$\frac{dy}{dx} = 5y + 3$ X

$\frac{dy}{dx} = \frac{5x}{y}$

) $\frac{dy}{dx} = -\frac{5x}{y}$

$\frac{dy}{dx} = \frac{1}{2}x + 3$ X

$\frac{dy}{dx} = 5x - y$ X

$\frac{dy}{dx}$ = slope of tangent line

$(x=1,\ y=2)$

$\left.\frac{dy}{dx}\right|_{x=1,\,y=2} = 13$

#11

The half-life of radium-226 is 1620 years. How long will it take for the original amount to be reduced by 60%?

$$K = \frac{\ln(1/2)}{1620}$$

40% rem.

$$.4 \cancel{A_0} = \cancel{A_0} e^{\frac{\ln(1/2)}{1620} t}$$

4) Which of the following functions is a solution to the differential equation: $-2y' + 6x^2 = 0$

$$-2y' = -6x^2$$

$$2\frac{dy}{dx} = 6x^2$$

$$\int dy = \int 3x^2\,dx$$

$$y = x^3 + C$$

any const.

10) The rate of decay of a radioactive substance is proportional to the amount of substance present. What is the half-life of a radioactive substance if it takes 11 years for one-third of the substance to decay?

$$A(t) = A_0 e^{Kt}$$

$$\frac{2}{3}\cancel{A_0} = \cancel{A_0} e^{K(11)}$$

$$\ln(2/3) = 11K$$

$$K = \frac{\ln(2/3)}{11}$$

$$\frac{1}{2}\cancel{A_0} = \cancel{A_0} e^{\left(\frac{\ln 2/3}{11}\right)t}$$

$$\ln(1/2) = \frac{\ln(2/3)}{11} t$$

$$\frac{11 \ln(1/2)}{\ln(2/3)} = t$$

18.8 yrs.

Q8

8) According to the Bureau of the Census, the population of the United States in 1990 was approximately 249 million and in 2000, 281 million. Use this information to predict the population for 2028.

$t=0$

$A_0 = 249\text{ m.}$

$t=10$ $t=38$

$$281 = 249\,e^{k(10)}$$

solve for k

$$A(t) = 249\,e^{(\quad)t}$$

$$A(38)$$

7.2

$v_0 = 0$ $\quad$ $v(4)$ $\quad$ av. vel. $t=0$ to $t=4$

30. A stone falls from rest for 4 seconds. Compare its terminal velocity to its average velocity. Compare its average velocity during the first 2 seconds to its average velocity during the next 2 seconds.

$$y(t) = -\tfrac{1}{2} g t^2 + v_0 t + y_0$$

$g$: 9.8 m/s

$$y'(t) = v(t) = -9.8t + v_0 = -9.8t$$

$$v(t) = -9.8t$$

avg. velocity during 1st 2 sec?

$t = 0$ to $t = 2$

$$AV = \frac{1}{2-0} \int_0^2 -9.8t \, dt$$

next 2 sec.

$t = 2$ to $t = 4$

Sec 3.1

# 7.5

10. $h(x) = \ln(\cos x)$, $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

$$h'(x) = \frac{-\sin x}{\cos x} = -\tan x$$

$$L = \int_{\pi/6}^{\pi/3} \sqrt{1 + (-\tan x)^2}\, dx$$

$$1 + \tan^2 x = \sec^2 x$$

$$+ \int \sec x \, dx = \ln|\sec x + \tan x| + C$$

Q9 #7

$$\int_{-\infty}^{\infty} \frac{7}{x^2}\,dx = \int_{-\infty}^{0} \frac{7}{x^2}\,dx + \int_{0}^{\infty} \frac{7}{x^2}\,dx$$

also imp. at $x = 0$

$$\int_0^1 \frac{7}{x^2}\,dx + \int_1^{\infty} \frac{7}{x^2}\,dx$$

$$\lim_{a\to 0+} \int_a^1 7x^{-2}\,dx + \lim_{b\to\infty} \int_1^b 7x^{-2}\,dx$$

$$= \lim_{a\to 0+} \left[\frac{-7}{x}\right]_a^1 + \lim_{b\to\infty} \left[\frac{-7}{x}\right]_1^b$$

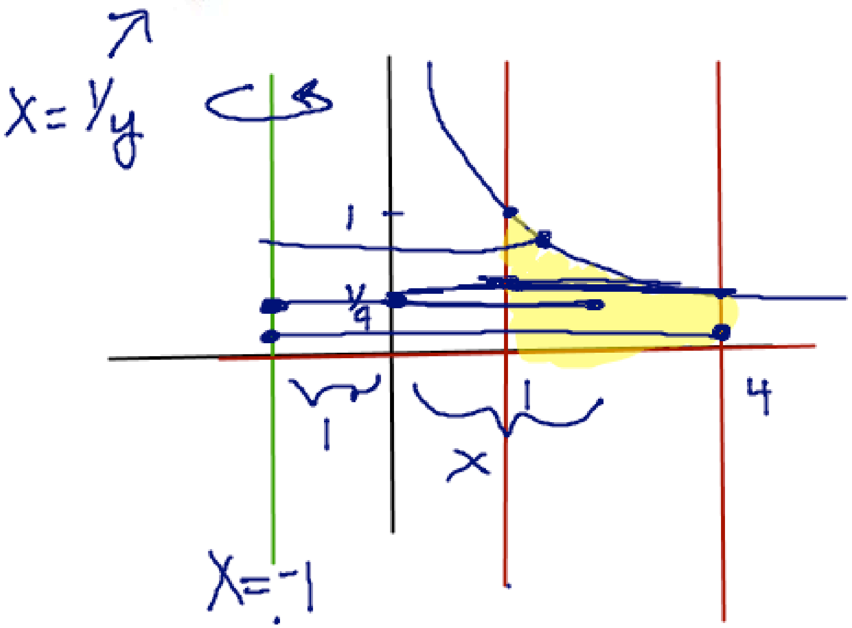
$$= \lim_{a\to 0+} \left[-7 - \frac{-7}{a}\right] + \lim_{b\to\infty} \left[\frac{-7}{\boxed{b}} - -7\right]$$

0

7

diverges

48. $y = \frac{1}{x}$, $y = 0$, $x = 1$, $x = 4$; about $x = -1$.




Shell: $\displaystyle\int_1^4 2\pi\overbrace{(x - -1)}^{\text{dist from axis to any pt in region}}(\text{top} - \text{bottom})\,dx$

Washer: $\displaystyle\int_0^{1/4} \pi\left[R^2 - r^2\right] dy + \int_{1/4}^{1} \pi\left[R^2 - r^2\right] dy$

$R = 5$ $\quad 2 = r$

$R = 1/y - (-1)$ $\quad r = 2$