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Office Hours (starting next Monday):

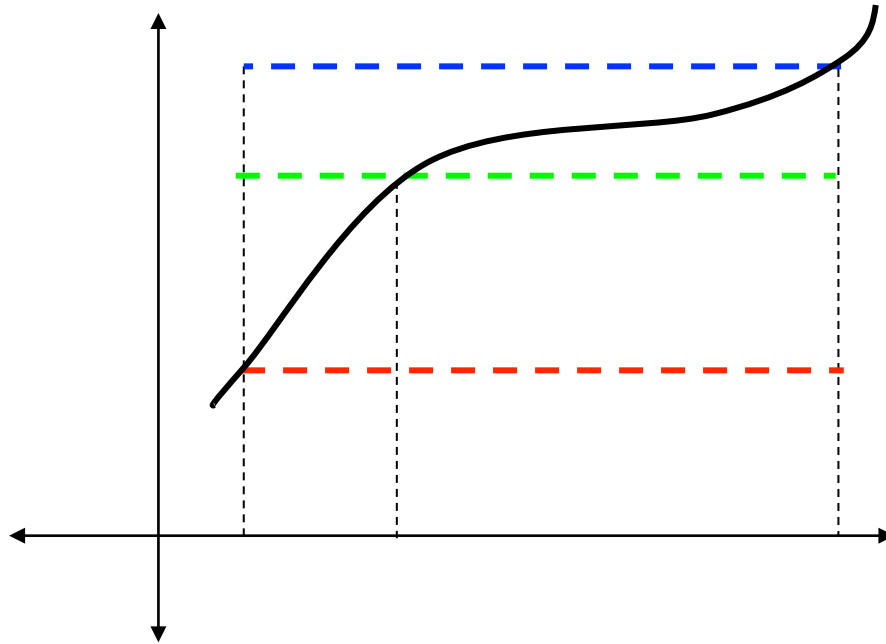
Mondays 1-2pm,
Fridays noon-1pm
(also available by appointment)

Class webpage:

<http://www.math.uh.edu/~bekki/Math1432.html>

Section 7.2

Average Value (Mean Value Theorem for Integrals)



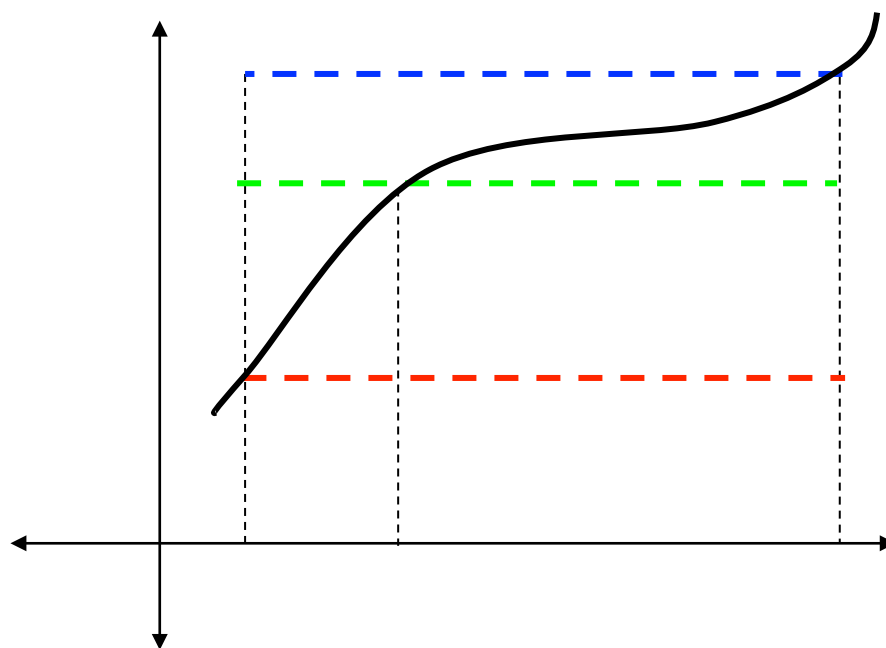
First Mean Value Theorem for Integrals:

If f is continuous on $[a, b]$, then there is at least one number c in (a, b) for which

$$\int_a^b f(x) dx = f(c)(b - a)$$

The number $f(c)$ is called the average (mean) value of f on $[a, b]$.

The area of the region under the graph of f is equal to the area of the rectangle whose height is the average value.



So....

If f is integrable on $[a, b]$, then the average value of f on the interval is:

$$\text{Average value} = f(c) = \frac{1}{(b-a)} \int_a^b f(x) dx$$

1. Find the average value of the function over the interval and find the value(s) of x (the value(s) of c) in the interval for which the function equals its average value:

$$f(x) = x^2 - 2 \quad [0, 2]$$

2. Find the average value of the function over the interval.

$$f(x) = 2x^3 + 3x^2 \quad [1, 4]$$

3. The average value of $\cos x$ over the interval $\frac{\pi}{6} \leq x \leq \frac{\pi}{2}$ is

4. Find the average value of $y = 4t^3 - 3t^2$ over $-1 \leq t \leq 2$.

5. Find the average value: $f(x) = e^x - \sin x$, $x \in \left[0, \frac{\pi}{2}\right]$

6. Given that the average value of an **even** function $f(x)$ over the interval $[-2,2]$ is

3, find $\int_0^2 f(x) dx$.

7. Suppose f is an **odd** function with $\int_0^2 f(x) dx = 3$. Give the average value over the interval $[-2, 2]$.

8. Suppose that $f(1)=6$ and that $f'(x) = x + 1$. Find $f(3)$.

9. What is the approximate instantaneous rate of change for

$$f(t) = \int_0^{2t} x \sin x \, dx \text{ at } t = \frac{\pi}{3} ?$$

10. For what values of k is the following equation true?

$$\int_{-1}^k 4x \, dx = 0$$

11. The function f is differentiable and $\int_0^x (f(t) + 3t) dt = \cos(x)$. Determine the value of $f'\left(\frac{\pi}{3}\right)$