

Math 1432

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Office Hours:

Mondays 1-2pm,
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(also available by appointment)

Class webpage:

<http://www.math.uh.edu/~bekki/Math1432.html>

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1. Compute $\lim_{x \rightarrow 0} \frac{x - \sin x}{x - \tan x}$.
2. Compute $\lim_{x \rightarrow 0} \frac{e^x - e^{-2x}}{2 \sin x}$.
3. Compute $\lim_{x \rightarrow \infty} (\arctan(x))$.

Improper Integrals

The definition of the definite integral $\int_a^b f(x) dx$ requires that $[a, b]$ be finite and that $f(x)$ be bounded on $[a, b]$.

Also, the Fundamental Theorem of Calculus requires that f be continuous on $[a, b]$.

If one or both of the limits of integration are infinite or if f has a finite number of infinite discontinuities on $[a, b]$, then the integral is called an improper integral.

Types of improper integrals:

A. (one or both bounds are infinite)

$\int_1^{\infty} \frac{dx}{x}$, $\int_{-\infty}^1 \frac{3dx}{x^4 + 5}$ and $\int_{-\infty}^{\infty} \frac{dx}{x^2 + 1}$ are improper because one or both bounds are infinite.

B. (infinite discontinuity at a boundary)

$\int_1^5 \frac{dx}{\sqrt{x-1}}$ is improper because $f(x) = \frac{1}{\sqrt{x-1}}$ has an infinite discontinuity at $x = 1$.

C. (infinite discontinuity in the interior)

$\int_{-2}^2 \frac{dx}{(x+1)^2}$ is improper because $f(x) = \frac{1}{(x+1)^2}$ has an infinite discontinuity at $x = -1$, and -1 is between -2 and 2 .

For the first type of improper integrals:

1) If f is continuous on $[a, \infty)$, then

$$\int_a^{\infty} f(x) dx = \lim_{b \rightarrow \infty} \int_a^b f(x) dx.$$

2) If f is continuous on $(-\infty, b]$, then

$$\int_{-\infty}^b f(x) dx = \lim_{a \rightarrow -\infty} \int_a^b f(x) dx.$$

3) If f is continuous on $(-\infty, \infty)$, then

$$\int_{-\infty}^{\infty} f(x) dx = \int_{-\infty}^c f(x) dx + \int_c^{\infty} f(x) dx.$$

If the limit exists, then the improper integral is said to converge.
Otherwise, it diverges.

Examples for the first type of improper integral.

1. $\int_1^{\infty} \frac{dx}{x}$

2. $\int_2^{\infty} e^{-x} dx$

$$3. \int_0^{\infty} \frac{1}{x^2 + 1} dx$$

$$4. \int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}} dx$$

The second and third type of improper integral:

1. If f is continuous on $[a, b)$ but has an infinite discontinuity at b , then

$$\int_a^b f(x) dx = \lim_{c \rightarrow b^-} \int_a^c f(x) dx.$$

2. If f is continuous on $(a, b]$ but has an infinite discontinuity at a , then

$$\int_a^b f(x) dx = \lim_{c \rightarrow a^+} \int_c^b f(x) dx.$$

3. If f is continuous on $[a, b]$ except for some c in (a, b) at which f has an infinite discontinuity, then

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx,$$

provided **both** integrals on the right converge. If either integral on the right diverges, we say that the integral on the left diverges.

Examples for the second type of improper integral.

1. $\int_0^1 \frac{dx}{\sqrt[3]{x}}$

$$2. \int_0^2 \frac{dx}{x^3}$$

3. $\int_0^{27} \frac{dx}{\sqrt[3]{27-x}}$

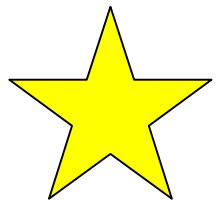
$$4. \int_1^4 \frac{dx}{x-2}$$

Important examples:

$$\int_1^{\infty} \frac{dx}{x^p} \quad p = 1$$

$$\int_1^{\infty} \frac{dx}{x^p} \quad p > 1$$

$$\int_1^{\infty} \frac{dx}{x^p} \quad 0 < p < 1$$



$$\int_1^{\infty} \frac{dx}{x^p}$$

Diverges for $p \leq 1$
Converges for $p > 1$

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Which of the following are improper integrals?