Math 1432

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Office Hours:

Mondays 1-2pm,
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(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

To summarize trig sub:

Given: **Use:**

$$\sqrt{a^2 + x^2}$$

$$\sqrt{a^2 - x^2}$$

$$\sqrt{x^2 - a^2}$$

Examples:

$$\int \sqrt{16 - x^2}\, dx$$

$$\int \frac{1}{x^2\sqrt{49+x^2}}\,dx$$

$$\int_0^{1/2} \frac{x^2}{\left(1-x^2\right)^{3/2}}\,dx$$

$$\int \sqrt{2 - x^2 + 4x}\, dx$$

Popper12

1. $$\int \frac{x^2 + 1}{x^3 + 3x - 4}\,dx$$

2. $$\int \frac{x^3 - 5}{x}\,dx$$

8.4 Rational Functions and Partial Fraction Decomposition

Rational functions are defined as functions in the form $R(x) = \frac{F(x)}{G(x)}$,

where F(x) and G(x) are polynomials.

Rational functions are said to be *proper* if the degree of the numerator is less then the degree of the denominator (otherwise they are improper).

Theorem:

If F(x) and G(x) are polynomials and the degree of F(x) is larger than or equal to the degree of G(x), then there are polynomials *q*(x) (quotient) and r(x) (remainder) such that

$$\frac{F(x)}{G(x)} = \mathrm{q}(x) + \frac{r(x)}{G(x)}$$

where the degree of r(x) is smaller than the degree of G(x).

Example:

Write $\dfrac{x^5+1}{x^3-x^2-2x}$ in terms of its quotient and remainder.

Write $\dfrac{x^2 + x - 1}{x^2 + 1}$ in terms of its quotient and remainder.

Compute: $\displaystyle\int \frac{x^2 + x - 1}{x^2 + 1}\,dx$

Compute: $\int \frac{3x^3 - 2}{x^2 + 4} dx$

Partial Fractions:

Example: $\dfrac{3}{x}+\dfrac{4}{x+1}=\dfrac{3(x+1)+4x}{x(x+1)}=\dfrac{7x+3}{x(x+1)}$

What if we have $\dfrac{7x+3}{x(x+1)}$ and want the original two fractions?

$$\frac{7x+3}{x(x+1)}=\frac{A}{x}+\frac{B}{x+1}=\frac{3}{x}+\frac{4}{x+1}$$

How do we find A and B?

In general, each linear factor of the form $(x - \alpha)^k$ in the denominator gives rise to an expression of the form

$$\frac{A_1}{(x-a)} + \frac{A_2}{(x-a)^2} + \ldots + \frac{A_k}{(x-a)^k}$$

Give the form of the partial fraction decomposition for:

1a. $\dfrac{5x^2 - 6x + 1}{(x+1)(x+2)(x+3)}$

1b. $\dfrac{2x^2 - 3x + 1}{(x+1)(x+2)^2(x+3)}$

Rewrite using partial fractions:

$$\frac{5x-10}{\left(x-4\right)\left(x+1\right)}$$

$$\frac{3x^2+20x+25}{(x-1)(x+2)(x+3)}$$

3. Rewrite $\dfrac{3}{(x-1)(x+2)}$ using partial fractions:

4. Give the form of the partial fraction decomposition for:

$$\frac{3x^2+20x+25}{(x-1)(x+2)(x+3)^2}$$