Math 1432

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Office Hours:

Mondays 1-2pm,
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Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

The sequence $\{a_n\}$ is said to be

- *increasing* if $a_n < a_{n+1}$ for all n,
- *non-decreasing* if $a_n \leq a_{n+1}$ for all n,
- *decreasing* if $a_n > a_{n+1}$ for all n,
- *non-increasing* if $a_n \geq a_{n+1}$ for all n.

A sequence that satisfies any of these conditions is called monotonic.

Increasing Sequence: $a_1 < a_2 < a_3 < \ldots < a_n < a_{n+1} < \ldots$

Non-decreasing Sequence: $a_1 \leq a_2 \leq a_3 \leq \ldots \leq a_n \leq a_{n+1} \leq \ldots$

Decreasing Sequence: $a_1 > a_2 > a_3 > \ldots > a_n > a_{n+1} > \ldots$

Non-increasing Sequence: $a_1 \geq a_2 \geq a_3 \geq \ldots \geq a_n \geq a_{n+1} \geq \ldots$

In an increasing sequence, the GLB is the first term, and if there is a limit, then it is the LUB.

In a decreasing sequence, the LUB is the first term, and if there is a limit, then it is the GLB.

If a sequence has a limit, then the sequence is bounded. BUT, if a sequence is bounded, it does not necessarily have a limit.

The limit of a sequence is a number that the sequence values a_n tend towards as $n \to \infty$.

Find the GLB and LUB if they exist:

1. $S = \{1, 2, 3, 4\}$

2. $[-4, 2]$

3. $(-\infty, 8)$

4. $(5, \infty)$

5. $S = \{x : x \leq 16\}$

6. $\left\{1 - \frac{2}{n}\right\}_{n=1}^{\infty}$

If a sequence has a limit, then it is bounded.

WARNING!!! The converse is not necessarily true!!!

Determine the boundedness and monotonicity.

$$\left\{\frac{1}{n^2}\right\}_{n=1}^{\infty}$$

$$\left\{(-1)^n\right\}$$

$$\left\{2^{n+1}\right\}$$

$$\left\{(-1)^{2n+1}\sqrt{n}\right\}$$

$$\left\{\frac{2n-1}{3n+2}\right\}_{n=1}^{\infty}$$

$$\left\{\frac{1}{2^{n}}\right\}_{n=1}^{\infty}$$

Using Geogebra (https://www.geogebra.org/)

Sequence[<Expression>, <Variable>, <Start Value>, <End Value>]

Sequence[(n, formula), n, 1, , <End Value>]

$\{sin(n)\}_{n=1}^{\infty}$.

Give the limit of the sequence $\left\{ sin(n\pi) \right\}_{n=1}^{\infty}$.

Give the LUB and GLB for $\left\{ n\, sin(n) \right\}_{n=1}^{\infty}$.

Determine whether the sequence $\left\{\frac{2n+(-1)^n}{n}\right\}_{n=1}^{\infty}$ is

a. bounded

b. monotone

c. then give the limit if it exists.

Determine whether the sequence $\left\{ \frac{\sqrt{n+1}}{\sqrt{n}} \right\}_{n=1}^{\infty}$ is

a. bounded

b. monotone

c. then give the limit if it exists.

Give the limit (if it exists) of $\left\{\frac{\sin(n)}{n}\right\}_{n=1}^{\infty}$.

Give the limit (if it exists) of $\left\{\left(\frac{2}{n}\right)^{n}\right\}_{n=1}^{\infty}$

Give the limit (if it exists) of $\left\{\left(\frac{2n^2-3n+6}{3n-16n^2+12}\right)\right\}_{n=1}^{\infty}$.

Give the limit (if it exists) of $\left\{n^{n}\right\}_{n=1}^{\infty}$.

Find a formula for the general term a_n of $\left\{\frac{3}{2}, -\frac{9}{4}, \frac{27}{8}, -\frac{81}{16}, \ldots\right\}$ assuming the pattern of the first few terms continues.

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1. Give the limit (if it exists) of $\left\{ \dfrac{2n^2 + 6n}{\sqrt{2}\, n - n^2} \right\}_{n=1}^{\infty}$

2. Give the limit (if it exists) of $\left\{n^{2n}\right\}_{n=1}^{\infty}$

Consider the sequence defined by $a_n = \left(\frac{2}{3}\right)^n$. ($n$ starts at 1)

a. Write the first five terms of the sequence.

b. Determine the limit of the sequence.

c. Let $b_n = \frac{a_{n+1}}{a_n}$. Write the first five terms of this sequence.

d. Determine the limit of b_n.

Consider the sequence defined by $a_n = \left(\frac{-3}{2} \right)^n$. ($n$ starts at 1)

a. Write the first five terms of the sequence.

b. Determine the limit of the sequence.

c. Let $b_n = \frac{a_{n+1}}{a_n}$. Write the first five terms of this sequence.

d. Determine the limit of b_n.

Are the following increasing, decreasing, or not monotonic?

$$a_n = \frac{3n+4}{2n+5}$$

$$a_n = \frac{3+(-1)^n}{n}$$

$$a_n = \frac{\sqrt{n+1}}{5n+3}$$

Give an upper bound for the set of negative real numbers.

Give a lower bound for the set of negative real numbers.

Give the LUB and GLB for the set of negative real numbers.

Give the LUB and GLB of $\left\{ \dfrac{(-1)^n}{n} \right\}_{n=3}^{\infty}$

Determine whether $\left\{ \ln\left(\dfrac{2n-1}{3n+7} \right) \right\}_{n=1}^{\infty}$ is bounded.

Sequences can be defined *recursively*: one or more terms are given explicitly; the remaining ones are then defined in terms of their predecessors. Give the first six terms
of the sequence and then give the nth term.

$a_1 = 1$; $a_{n+1} = \frac{1}{2} a_n + 1$.

3. Give the limit (if it exists) of $\left\{ \dfrac{(-1)^n}{n} \right\}_{n=1}^{\infty}$.

4. Give the limit (if it exists) of $\left\{ \dfrac{1+\sin(n)}{n} \right\}_{n=1}^{\infty}$

5. Give the LUB for $\{x \mid x^2 - 2x < 3\}$