

Math 1432

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Office Hours:

Mondays 1-2pm,
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Class webpage:

<http://www.math.uh.edu/~bekki/Math1432.html>

With sequences, we are concerned with limits of sequences as n approaches infinity.

A sequence that has a limit is said to be **convergent**.

A sequence that has no limit is said to be **divergent**.

Every convergent sequence is bounded and every unbounded sequence is divergent.

“The sequence converges”

means

“The sequence has a limit”.

“The sequence diverges”

means

“The sequence does not have a limit”.

Important Limits:

A. For each $a > 0$, $\frac{1}{n^a} \rightarrow 0$ as $n \rightarrow \infty$

B. For each real x , $\frac{x^n}{n!} \rightarrow 0$ as $n \rightarrow \infty$

C. If $|x| < 1$, then $x^n \rightarrow 0$ as $n \rightarrow \infty$

D. $\frac{\ln n}{n} \rightarrow 0$ as $n \rightarrow \infty$

E. If $x > 0$, then $x^{\frac{1}{n}} \rightarrow 1$ as $n \rightarrow \infty$

F. $n^{\frac{1}{n}} \rightarrow 1$ as $n \rightarrow \infty$

G. For each real x , $\left(1 + \frac{x}{n}\right)^n \rightarrow e^x$ as $n \rightarrow \infty$

Examples:

Give the limit of $\left\{(-1)^n\right\}_{n=1}^{\infty}$

Give the limit of $\left\{\frac{2n-6}{3n^2+2}\right\}_{n=1}^{\infty}$

Give the limit (if it exists) of $\left\{ \ln(n+1) - \ln(n) \right\}_{n=1}^{\infty}$

Give the limit (if it exists) of $\left\{ \ln(2n+1) - \ln(n) \right\}_{n=1}^{\infty}$

Give the limit (if it exists) of $\left\{ \frac{\ln(n+1)}{n} \right\}_{n=1}^{\infty}$

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1. Give the limit of the sequence: $\left\{ \frac{\ln(n+2)}{\sqrt{2} + n} \right\}_{n=1}^{\infty}$

Give the limit (if it exists) of $\left\{ \frac{3^n}{4^n} \right\}_{n=1}^{\infty}$

Give the limit (if it exists) of $\left\{ n^{\frac{1}{n+2}} \right\}_{n=1}^{\infty}$

Note: $\left\{ \left(e^n \right)^{\frac{1}{n}} \right\}_{n=1}^{\infty} \rightarrow e$

$$\left\{ (stuff)^{\frac{1}{n}} \right\}_{n=1}^{\infty}$$

may not go to 1 as n approaches infinity if “*stuff*” overpowers the exponent.
Be careful!

Give the limit (if it exists) of $\left\{ \left(1 - \frac{1}{n} \right)^n \right\}_{n=1}^{\infty}$

2. Give the limit of $\left\{ \left(1 - \frac{2}{n} \right)^n \right\}_{n=1}^{\infty}$

3. Give the limit of $\left\{ \frac{3n^3 - 2n + 1}{1000n^2 - n^3 + 3} \right\}_{n=1}^{\infty}$

4. Give the limit of $\left\{ \cos(n\pi) \right\}_{n=1}^{\infty}$

5. Give the limit (if it exists) of the sequence $\left\{ 3^{\frac{2}{n}} \right\}_1^{\infty}$