Math 1432

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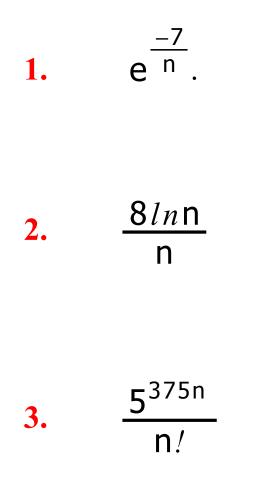
Office Hours:

Mondays 1-2pm, Fridays noon-1pm (also available by appointment)

Class webpage: http://www.math.uh.edu/~bekki/Math1432.html

POPPER 19

State whether the **sequence** converges as $n \rightarrow \infty$.



4.
$$\frac{8^{n+1}}{9^{n-1}}$$

5.
$$\int_0^n e^{-8x} dx$$

6.
$$\int_{-n}^{n} \frac{8}{1+x^2} dx$$

<u>Sequences:</u> Let $\{a_n\}$ be a sequence of real numbers.

Possibilities:

1) If
$$\lim_{n \to \infty} a_n = \infty$$
 then $\{a_n\}$ diverges to infinity.

2) If
$$\lim_{n \to \infty} a_n = -\infty$$
, then $\{a_n\}$ diverges to negative infinity.

- 3) If $\lim_{n \to \infty} a_n = c$, a finite real number, then $\{a_n\}$ converges to c.
- 4) If $\lim_{n \to \infty} a_n$ oscillates between two numbers, then $\{a_n\}$ diverges by oscillation.

If a sequence has a finite limit as n approaches infinity, we say that the sequence **converges**.

If a sequence does not have a finite limit as n approaches infinity, then it **diverges**.

9.3 Infinite Series

Series vs. Sequence

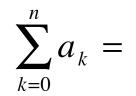
First and most important – a sequence is a LIST and a series is a SUM.

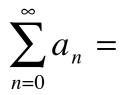
Sequence: $\{a_n\} = a_1, a_2, a_3, \dots, a_n, \dots$

Series:
$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \dots + a_n$$

Notation for series:

 \sum Sigma – means summation





Examples:

 $\sum_{n=1}^{\infty} \frac{1}{n} =$

 $\sum_{n=1}^{\infty} \frac{1}{n^2} =$

Properties:

$$\sum_{k=0}^{n} \alpha a_{k} = \alpha \sum_{k=0}^{n} a_{k}$$

$$\sum_{k=0}^{n} (a_{k} + b_{k}) = \sum_{k=0}^{n} a_{k} + \sum_{k=0}^{n} b_{k}$$

$$\sum_{k=0}^{m} a_{k} + \sum_{k=m+1}^{n} a_{k} = \sum_{k=0}^{n} a_{k}$$

The sum of a finite series is denoted by S_n where $S_n = \sum_{k=0}^n a_k$

For an *infinite* series, we are taking an infinite sequence $(a_0, a_1, a_2,)$ and adding all the terms together. But since there are an infinite amount of terms, doing this (literally) would be impossible so we will begin by looking at the *partial sums*:

$$S_{0} = \sum_{k=0}^{6} a_{k} = a_{0}$$

$$S_{1} = \sum_{k=0}^{1} a_{k} = a_{0} + a_{1}$$

$$S_{2} = \sum_{k=0}^{2} a_{k} = a_{0} + a_{1} + a_{2}$$

$$\vdots$$

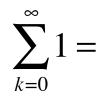
$$S_{n} = \sum_{k=0}^{n} a_{k} = a_{0} + a_{1} + a_{2} + \dots + a_{n}$$

$$\vdots$$

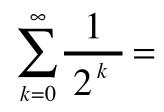
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If we determine that the sequence of these partial sums has a limit, then that limit would be the sum of the infinite series $\sum_{k=0}^{\infty} a_k$. In other words, $\sum_{k=0}^{\infty} a_k = \lim_{n \to \infty} \{S_n\}$

What is the sequence of partial sums for each?



 $\sum_{k=0}^{\infty} r =$



If the sequence of partial sums converges to a finite limit *L*, we write $\sum_{k=0}^{\infty} a_k = L \text{ and say that the series } \sum_{k=0}^{\infty} a_k \text{ converges to } L. \text{ We call } L \text{ the sum of the series. If the sequence of partial sums diverges, we say that the series } \sum_{k=0}^{\infty} a_k \text{ diverges.}$

THEOREM: The *kth term* of a convergent series tends to 0; namely,

if
$$\sum_{k=0}^{\infty} a_k$$
 converges, then $a_k \to 0$ as $k \to \infty$

THEOREM: BASIC DIVERGENCE TEST
If
$$a_k \neq 0$$
 as $k \neq \infty$, then $\sum_{k=0}^{\infty} a_k$ diverges.

****The Basic Divergence Test only proves <u>divergence</u>.****

More General Properties:

• If
$$\sum_{k=0}^{\infty} a_k$$
 converges and $\sum_{k=0}^{\infty} b_k$ converges,
then $\sum_{k=0}^{\infty} (a_k + b_k)$ converges.
• If $\sum_{k=0}^{\infty} a_k$ converges, then $\sum_{k=0}^{\infty} \alpha a_k$ converges.

Ex. 1 Does
$$\sum_{n=1}^{\infty} \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right)$$
 converge or diverge?

Telescoping Series:

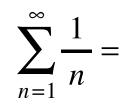
A series such as
$$\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \dots$$
 is called

a telescoping series because it collapses to one term or a few terms. If a series collapses to a finite sum, then it converges.

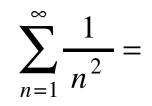
Ex. 2. Does the series 1 - 1 + 1 - 1 + 1 ... converge or diverge? How would we write the series using sigma notation?

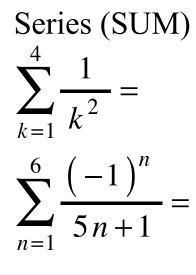
Important examples:

$$\int_{1}^{\infty} \frac{dx}{x}$$



$$\int_{1}^{\infty} \frac{dx}{x^2}$$





 $\sum_{n=0}^{5} \left(\frac{1}{2}\right)^n =$

 $\sum_{n=1}^{\infty} \left(\frac{-1}{2} \right)^n =$

 $\sum_{n=1}^{\infty} (-1)^n =$

POPPER 19

7. Which of the following diverge by the BDT?

