Math 1432

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Office Hours:

Mondays 1-2pm,
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(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

Does the series $\sum_{n=1}^{\infty}\left(\frac{1}{n+2}-\frac{1}{n+4}\right)$ converge or diverge?

★ Does the series $\sum_{n=1}^{\infty} \frac{1}{n}$ converge or diverge?

★ Does the sequence $\left\{ \frac{1}{n} \right\}_{1}^{\infty}$ converge or diverge?

Important reminders:

- A sequence is a LIST { }
  - A sequence converges if it has a limit as $n -> \infty$
- A series is a SUM Σ
  - Converges if the sequence of partial sums $\{S_0, S_1, S_2, \ldots\}$ converges
  - If terms do not approach 0 then it diverges

Geometric Series Test

A geometric series is in the form $\sum_{n=0}^{\infty} a_1 r^n, \; a_1 \neq 0$.

$$\sum_{n=0}^{\infty} a_1 r^n = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \ldots + a_1 r^n + \ldots$$

The geometric series **diverges** if $|r| \geq 1$.

The geometric series **converges** if $|r| < 1$.

If $|r| < 1$, then the series **converges** to the sum $S = \dfrac{a_1}{1-r}$.

The sum of the first n terms of a geometric series is

$$S_n = \frac{a_1\left(1-r^n\right)}{1-r}.$$

Examples: Determine whether the following infinite series converge or diverge. If they converge, what is the sum of the series?

1) $\sum_{n=1}^{\infty} \frac{3}{2^n}$

2) $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$

3) 0.080808….

4) $\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{n-1}$

5) $$1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots+\left(\frac{-1}{2}\right)^{n-1}+\ldots$$

6) $$\frac{\pi}{2}+\frac{\pi^2}{4}+\frac{\pi^3}{8}+\ldots$$

We can typically only determine whether an infinite series (sum) converges or diverges. Quite often, we cannot find the actual sum.

Basic Divergence Test

If $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.

Note: This does **NOT** say that if $\lim_{n\to\infty} a_n = 0$, the series converges.

******The Basic Divergence Test only proves divergence.******

If $\lim_{n\to\infty} a_n = 0$, then the test doesn't tell us anything, and we need to use another test.

Examples:

1) $$\sum_{n=1}^{\infty} \frac{2n+3}{3n-5}$$

2) $$\sum_{n=1}^{\infty} \frac{n}{2n+1}$$

3) $$\sum_{n=1}^{\infty} \frac{3^n - 2}{3^n}$$

4) $$\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$$

Note: (2n)! is not the same as 2n!.

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1. $$\sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^{n}$$

2. $$\sum_{n=1}^{\infty}\frac{1}{n}$$

3. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{6}{7}\right)^{n}$

4. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}$

5. $$\sum_{n=1}^{\infty}\left(\frac{8.0001}{8}\right)^{n}$$

6. Find $$\lim_{x\to\infty}\frac{x^{25}}{3^{x}}$$

7. Give the value of $\sum_{n=1}^{\infty} \frac{1}{3^n}$

Some more examples:

Find the sequence of partial sums for $\sum_{n=1}^{\infty} \frac{2n}{n^2+1}$

Looking at the sequence of partial sums for $\sum_{n=1}^{\infty} \frac{2n}{n^2+1}$, we can say that this series

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8. Determine whether the following *sequence* converges or diverges. If it converges, find its limit.

$$a_n = \left(1 + \frac{2}{5n}\right)^n$$

9. Determine whether the following *series* converges or diverges.

$$\sum_{n=1}^{\infty} \left(1 + \frac{2}{5n}\right)^n$$