Math 1432

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Office Hours:

Mondays 1-2pm,
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Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

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1. $\left\{ \dfrac{2n^2}{n^2+6n} \right\}_{n=1}^{\infty}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2}{n^2+6n}$

3. $\left\{ \left(1-\dfrac{1}{n}\right)^n \right\}_{n=1}^{\infty}$

4. $\displaystyle\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^n$

Section 9.4

The Integral Test; Comparison Tests

Integral Test ("hardest" test – be careful!):

If f is **positive**, **continuous**, and (*ultimately)* **decreasing**

for $x \geq 1$ and $a_n = f(n)$, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)\,dx$

either **both** converge or both diverge.

Note: When we use the Integral Test it is not necessary to start the series or the integral at n = 1.

Also, it is not necessary that f be always decreasing. What is important is that f be *ultimately* decreasing.

That is, decreasing for x larger than some number N, since a finite number of terms doesn't affect the convergence or divergence of a series.

Examples: Determine whether the following series converge or diverge. Show that the series meets the requirements of the integral test before you use it.

1) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

2) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

3) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

4) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

5) Use the integral test to determine the values of p for which

$$\sum_{n=1}^{\infty} \frac{1}{n^{p}}$$ converges

p-Series Test:

A series of the form

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots + \frac{1}{n^p} + \ldots$$

is called a **p-series**, where p is a positive constant.

For p = 1, the series $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \ldots$ is called the **harmonic series**.

The **harmonic series diverges**.

The p-series **diverges** if $0 < p \leq 1$.

The p-series **converges** if $p > 1$.

Examples: Determine whether the following series converge or diverge.

1) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$

2) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt[4]{n}}$

3) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^{3}\sqrt{n}}$

Basic Comparison Test:

If $a_n \geq 0$ and $b_n \geq 0$ and

1) If $\sum_{n=1}^{\infty} b_n$ converges and $0 \leq a_n \leq b_n$, then $\sum_{n=1}^{\infty} a_n$ converges.

2) If $\sum_{n=1}^{\infty} a_n$ diverges and $0 \leq a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

So….

Let $a_n \geq 0$ and $b_n \geq 0$,

If A diverges and B < A, what happens?

If A converges and B > A, what happens?

If A converges and B < A, what happens?

If A diverges and B > A, what happens?

Examples: Determine whether the following series converge or diverge.

1) $\sum_{n=1}^{\infty} \frac{1}{n^3 + 1}$

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2) $\sum_{n=1}^{\infty} \frac{1}{3^n + 2}$

3) $$\sum_{n=10}^{\infty} \frac{1}{\sqrt{n} - 3}$$

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5. Use the fact that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ to find $\sum_{n=3}^{\infty} \frac{1}{n^2}$

6. $\sum_{n=1}^{\infty} \frac{1}{n^7}$

7. $\sum_{n=1}^{\infty} \frac{1}{n+3}$