Math 1432

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Office Hours:

Mondays 1-2pm,
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Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

Limit Comparison Test:

Let $\sum a_n$ and $\sum b_n$ be series with positive terms $(a_n > 0, b_n > 0)$ and

$\lim_{n \to \infty} \frac{a_n}{b_n} = L$, where L is both **finite and positive**.

Then the two series $\sum a_n$ and $\sum b_n$ either **both** converge or **both** diverge.

The Limit Comparison Test works well for comparing "messy" algebraic series to a p-series. Choose a p-series with an nth term of the same magnitude as the nth term of the given series.

Examples: Determine whether the following series converge or diverge.

1) $$\sum_{n=1}^{\infty} \frac{1}{3n^2 - 4n + 5}$$

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2) $$\sum_{n=1}^{\infty} \frac{n^2 + 10}{4n^3 - n^2 + 7}$$

3) $$\sum_{n=2}^{\infty} \frac{1}{n^3 - 2}$$

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4) $$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^3} + 1}$$

5) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{3n-2}}$

Popper 22

1. $\left\{ \frac{1}{\sqrt{n}} \right\}_{n=1}^{\infty}$

2. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

3. $\sum_{n=1}^{\infty} \frac{1}{n^7}$

4. $\sum_{n=1}^{\infty} \frac{1}{n+3}$

Section 9.5

The Root Test; The Ratio Test

Root Test Let $\sum_{n=b}^{\infty} a_n$ be a series with nonzero terms.

1. $\sum_{n=b}^{\infty} a_n$ converges if $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$.

2. $\sum_{n=b}^{\infty} a_n$ diverges if $\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$.

3. The Root Test in inconclusive if $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$.
   (Use another test.)

The Root Test works well for series involving an *n*th power.

Examples: Determine whether the following series converge or diverge.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{2n}}{n^{n}}$

2. $\sum_{n=1}^{\infty}\left(\frac{3n+4}{2n}\right)^{n}$

$$\lim_{k\to\infty} k^{\frac{1}{k}} =$$

3. $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$

Ratio Test Let $\sum_{n=b}^{\infty} a_n$ be a series with nonzero terms.

1. $\sum_{n=b}^{\infty} a_n$ converges if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$.

2. $\sum_{n=b}^{\infty} a_n$ diverges if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| > 1$.

3. The Ratio Test is inconclusive if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$

   (Use another test.)

Series involving factorials and exponential functions work especially well in the Ratio Test.

Examples: Determine whether the following series converge or diverge.

1. $\sum_{n=0}^{\infty} \frac{2^n}{n!}$

2. $$\sum_{n=0}^{\infty} \frac{n^2\, 2^{n+1}}{3^n}$$

3. $$\sum_{n=0}^{\infty} \frac{(n+1)!}{3^n}$$

So here is what we know so far:

$\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n \neq 0$

$\sum_{n=1}^{\infty} \frac{1}{n}$ Harmonic Series – diverges

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ P-Series – converges if p>1, diverges otherwise

$\sum_{n=0}^{\infty} (r)^n$ Geometric – converges if $|r| < 1$ to $\frac{a_1}{1-r}$ and diverges if $|r| \geq 1$

Basic Comparison Test: $\sum_{n=1}^{\infty} a_n, a_n > 0$

1. If $a_n \geq b_n$ and $\sum_{n=1}^{\infty} b_n, b_n > 0$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges
2. If $a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n, b_n > 0$ converges, then $\sum_{n=1}^{\infty} a_n$ converges

Limit Comparison Test: $\sum_{n=1}^{\infty} a_n, a_n \geq 0$

If you know $\sum_{n=1}^{\infty} b_n, b_n \geq 0$

1. If $\sum_{n=1}^{\infty} b_n$ converges and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ (L is any finite number),then $\sum_{n=1}^{\infty} a_n$ converges

2. If $\sum_{n=1}^{\infty} b_n$ diverges and $\lim_{n\to\infty} \frac{a_n}{b_n} > 0$, then $\sum_{n=1}^{\infty} a_n$ diverges

The Integral Test:
If f is positive, continuous and decreasing for $x \geq 1$ and $a_n = f(n)$, then $\sum_{n=1}^{\infty} a_n$ *and* $\int_1^{\infty} f(x)\,dx$ either both converge or both diverge.

The Root Test:

Let $\sum a_k$ be a series with nonnegative terms. Suppose $(a_k)^{1/k} \to \rho$, then

1. $\sum a_k$ converges if $\rho < 1$
2. $\sum a_k$ diverges if $\rho > 1$
3. The test is inconclusive if $\rho = 1$

The Ratio Test:

Let $\sum a_k$ be a series with positive terms. Suppose $\frac{a_{k+1}}{a_k} \to \lambda$, then

1. $\sum a_k$ converges if $\lambda < 1$
2. $\sum a_k$ diverges if $\lambda > 1$
3. The test is inconclusive if $\lambda = 1$

5. $\sum_{n=1}^{\infty} 5\cos(n\pi)$

6. $\sum_{n=1}^{\infty} 3n^{-2/3}$

7. $\sum_{n=1}^{\infty} \frac{n+1}{n^3}$

8. $\sum_{n=1}^{\infty} \left(\frac{-1}{5}\right)^n$