Math 1432

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Office Hours:

Mondays 1-2pm,
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(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

Review of test for series convergence:

$\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n \neq 0$

$\sum_{n=1}^{\infty} \frac{1}{n}$ Harmonic Series – diverges

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ P-Series – converges if p>1, diverges otherwise

$\sum_{n=0}^{\infty} (r)^n$ Geometric – converges if $|r| < 1$ to $\frac{a_1}{1-r}$ and diverges if $|r| \geq 1$

Basic Comparison Test: $\sum_{n=1}^{\infty} a_n, a_n > 0$

1. If $a_n \geq b_n$ and $\sum_{n=1}^{\infty} b_n, b_n > 0$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges
2. If $a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n, b_n > 0$ converges, then $\sum_{n=1}^{\infty} a_n$ converges

Limit Comparison Test: $\sum_{n=1}^{\infty} a_n, a_n \geq 0$

If you know $\sum_{n=1}^{\infty} b_n, b_n \geq 0$

1. If $\sum_{n=1}^{\infty} b_n$ converges and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ (L is any finite number),then $\sum_{n=1}^{\infty} a_n$ converges

2. If $\sum_{n=1}^{\infty} b_n$ diverges and $\lim_{n\to\infty} \frac{a_n}{b_n} > 0$, then $\sum_{n=1}^{\infty} a_n$ diverges

The Integral Test:
If f is positive, continuous and decreasing for $x \geq 1$ and $a_n = f(n)$, then $\sum_{n=1}^{\infty} a_n$ *and* $\int_1^{\infty} f(x)\,dx$ either both converge or both diverge.

The Root Test:

Let $\sum a_k$ be a series with nonnegative terms. Suppose $(a_k)^{1/k} \to \rho$, then

1. $\sum a_k$ converges if $\rho < 1$
2. $\sum a_k$ diverges if $\rho > 1$
3. The test is inconclusive if $\rho = 1$

The Ratio Test:

Let $\sum a_k$ be a series with positive terms. Suppose $\frac{a_{k+1}}{a_k} \to \lambda$, then

1. $\sum a_k$ converges if $\lambda < 1$
2. $\sum a_k$ diverges if $\lambda > 1$
3. The test is inconclusive if $\lambda = 1$

More Examples:

1. $\sum \frac{n}{3n+1}$

2. $\sum 2\left(\frac{4}{5}\right)^n$

3. $\sum \frac{\sqrt{n}}{n}$

4. $\sum \frac{1}{n^{1.1}}$

5. $\sum \frac{5n}{3n^2 - 6n + 2}$

6. $\sum \frac{k \cdot 2^k}{3^k}$

7. $\sum \frac{3^n}{(n+1)!}$

8. $\sum \frac{(n+1)!}{(n+4)!}$

Popper 23

1. Use the Root test to determine if the following are convergent or divergent (or if test is inconclusive).

$$\sum \frac{k^6}{e^{3k}}$$

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2. $\displaystyle\sum_{n=0}^{\infty}\left(\frac{1}{3^n}-\frac{5}{6^n}\right)$

3. $\displaystyle\sum_{n=1}^{\infty}\frac{n-1}{n!}$

4. $\displaystyle\sum_{n=1}^{\infty}\frac{n+3}{n}$

5. $$\sum_{n=1}^{\infty}\left(\frac{3}{2}\right)^{n}$$

6. $$\sum_{n=1}^{\infty}\frac{1}{n^{2}+4n+3}$$

7. $$\sum_{n=1}^{\infty}\frac{1}{n^{5}}$$

8. $$\sum_{n=1}^{\infty}\frac{1}{n}$$

9. $$\sum_{k=1}^{\infty}\frac{1}{5^{k-1}}$$

10. $\displaystyle\sum_{n=2}^{\infty} \frac{3n^2 - 1}{10n + 5n^2}$

11. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$

12. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$

13. Choose the series that converges:

EMAIL ME ANY TEST REVIEW QUESTIONS BY 5PM THURSDAY!